Name: Solution Key

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

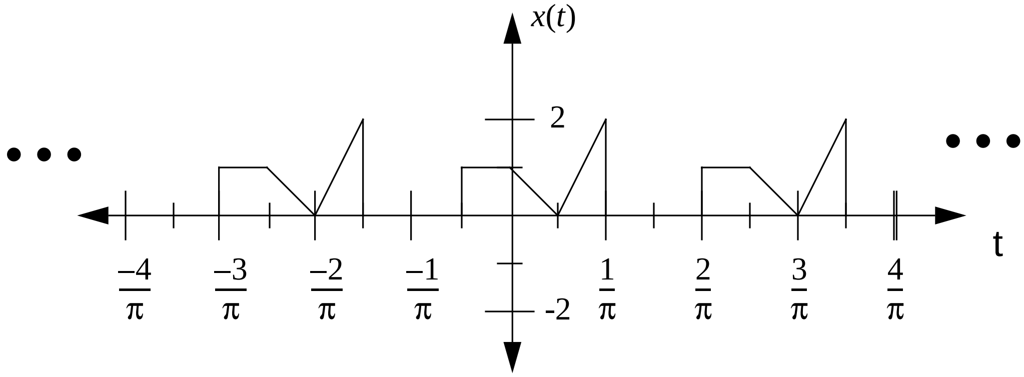
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

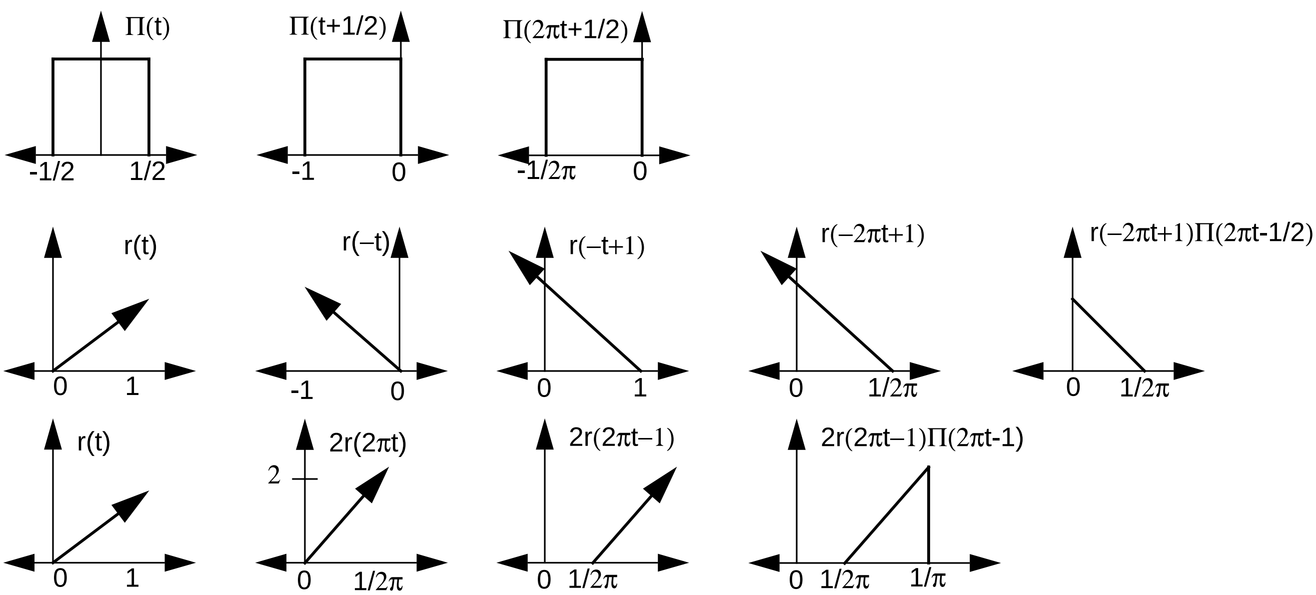
3. Please indicate clearly your answer to the problem.

Problem No. 1: Signal Models



(a) Express the waveform shown above in terms of $u(t), r(t), \Pi(t)$:

One possible solution is:



$$\therefore\ g(t) = \Pi(2\pi t + \tfrac{1}{2}) + r(-2\pi t + 1)\Pi(2\pi t - \tfrac{1}{2}) + 2r(2\pi t - 1)\Pi(2\pi t - 1)$$

To make the function periodic,

$$x(t) = \sum_{n=-\infty}^{\infty} g\left(t - \frac{5n}{2\pi}\right)$$

(b) Compute the power in $x(t)$.

$$Power = \frac{1}{T_0}\int |x(t)|^2 dt$$

Note that we can break the function into three distinct sections, and shift each to start at $t = 0$ so that the integral is simpler:

$-\frac{1}{2\pi} < t < 0$: $$P_1 = \int_0^{\frac{1}{2\pi}} (1)^2 dt = \frac{1}{2\pi}$$

Note that we can time-reverse the waveform:

$0 < t < \frac{1}{2\pi}$: $$P_2 = \int_0^{\frac{1}{2\pi}} (2\pi t)^2 dt = (4\pi^2)\left[\frac{t^3}{3}\right]\Bigg|_0^{\frac{1}{2\pi}} = 4\pi^2\left(\frac{1}{24\pi^3}\right) = \frac{1}{6\pi}$$

$\frac{1}{2\pi} < t < \frac{1}{\pi}$: $$P_3 = \int_0^{\frac{1}{2\pi}} (4\pi t)^2 dt = 4\int_0^{\frac{1}{2\pi}} (2\pi t)^2 dt = \frac{2}{3\pi}$$

$\therefore$ Total Power = $$Total\ Power = \frac{1}{\left(\frac{5}{2\pi}\right)}\left[\frac{1}{2\pi} + \frac{1}{6\pi} + \frac{2}{3\pi}\right] = \frac{4}{15} = 0.27 \quad Watts$$

(c) The signal $y(t)$ is given as: $y(t) = x(t) + \sin 2\pi t$.

Note: $f_0^x = \frac{2\pi}{5}$ for $x(t)$ and $f_0^s = 1$ for $\sin 2\pi t$, which implies $\frac{f_0^x}{f_0^s} = \frac{2\pi}{5} \neq \frac{n_1}{n_2}$

Is $y(t)$ (circle all that apply):

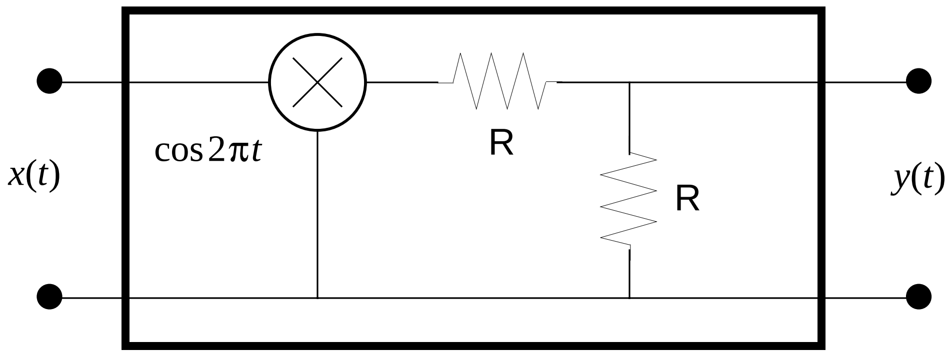
(2 pts) Continuous-time Discrete-time

(2 pts) Continuous amplitude Quantized in amplitude

(6 pts) Periodic Aperiodic

Problem No. 2: Linear Systems

(a) Is the system shown below:



Linear? Explain.

$$y(t) = \frac{R}{(R+R)}(x(t)\cos 2\pi t)$$

Hence,

$x_1(t)$: $\quad y_1(t) = \frac{1}{2}x_1(t)\cos 2\pi t$

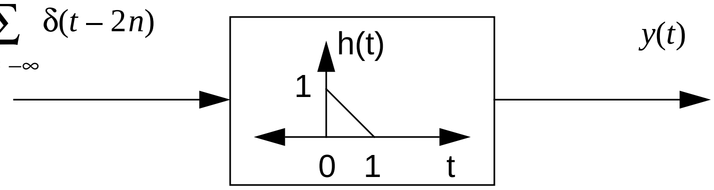
$x_2(t)$: $\quad y_2(t) = \frac{1}{2}x_2(t)\cos 2\pi t$

$a_1x_1(t) + a_2x_2(t)$: $\quad y_3(t) = \frac{1}{2}(a_1x_1(t) + a_2x_2(t))\cos 2\pi t = a_1y_1(t) + a_2y_2(t)$

Therefore, the system is linear!

(b) Find $y(t)$:

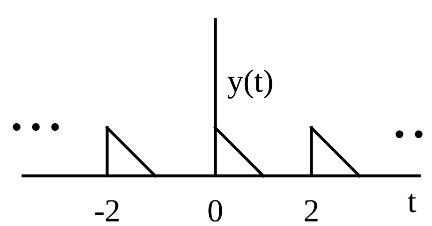
$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t-2n)$$



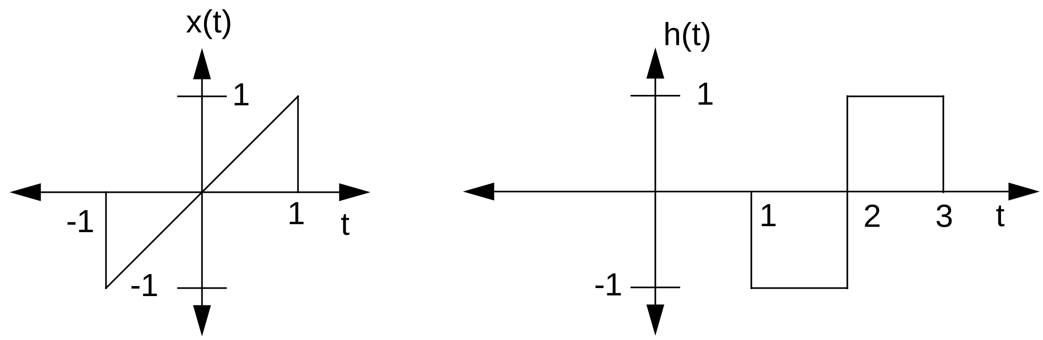
The input signal is a periodic pulse train with a period of two seconds. The output to each pulse will be the system impulse response. Hence, the output is:

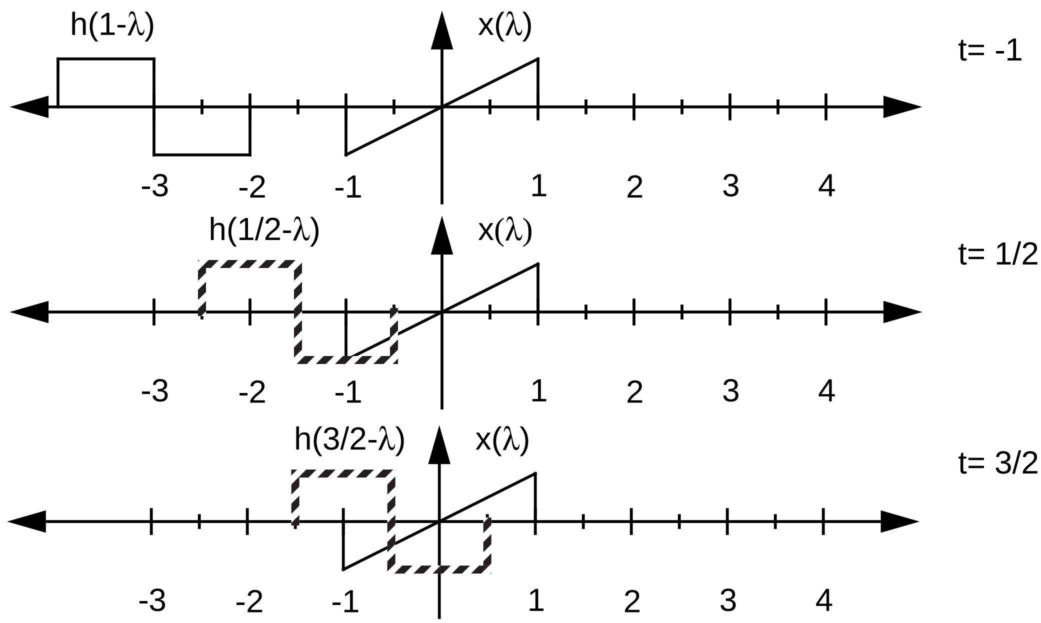
$$y(t) = \int_{-\infty}^{\infty} x(t)h(t-\lambda)d\lambda = \int_{-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty}\delta(t-2n)\right)h(t-\lambda)d\lambda$$

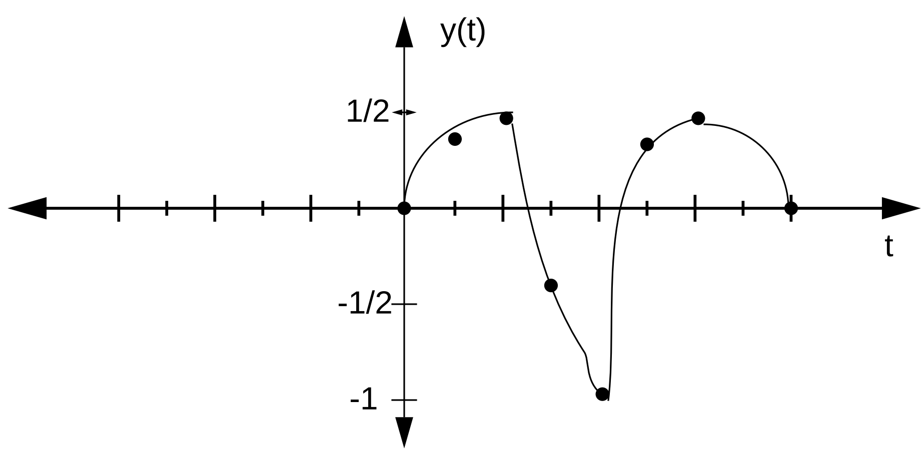
$$= \sum_{n=-\infty}^{\infty} h(t-2n)$$



(c) Sketch the output of the system show below:

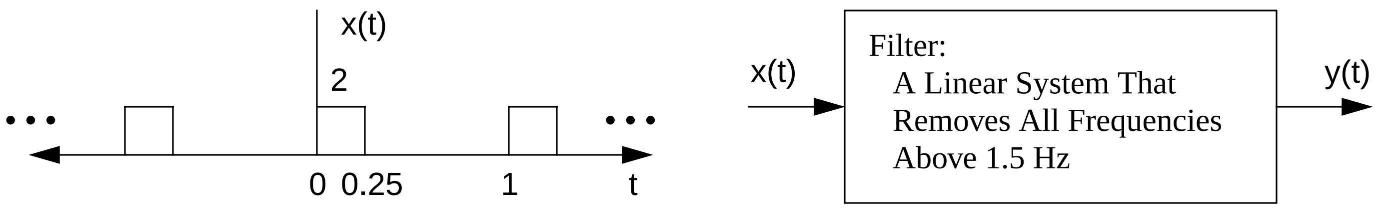






Problem No. 3: Fourier Series

For the signal and system shown below:



(a) Compute the DC value of the output:

$$DC\ Value = a_0 = \frac{1}{T}\int_0^{1/4}(1)dt = \left(\frac{1}{4}\right)(2) = \frac{1}{2}$$

(b) Compute the output y(t):

Use a Fourier Series. Note that the fundamental frequency is 1 Hz
⇨ only need to compute the first harmonic:

$$a_1 = \frac{1}{1}\int_0^{0.25}(2)\cos 2\pi t = (2)\left(\frac{1}{2\pi}\right)\sin 2\pi t\Big|_0^{0.25} = \frac{1}{\pi}\sin\frac{\pi}{2} = \frac{1}{\pi}$$

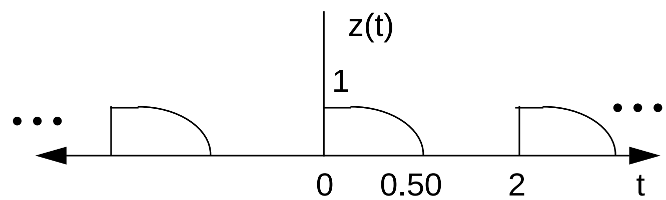
$$b_1 = \frac{1}{1}\int_0^{0.25}(2)\sin 2\pi t = (2)\left(\frac{-1}{2\pi}\right)\cos 2\pi t\Big|_0^{0.25} = \frac{1}{\pi}$$

∴ $$y(t) = \frac{1}{2} + \frac{1}{\pi}\cos 2\pi t + \frac{1}{\pi}\sin 2\pi t$$

(c) Compute the energy of y(t):

The output is a periodic signal, which is a power signal. Hence,

$$E = \infty$$



(d) Discuss the differences in the spectra of the signal shown below and x(t).

- The period of z(t) is twice as long as x(t), therefore the harmonics of z(t) will occur at multiples of 1/2 Hz, rather than 1 Hz. This means in a given interval, there will be twice as many harmonics of z(t) as x(t).

- The "duty cycle" of z(t), given by $\left(\frac{\tau}{T} = \frac{0.5}{2} = \frac{1}{4}\right)$, is the same as the duty cycle for x(t). Therefore, the rate at which the spectrum attenuates as a function of frequency (not the harmonic number) will be the same. Hence, the effective bandwidth will be comparable (even though one signal has more harmonics than the other).

- The rounded edge of z(t) will reduce the high frequency content compared to x(t).

Hence, the overall effect will be to give z(t) a spectrum in which energy is concentrated at lower frequencies when compared to x(t).