Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

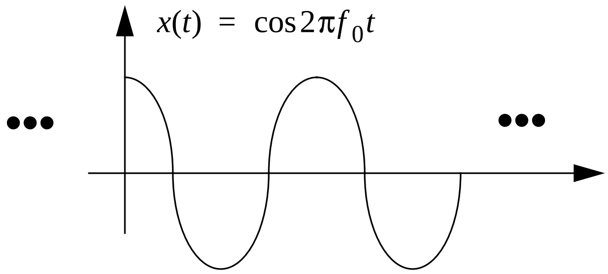
Notes:

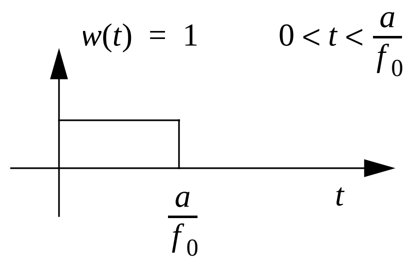
1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

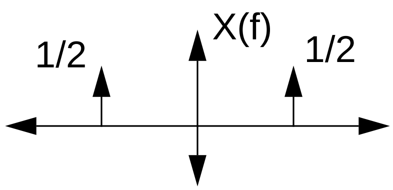
3. Please indicate clearly your answer to the problem.

Problem No. 1:





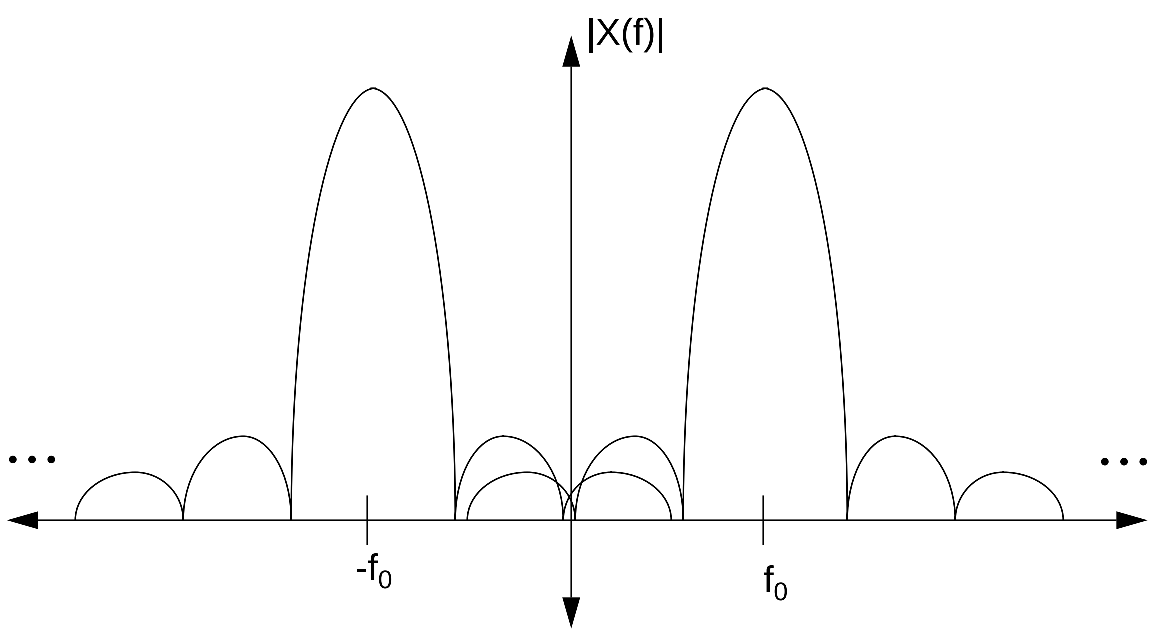
(a) Sketch the magnitude spectrum of $z(t) = x(t)w(t)$ for $a = 1.5$:



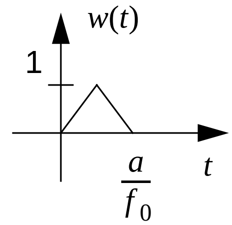
$$W(f) = \left[\frac{a}{f_0}\operatorname{sinc}\left(\frac{a}{f_0}f\right)\right]e^{-j2\pi f\left(\frac{a}{2f_0}\right)}$$

$z(t) = x(t)w(t)$ is equivalent to convolution in the frequency domain: $Z(f) = X(f) \otimes W(f)$.

Hence, the magnitude spectrum is the sum of two OVERLAPPING sinc functions:



(b) Compare the result in part (a) to the result that would be obtained if $w(t)$ is changed from a rectangular function to a triangle-shaped function:



The triangle function has a spectrum that is proportional to $\mathrm{sinc}^2(f)$, which has a smaller effective bandwidth (narrower spectrum) than the rectangular function. Hence, the spectrum will be two overlapping sinc functions, and the amount of overlap will be less.

(c) Name your favorite Fourier transform theorem:

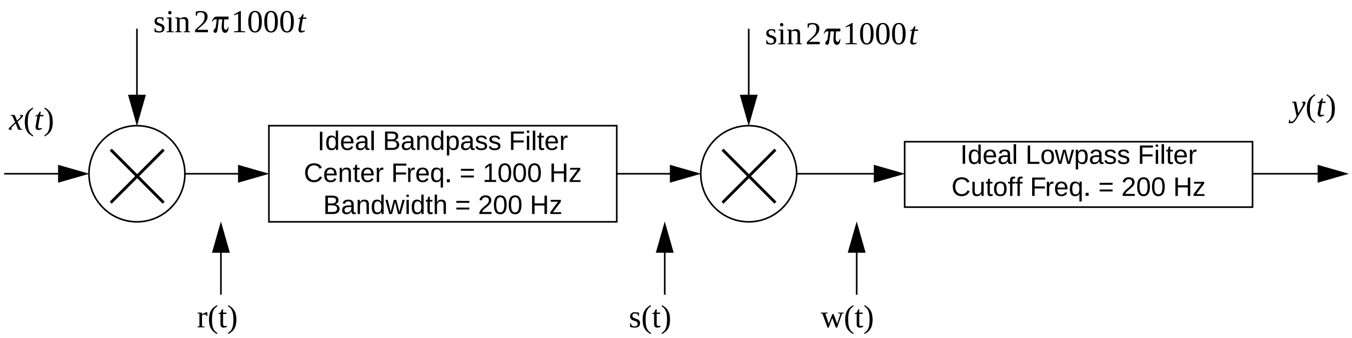
Linearity: $y(t) = ax(t) \Leftrightarrow Y(f) = aX(f)$.

(d) Prove the theorem described in (c):

$$Y(f) = \int_{-\infty}^{\infty} ax(t)e^{-j2\pi ft}dt = a\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt = aX(f)$$

Problem No. 2:

(a) $x(t)$ is a data communications signal that has frequency content ranging from -75 Hz to 75 Hz. Compute $y(t)$ for the system shown below:



(1) Applying the modulation theorem:

$$R(f) = \frac{1}{2j}X(f-1000) - \frac{1}{2j}X(f+1000$$

(2) Apply the ideal bandpass filter (think of this graphically):

$$S(f) = R(f)$$

(3) Apply the modulation theorem again:

$$W(f) = \frac{1}{2j}R(f-1000) - \frac{1}{2j}R(f+1000)$$

$$= \frac{1}{2j}\left[\frac{1}{2j}X(f-2000) - \frac{1}{2j}X(f)\right] - \frac{1}{2j}\left[\frac{1}{2j}X(f) - \frac{1}{2j}X(f+2000$$

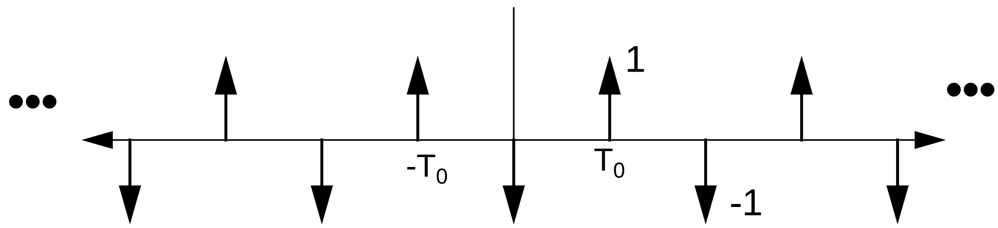
$$= \frac{1}{2}X(f) - \frac{1}{4}X(f-2000) - \frac{1}{4}X(f+2000)$$

(4) Apply the ideal lowpass filter (think of this graphically):

$$Y(f) = \frac{1}{2}X(f)$$

$$y(t) = \frac{1}{2}x(t)$$

(b) Describe the shape of the magnitude spectrum of the impulse train shown below:



Think of this as two pulse trains:

$x(t) = g(t) - g(t - T_0)$, and $g(t) = (-1)\left(\sum_{m=-\infty}^{\infty} \delta(t - m2T_0)\right)$, and $G(f) = \frac{-1}{T_0}\sum_{m=-\infty}^{\infty} \delta(f - \frac{m}{2T_0})$.

Using the time delay theorem:

$X(f) = G(f)[1 - e^{j2\pi f T_0}]$. But, $G(f) \neq 0$ when $f = \frac{m}{2T_0}$, which implies:

$$X(\frac{m}{2T_0}) = \begin{cases} 2G(\frac{m}{2T_0}) & m = odd \\ 0 & m = even \end{cases}.$$

Hence, the spectrum is an impulse train spaced at frequencies $f = \frac{m}{2T_0}$ $m\ odd$.

This is really the spectrum of cosine of frequency $\frac{1}{2T_0}$ sampled at $t = \frac{1}{T_0}$.

(c) Prove that the principles of linearity and superposition hold for the Fourier Transform:

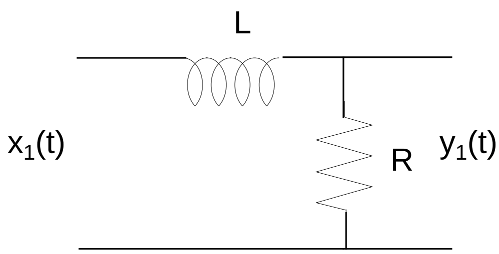
Linearity and Superposition: $y(t) = a_1x_1(t) + a_2x_2(t)$ implies $Y(f) = a_1X_1(f) + a_2X_2(f)$.

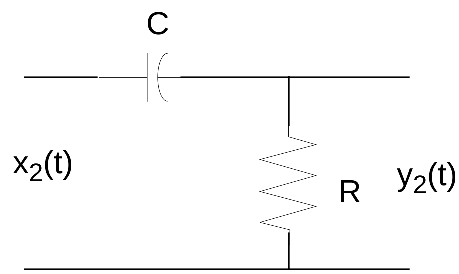
$$Y(f) = \int_{-\infty}^{\infty} (a_1x_1(t) + a_2x_2(t))e^{-j2\pi ft}dt$$

$$= a_1\int_{-\infty}^{\infty} x_1(t)e^{-j2\pi ft}dt + a_2\int_{-\infty}^{\infty} x_2(t)e^{-j2\pi ft}dt$$

$$= a_1X_1(f) + a_2X_2(f)$$

Problem No. 3:





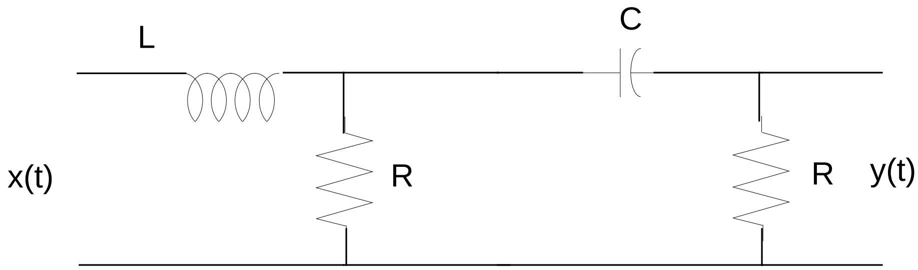
System H_1 **System H_2**

(a) Compute $H_1(s)$, $H_2(s)$, and $H_1(s)H_2(s)$. Assume all initial conditions are zero.

$$H_1(s) = \frac{R}{R + sL} \qquad H_2(s) = \frac{R}{R + \frac{1}{sC}} = \frac{sRC}{1 + sRC}$$

$$H_1(s)H_2(s) = \frac{sR^2C}{(R + sL)(1 + sRC)}$$

(b) Compute the transfer function H(s). Assume all initial conditions are zero.



$$H(s) = \left(\frac{R \,\|\, \left(R + \frac{1}{sC}\right)}{\left(R \,\|\, \left(R + \frac{1}{sC}\right)\right) + sL} \right) \left(\frac{R}{R + \frac{1}{sC}} \right)$$

(c) Explain the similarities and differences between $H_1(s)H_2(s)$ in (a) and the answer to (b). Are these systems linear?

The answers to (a) and (b) are different, because in (b), the RC circuit loads the RL circuit. A simple cascading of systems cannot be applied.

The systems are linear, however. They just don't have infinite input impedances (which would prevent loading).