Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

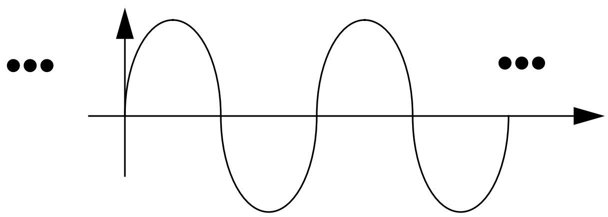
3. Please indicate clearly your answer to the problem.

I hereby promise not to discuss this exam with anyone in the MWF section of EE 3133.

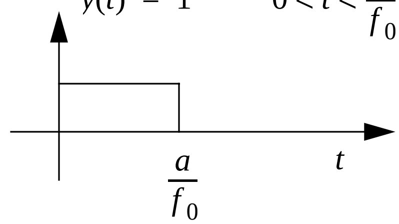
Signature: ____

Problem No. 1:

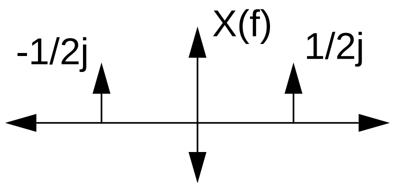
$x(t) = \sin 2\pi f_0 t$



$y(t) = 1 \qquad 0 < t < \frac{a}{f_0}$



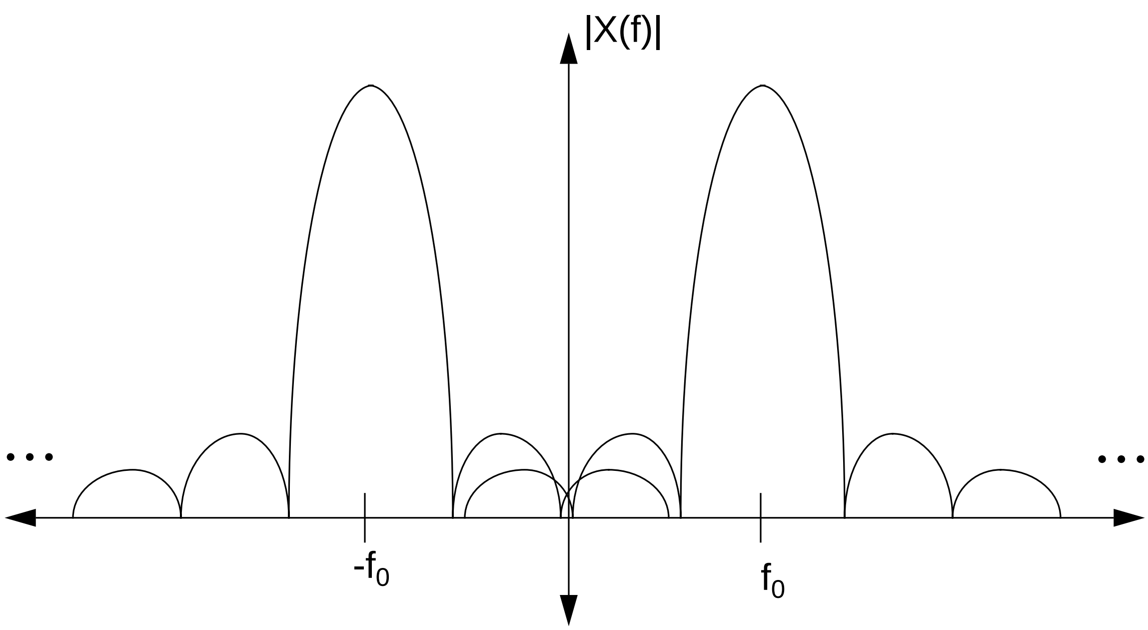
(a) Sketch the magnitude spectrum of $z(t) = x(t)y(t)$ for $a = 1.5$:



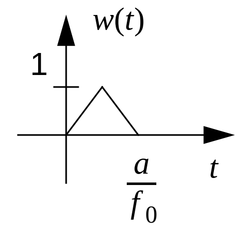
$$W(f) = \left[\frac{a}{f_0}\operatorname{sinc}\left(\frac{a}{f_0}f\right)\right]e^{-j2\pi f\left(\frac{a}{2f_0}\right)}$$

$z(t) = x(t)w(t)$ is equivalent to convolution in the frequency domain: $Z(f) = X(f) \otimes W(f)$.

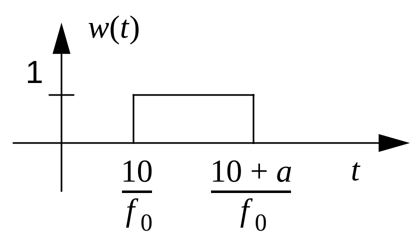
Hence, the magnitude spectrum is the sum of two OVERLAPPING sinc functions:



(b) Compare the result in part (a) to the result that woud be obtained if $w(t)$ is changed from a rectangular function to a triangle-shaped function:



The triangle function has a spectrum that is proportional to $\mathrm{sinc}^2(f)$,which has a smaller effective bandwidth (narrower spectrum) than the rectangular function. Hence, the spectrum will be two overlapping sinc functions, and the amount of overlap will be less.



(c) For $w(t)$ shown to the right, sketch the magnitude spectrum of $z(t) = x(t)w(t)$:

The magnitude spectrum is more or less the same as that for part (a). W(t) in (c) is just a time-delayed version of w(t) in part (a), which means the magnitude spectrum doesn’t change appreciably (there is actually a minor change depending on the frequency of the sinewave and the value of a). To be precise:

$$Z(f) = X(f) \otimes (W_a(f)e^{-j2\pi f t_0}$$

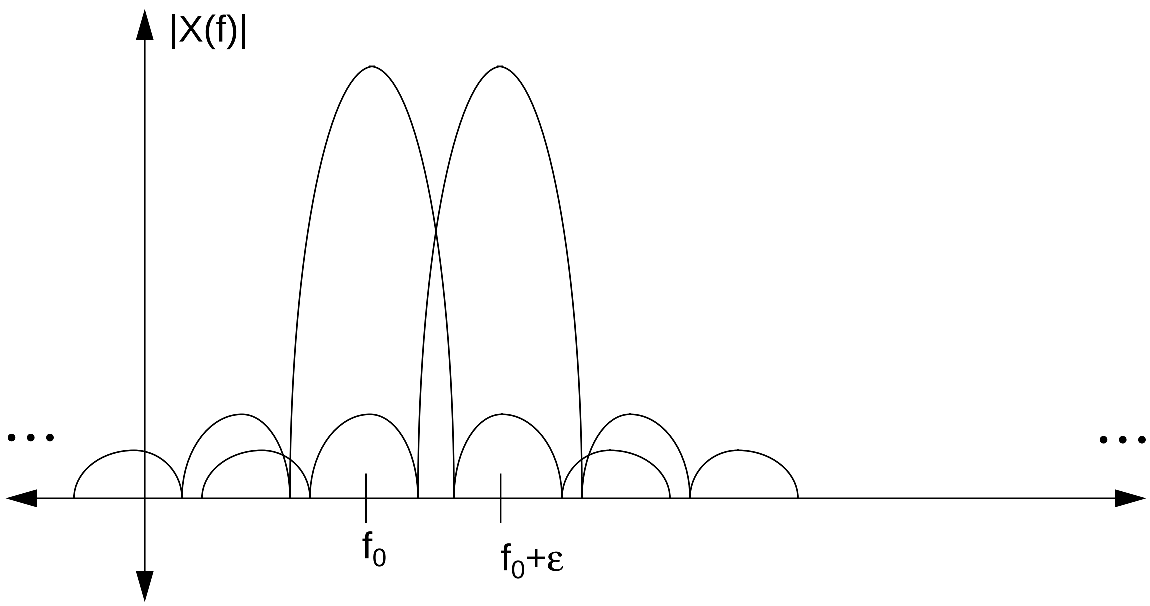
where $t_0 = 10$.

(d) Suppose $x(t)$ is given by:

$$x(t) = \sin 2\pi f_0 t + \sin 2\pi (f_0 + \varepsilon) t$$

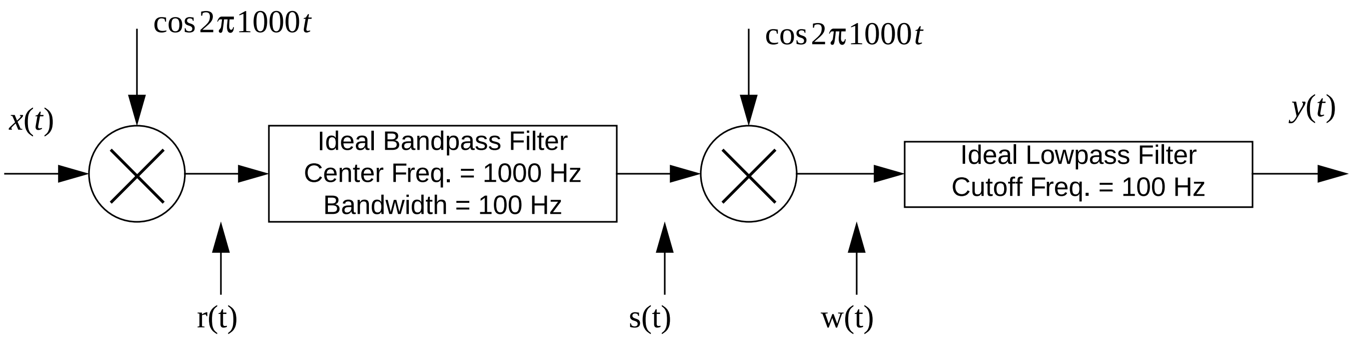
where $\varepsilon \ll f_0$. For $w(t)$ shown in part (a), sketch the magnitude spectrum and discuss the impact $w(t)$ has on the resulting spectrum. Compare this result to the spectrum of $x(t)$.

The composite spectrum is the convolution of the spectrum of the two sinewaves and a sinc function. If the sinewaves are close in frequency, the amount of overlap between the four sinc functions will be large, and the spectrum will be distorted:



Problem No. 2:

(a) $x(t)$ is a data communications signal that has frequency content ranging from -50 Hz to 50 Hz. Compute $y(t)$ for the system shown below:



(1) Applying the modulation theorem:

$$R(f) = \frac{1}{2}X(f - 1000) + \frac{1}{2}X(f + 1000)$$

(2) Apply the ideal bandpass filter (think of this graphically):

$$S(f) = R(f)$$

(3) Apply the modulation theorem again:

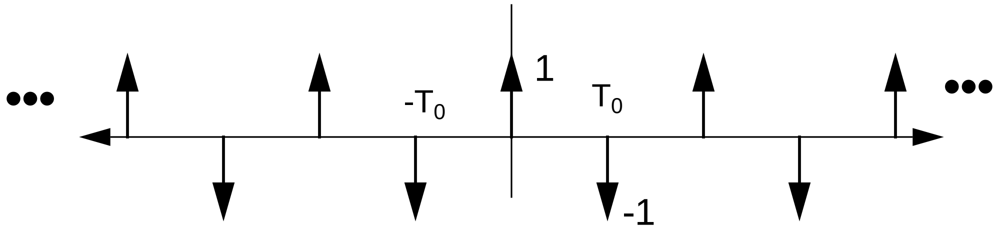
$$\begin{aligned} W(f) &= \frac{1}{2}R(f - 1000) + \frac{1}{2}R(f + 1000) \\ &= \frac{1}{2}\left[\frac{1}{2}X(f - 2000) + \frac{1}{2}X(f)\right] + \frac{1}{2}\left[\frac{1}{2}X(f) + \frac{1}{2}X(f + 2000)\right] \\ &= \frac{1}{2}X(f) + \frac{1}{4}X(f - 2000) + \frac{1}{4}X(f + 2000) \end{aligned}$$

(4) Apply the ideal lowpass filter (think of this graphically):

$$Y(f) = \frac{1}{2}X(f)$$

$$y(t) = \frac{1}{2}x(t)$$

(b) Describe the shape of the magnitude spectrum of the impulse train shown below:



Think of this as two pulse trains:

$x(t) = g(t) - g(t - T_0)$, and $g(t) = \sum_{m=-\infty}^{\infty} \delta(t - m2T_0)$, and $G(f) = \frac{1}{T_0} \sum_{m=-\infty}^{\infty} \delta(f - \frac{m}{2T_0})$.

Using the time delay theorem:

$X(f) = G(f)[1 - e^{j2\pi f T_0}]$. But, $G(f) \neq 0$ when $f = \frac{m}{2T_0}$, which implies:

$$X\left(\frac{m}{2T_0}\right) = \left\{\begin{array}{ll} 2G\left(\frac{m}{2T_0}\right) & m = odd \\ 0 & m = even \end{array}\right\}.$$

Hence, the spectrum is an impulse train spaced at frequencies $f = \frac{m}{2T_0} \quad m\ odd$.

This is really the spectrum of cosine of frequency $\frac{1}{2T_0}$ sampled at $t = \frac{1}{T_0}$.

(c) Prove the time delay theorem for the Fourier Transform:

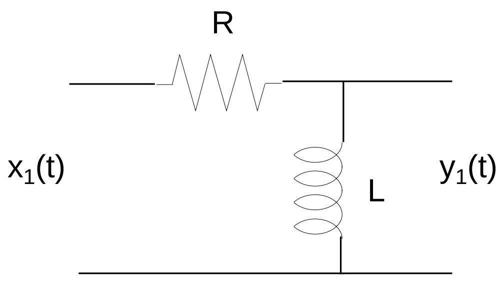
The time delay theorem is: $F\{x(t - t_0)\} = e^{-j2\pi f t_0} F\{x(t)\}$. (See the class notes.)

$$F\{x(t - t_0)\} = \int_{-\infty}^{\infty} x(t - t_0) e^{-j2\pi f t} dt$$

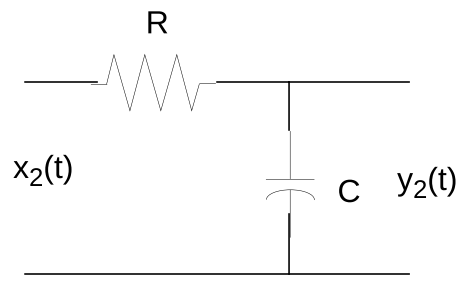
Make a change of variables: $\lambda = t - t_0$ and $d\lambda = dt$.

$$\begin{aligned} F\{x(t - t_0)\} &= \int_{-\infty}^{\infty} x(\lambda) e^{-j2\pi f(\lambda + t_0)} d\lambda \\ &= e^{-j2\pi f t_0} \int_{-\infty}^{\infty} x(\lambda) e^{-j2\pi f \lambda} d\lambda \\ &= e^{-j2\pi f t_0} F\{x(t)\} \end{aligned}$$

Problem No. 3:



System H_1



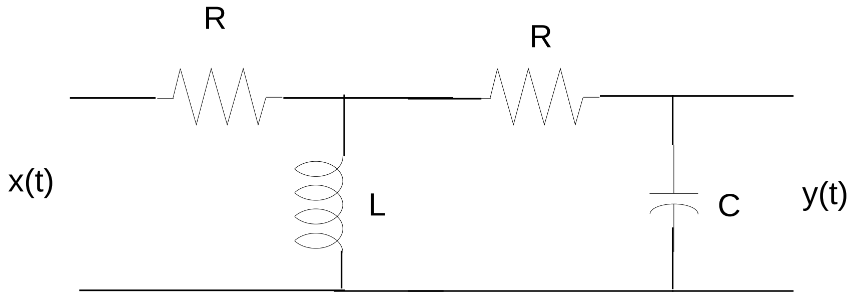
System H_2

(a) Compute $H_1(s)$, $H_2(s)$, and $H_1(s)H_2(s)$. Assume all initial conditions are zero.

$$H_1(s) = \frac{sL}{R + sL} \qquad H_2(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$

$$H_1(s)H_2(s) = \frac{sL}{(R + sL)(1 + sRC)}$$

(b) Compute the transfer function H(s). Assume all initial conditions are zero.



$$H(s) = \left(\frac{sL \parallel \left(R + \frac{1}{sC}\right)}{\left(sL \parallel \left(R + \frac{1}{sC}\right)\right) + R}\right)\left(\frac{\frac{1}{sC}}{R + \frac{1}{sC}}\right)$$

(c) Explain the similarities and differences between the answers to (a) and (b). Are these systems linear?

The answers to (a) and (b) are different, because in (b), the RC circuit loads the RL circuit. A simple cascading of systems cannot be applied.

The systems are linear, however. They just don’t have infinite input impedances (which would prevent loading).