Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1:

(a) For the signal $x(t) = e^{-\alpha t} + e^{-\beta t} + \mathrm{sinc}(100t)$ where $|\alpha| < 1$ and $|\beta| < 1$, compute the minimum sample frequency required for perfect reconstruction of the signal when it is converted from an analog signal to a discrete-time signal, and reconverted back to an analog signal.

The Fourier transform of $x(t)$ is:

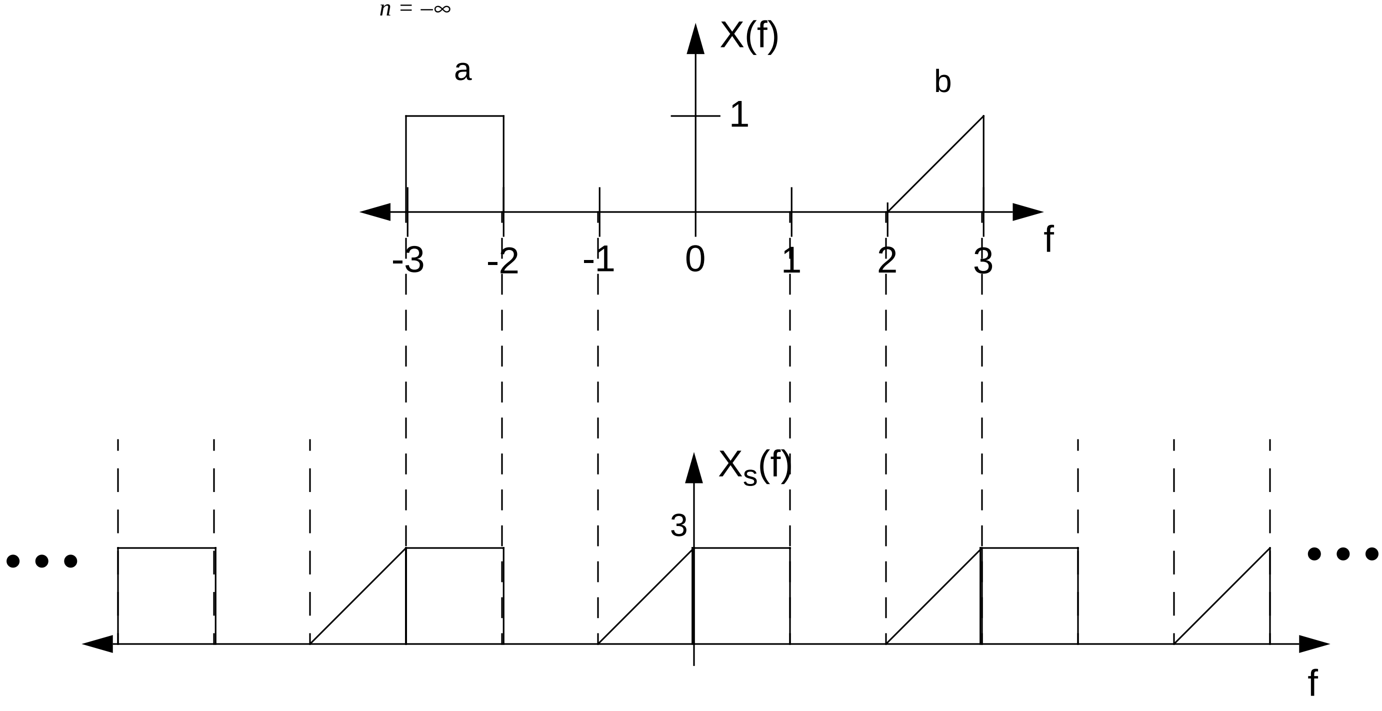
$$X(f) = \frac{1}{j2\pi f + \alpha} + \frac{1}{j2\pi f + \beta} + \frac{1}{100}\Pi\left(\frac{f}{100}\right)$$

This is not a bandlimited signal. It cannot be sampled without aliasing.

The minimum sampling frequency is ________∞____________________ Hz.

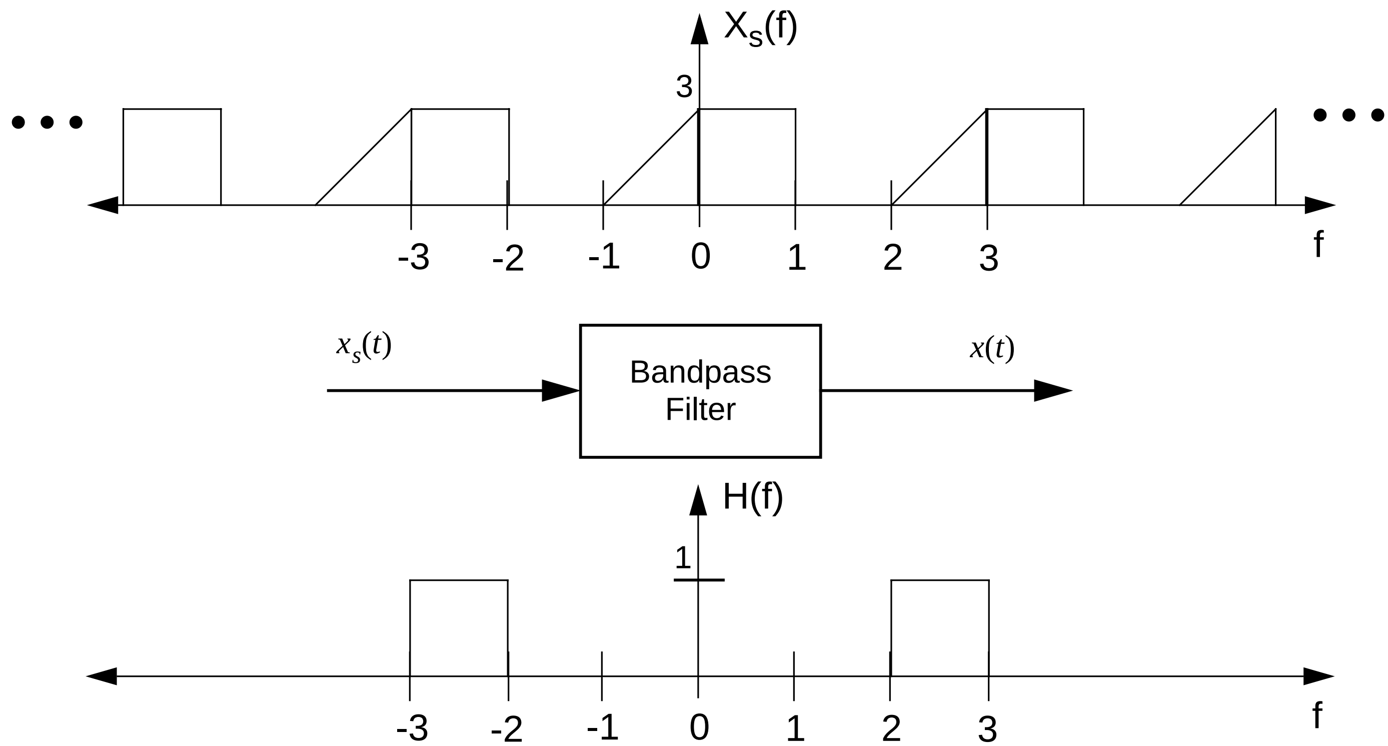
(b) The signal shown below is sampled using a sample frequency of 3 Hz. Plot the spectrum of the sampled signal.

Note: $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$



(c) Can the original signal be reconstructed from the sampled signal with no error? If so, explain how. Explain whether this violates the Sampling Theorem.

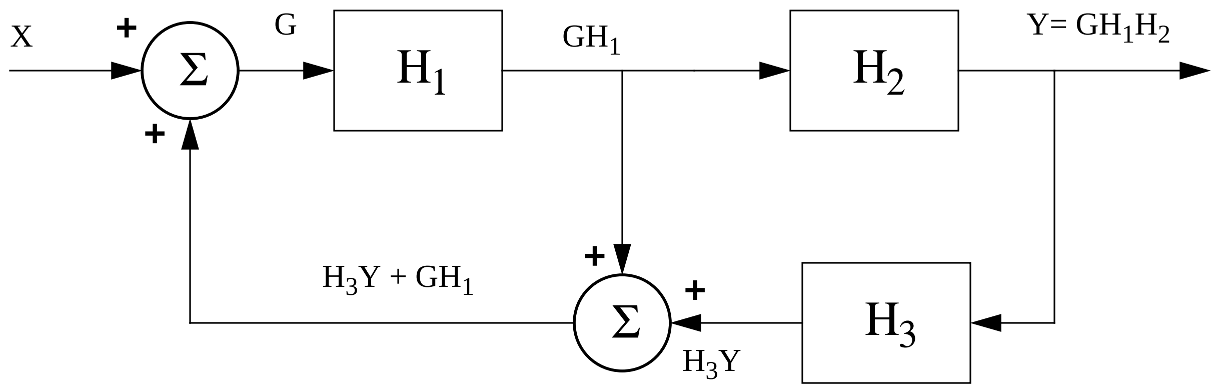
If not, explain why the signal cannot be recovered.



The ORIGINAL signal can be reconstructed by passing the sampled signal through a bandpass filter that removes the replicas of the spectrum created by the sampling process. This is a demonstration of the bandpass sampling theorem — which states that the minimum sampling frequency required to sample a signal with no aliasing is a function of the lowest and highest frequencies for which the signal has spectral energy. In this case, the signal has a bandwidth of 1 Hz, and the lower and upper frequencies are multiples of a common factor, so the minimum sample frequency is 2 Hz.

Problem No. 2:

(a) Compute the equivalent transfer function for the block diagram below:



From the first summation unit $X + H_3Y + GH_1 = G$, and, from the output equation, $G = \frac{Y}{H_1H_2}$

Hence,

$$X + H_3Y + \frac{Y}{H_2} = \frac{Y}{H_1H_2},$$

or, through simplification:

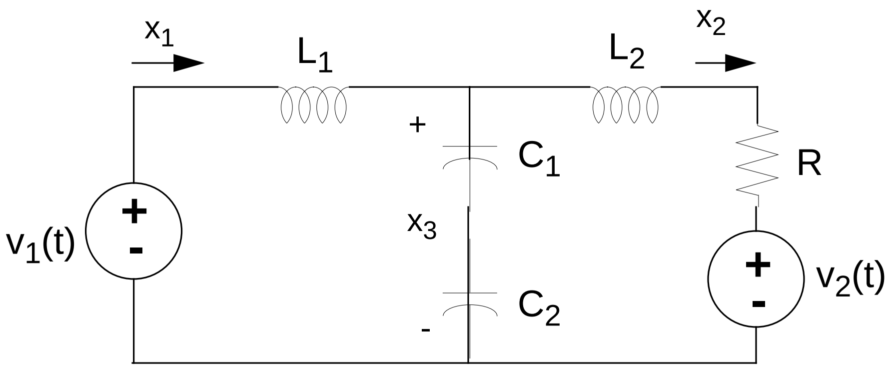
$$Y[1 - H_1 - H_1H_2H_3] = XH_1H_2$$

Therefore,

$$H = \frac{Y}{X} = \frac{H_1H_2}{1 - H_1 - H_1H_2H_3}$$

(b) Explain how you would use the Nyquist criterion to determine the stability of this system.

See the lecture notes. The denominator of the Laplace transform of the transfer function must be evaluated along a contour encircling the right-half plane. The complex values of this contour must be plotted in the $H(\hat{s})$ plane (real/imaginary axes). The number of counter-clockwise encirclements of the origin must be counted. If the number of net counterclockwise encirclements (R) is equal to the number of poles with positive real parts (P, which is usually zero), the system is stable.



(c) What is the minimum number of state variables needed to represent the circuit shown above? Explain.

This circuit can be represented by a third-order differential equation. It is true that there are four memory elements, but the two capacitors can be reduced to a single capacitor. Hence, to account for the two inductors, and one capacitor, we need three state variables.

Minimum number of state variables is 3.

(d) Derive a state variable representation for this circuit. Note that there are two inputs, $v_1(t)$ and $v_2(t)$. Assume there are two desired outputs, the voltage across the resistor, R, and the current through the resistor.

(1) $-v_1 + L_1\dot{x}_1 + x_3 = 0$

(2) $-x_3 + L_2\dot{x}_2 + Rx_2 + v_2 = 0$

(3) $x_2 - x_1 = C\dot{x}_3$ where $C = C_1 \| C_2$.

After rearrangement:

$$\dot{x}_1 = 0 + 0 + \left(-\frac{1}{L_1}\right)x_3 + \left(\frac{1}{L_1}\right)v_1$$

$$\dot{x}_2 = 0 + \left(-\frac{R}{L_2}\right)x_2 + \left(\frac{1}{L_2}\right)x_3 + \left(-\frac{1}{L_2}\right)v_2$$

$$\dot{x}_3 = \left(-\frac{1}{C}\right)x_1 + \left(\frac{1}{C}\right)x_2 + 0$$

Let $\bar{u} = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$.

In matrix form:

$$\dot{\bar{x}} = \begin{bmatrix} 0 & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R}{L_2} & \frac{1}{L_2} \\ -\frac{1}{C} & \frac{1}{C} & 0 \end{bmatrix} \bar{x} + \begin{bmatrix} \frac{1}{L_1} & 0 \\ 0 & \frac{-1}{L_2} \\ 0 & 0 \end{bmatrix} \bar{u}$$

The output equations are easy:

(1) $y_0 = v_r = x_2 R$
(2) $y_1 = x_2$

In matrix form:

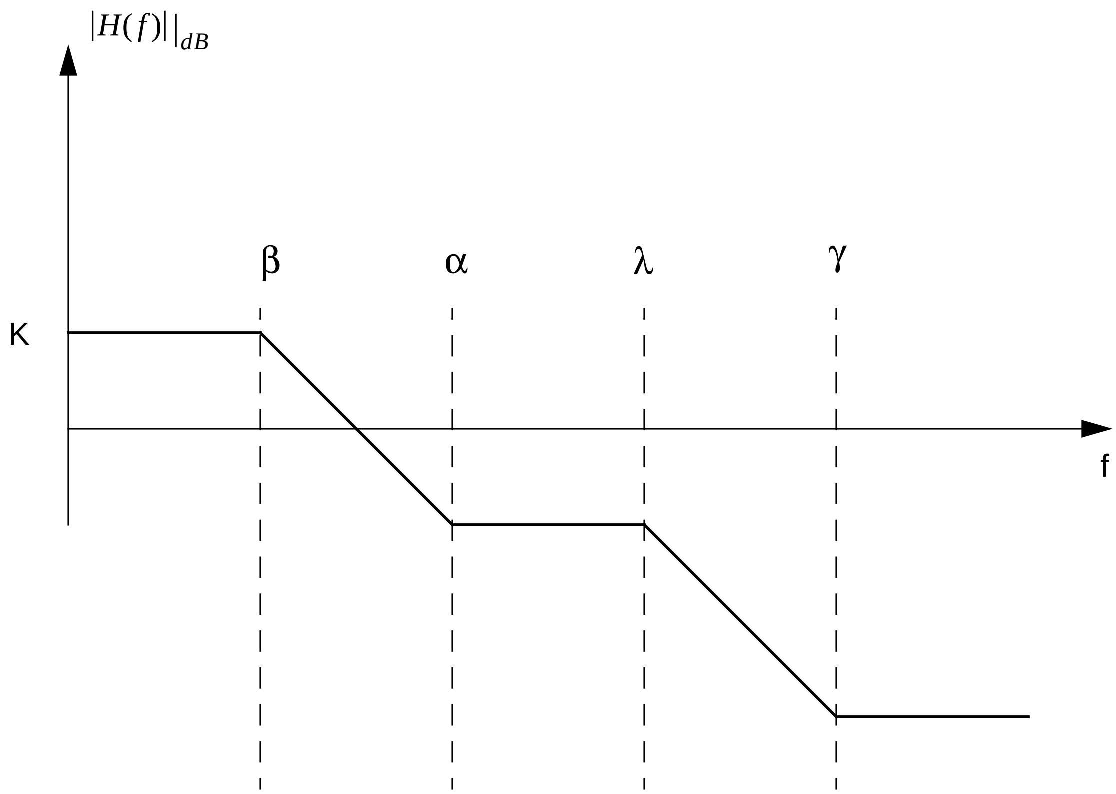
$$\bar{y} = \begin{bmatrix} 0 & R \\ 0 & 1 \end{bmatrix} \bar{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \bar{u}$$

Problem No. 3:

(a) For the transfer function shown below, sketch the Bode magnitude plot:

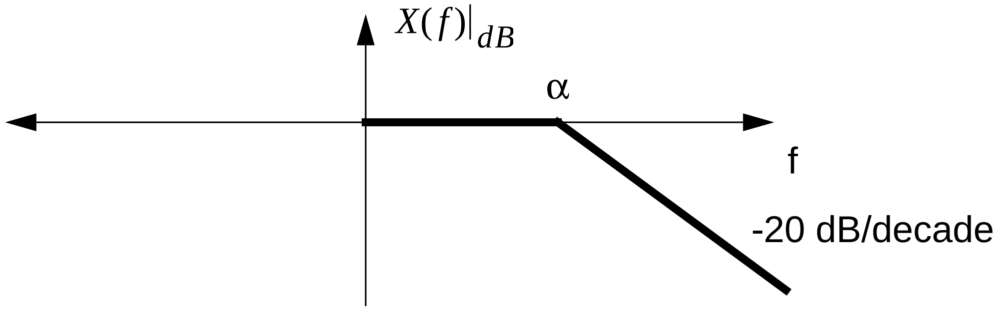
$$H(s) = \frac{C(s+\alpha)(s+\lambda)}{(s+\beta)(s+\gamma)}$$

Assume $\beta \ll \alpha \ll \lambda \ll \gamma$, and that $\alpha,\beta,\gamma,\lambda$ are real.



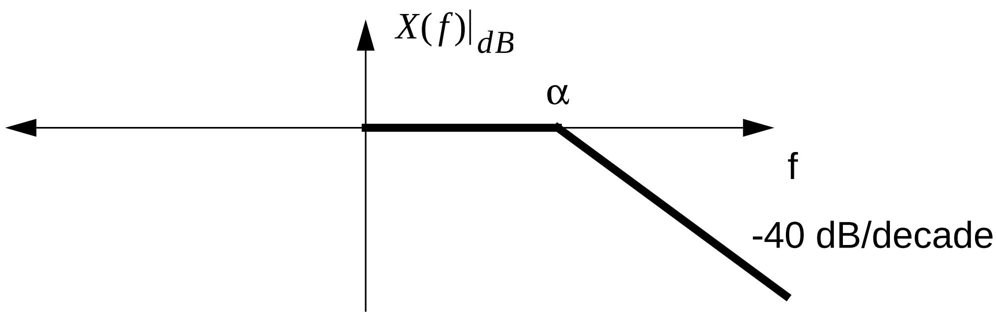
Note that $K = 20\ \log\left(\frac{C\alpha\lambda}{\beta\gamma}\right)$, and that the slopes of the lines are 20 dB/decade.

(b) Describe the transfer function for the system that has the following Bode plot (ignore $\angle X(f)$):



$$X(s) \approx \frac{K}{s + \alpha}$$

(c) Describe the transfer function for the system that has the following Bode plot (ignore $\angle X(f)$):



$$X(s) \approx \frac{K}{(s + \alpha)^2}$$