Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

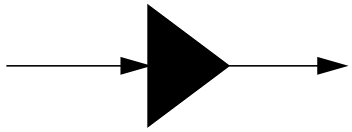
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Signal Models

(a) In the real world, a stereo amplifier shown below can be described as follows:

$x(t)$



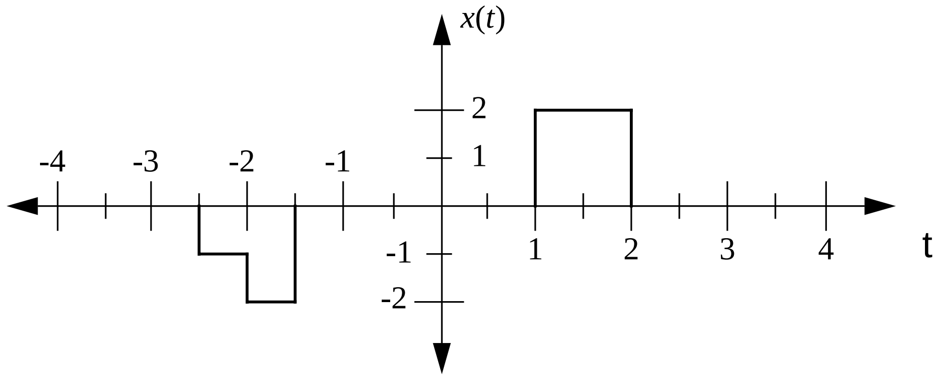
$$y(t) = \alpha(t)x(t) + \beta(t)x^2(t)$$

where $\alpha(t) = e^{-\rho t}u(t)$, and $\beta(t) = e^{-\sigma t}$, ρ and σ are constants.

Classify the system using as many concepts discussed in Chapters 1-3 as possible (for example, linearity, time-invariance, dynamic, etc.).

Continuous-time:	The input and output are functions of the continuous variable t (which denotes time).
Nonlinear:	The presence of $x^2(t)$ clearly results in $\alpha y(t) \neq F(\alpha x(t))$.
Instantaneous:	No terms depending on past values of the input (there are no derivatives in the system equation.
Causal:	The output does not depend on the input.
Time-Varying:	The weighting factors, $\alpha(t)$ and $\beta(t)$, depend on time. Hence, the output depends on what time the input starts.

(b) Represent the signal shown below in terms of the unit step function, $u(t)$:



Consider this signal the superposition of three pulses:

$$x(t) = -[u(t+2.5) - u(t+2)] + 2[u(t+2) - u(t+1.5)] + 2[u(t-1) - u(t-2)]$$

(c) Compute the power and the energy of the signal in part (b):

This is an energy signal:

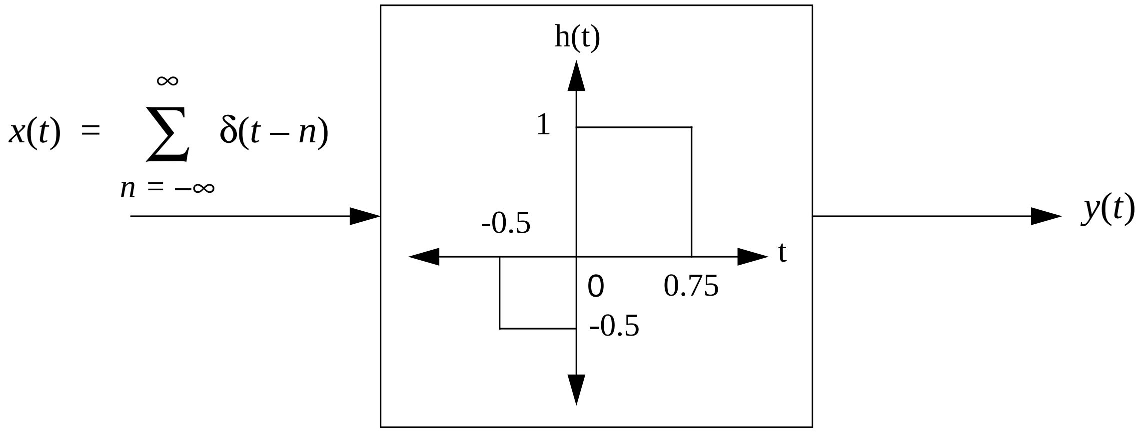
$$E = \lim_{T \to \infty} \int_{-T}^{T} x^2(t)dt = (-1)^2(0.5) + (-2)^2(0.5) + (2)^2(1) = 6.5 \ \ Joules$$

and,

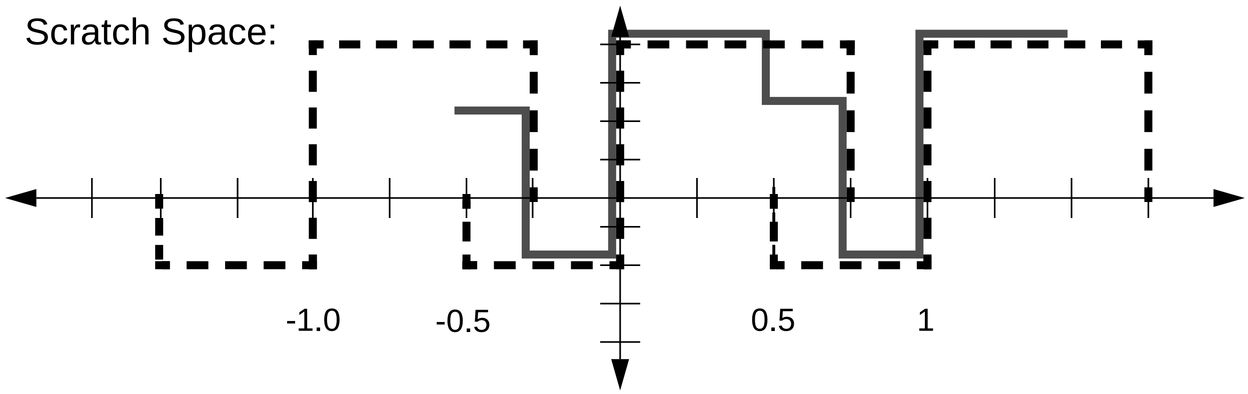
$$P = 0$$

Problem No. 2: Linear Systems

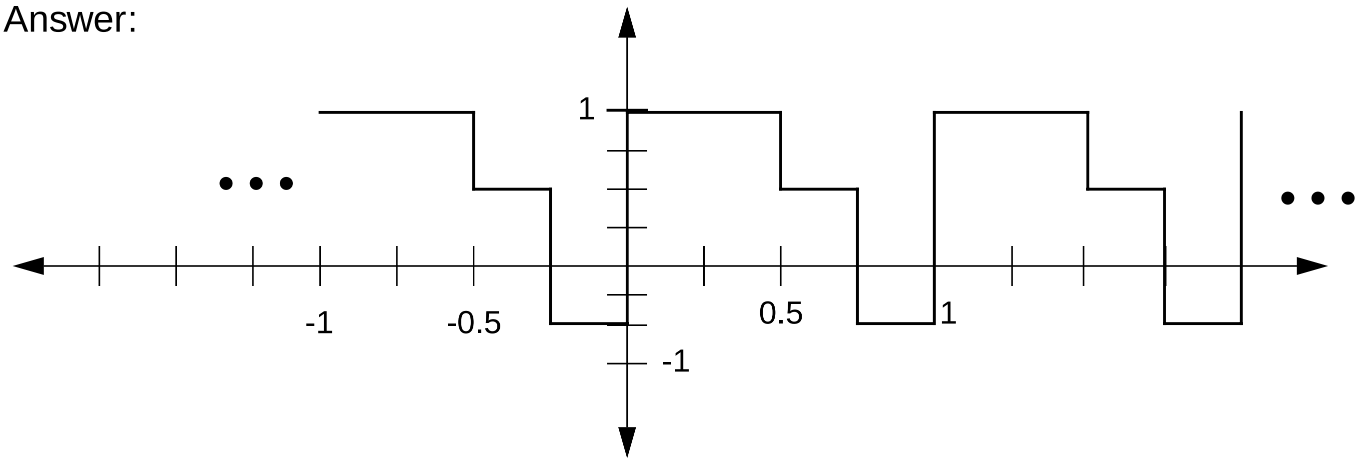
(a) Compute the output, $y(t)$, for the system shown below:



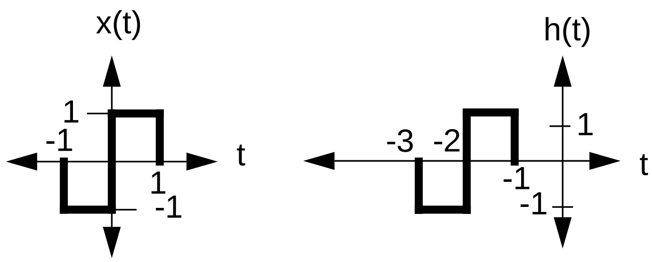
Scratch Space:

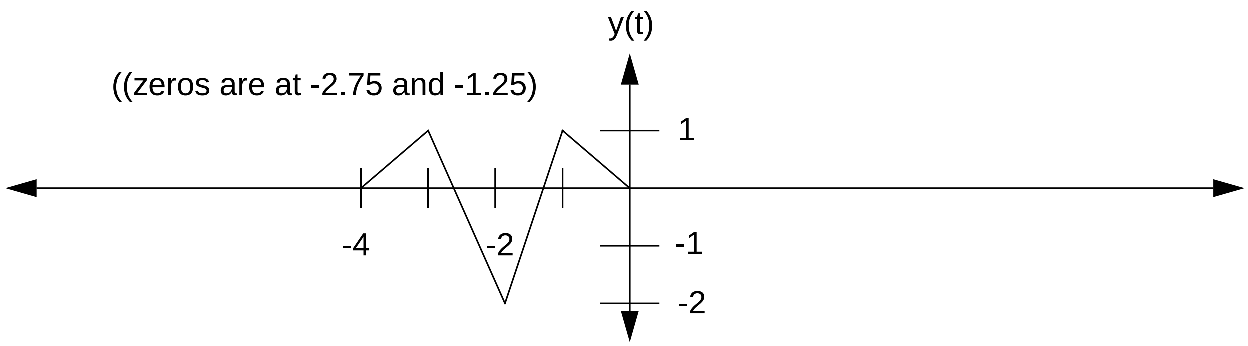


Answer:



(b) Compute the output, $y(t)$, for the system shown below:





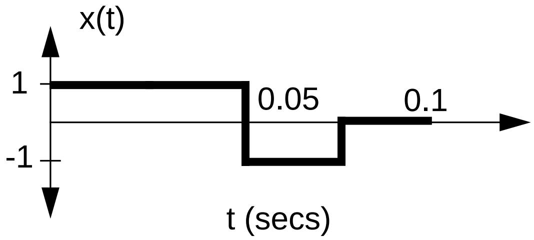
(c) Prove the commutative derivative identity for convolution ("⊗" denotes convolution):

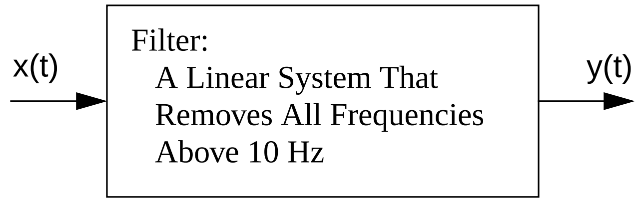
$$x(t) \otimes \frac{\partial}{\partial t} y(t) = \frac{\partial}{\partial t} x(t) \otimes y(t)$$

Even though this might be an interesting property, it is definitely not true!

In general, the derivatives of two signals is no way related to their convolution. (If one signal was the derivative of the other, such a property would obviously hold for this one special case.)

Problem No. 3: Expansions of Signals With Orthogonal Functions



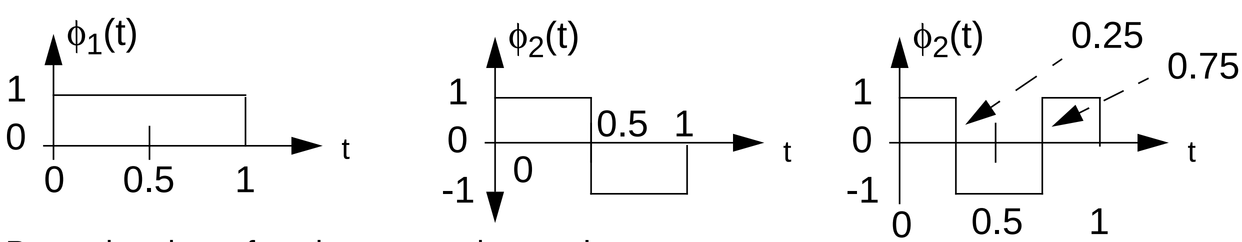


(a) x(t) is a periodic signal whose shape for one period is shown above. Describe the output signal, y(t). Be as specific as possible.

The fundamental frequency of the input is 10 Hz. The function is not symmetric. Hence, the DC value (a_0), and its first harmonics, a_1 and b_1, would be the only components that would pass the filter. Hence, the output would be of the form:

$$y(t) = a_0 + a_1 \cos 2\pi f_0 t + b_1 \sin 2\pi f_0 t$$

(b) A set of three functions are defined as follows:



Prove that these functions are orthogonal.

These functions are the first three functions in a family of functions known as Walsh functions. By inspection, we observe that the functions have equal positive and negative area. Hence, it is easy to show:

$$\int_0^1 \phi_n(t)\phi_m(t) = 0 \qquad n \neq m$$

(c) Consider approximating x(t) in (a) in terms of the functions in (b):

$$\tilde{x}(t) = a_1\phi_1(t) + a_2\phi_2(t) + a_3\phi_3(t)$$

Compute a_1:

This is just the DC value of the signal:

$$a_0 = \frac{1}{1}\int_0^1 x(t)dt = 1$$

(d) Do these functions form a complete set? (Hint: Can you approximate x(t) with zero error?). Explain how you would compute a_2.

To be a complete set, these functions must be capable of modeling all real signals that satisfy the Dirichlet conditions with zero error. Obviously, even the signal above can't be modeled with zero error, so this set of three functions are not complete.

However, these are drawn from a family of functions known as Walsh functions, which, in the limit do form a complete set. Walsh functions are useful in modeling binary data signals.

a_2 would be computed by multiplying $\tilde{x}(t)$ by $\phi_i(t)$, and integrating over one period. If the functions are orthogonal, only the portion of x(t) corresponding to $\phi_i(t)$ will remain.