Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

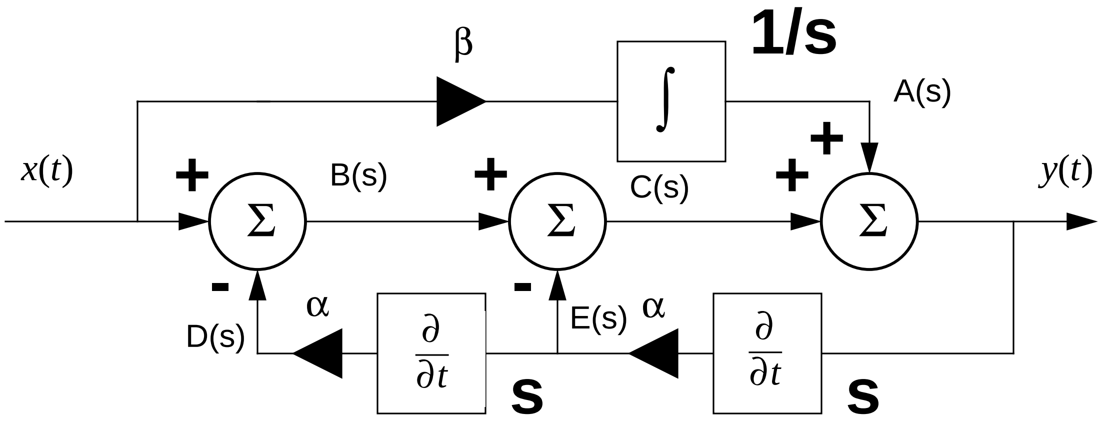
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1: For the system shown below:



(a) For what of values of α, assuming α is a real constant, is the system unstable?

Stategy: find the overall system transfer function. A few preliminaries:

$$A(s) = 1 + \beta\frac{1}{s}X(s)$$
$$E(s) = \alpha s Y(s)$$
$$D(s) = \alpha s E(s) = \alpha^2 s^2 Y(s)$$
$$B(s) = X(s) - D(s)$$
$$C(s) = B(s) - E(s)$$

Putting this all together, and writing and equation at the output summer:

$$Y(s) = \left(1 + \beta\frac{1}{s}\right)X(s) - (\alpha^2 s^2 + \alpha s + 1)Y(s)$$

The transfer function is:

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\left(1 + \beta\frac{1}{s}\right)}{1 + \alpha s + \alpha^2 s^2} = \frac{\left(1 + \beta\frac{1}{s}\right)\left(\frac{1}{\alpha^2}\right)}{\left(s^2 + \frac{1}{\alpha}s + \frac{1}{\alpha^2}\right)}$$

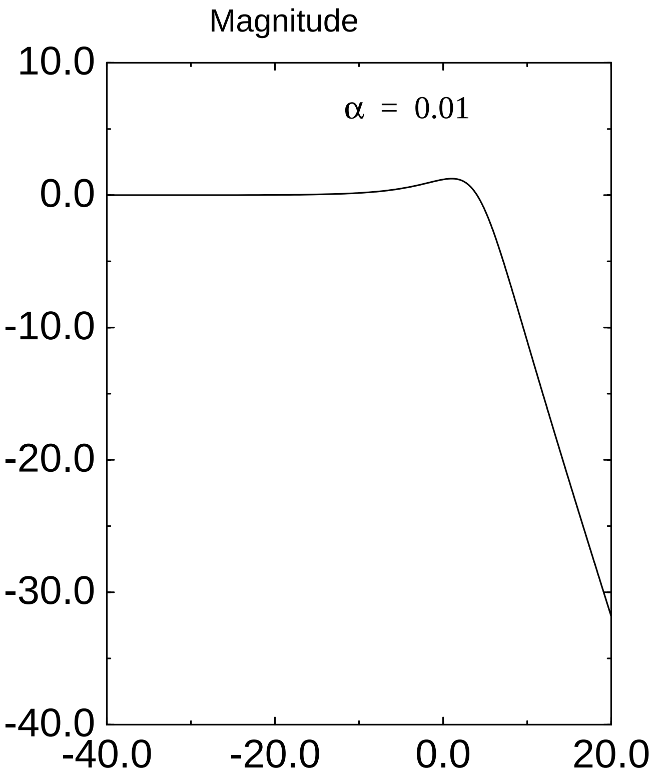
The system is unstable when the poles are in the right-half plane. The roots are given by the quadratic equation:

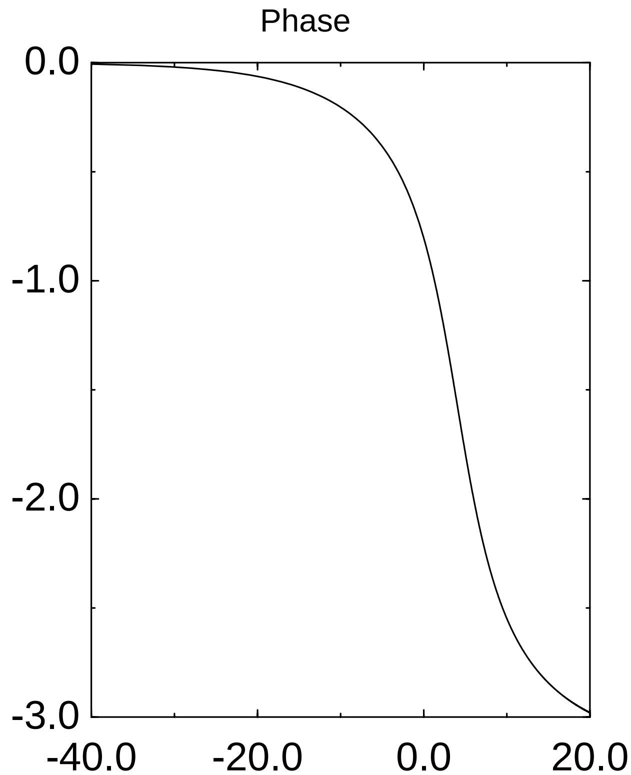
$$s_{1,2} = \frac{-\frac{1}{\alpha} \pm \sqrt{\frac{1}{\alpha^2} - \frac{4}{\alpha^2}}}{2} = \frac{-\frac{1}{\alpha} \pm \sqrt{-\frac{3}{\alpha^2}}}{2}$$

Obviously, the poles are in the right-half plane when $\alpha < 0$.

(b) Assume $\beta = 0$. Sketch the Bode plots for the magnitude and phase.

This is simply a two-pole second-order system with a complex conjugate pole pair — which implies potentially a bandpass filter or lowpass filter in the degenerate case.





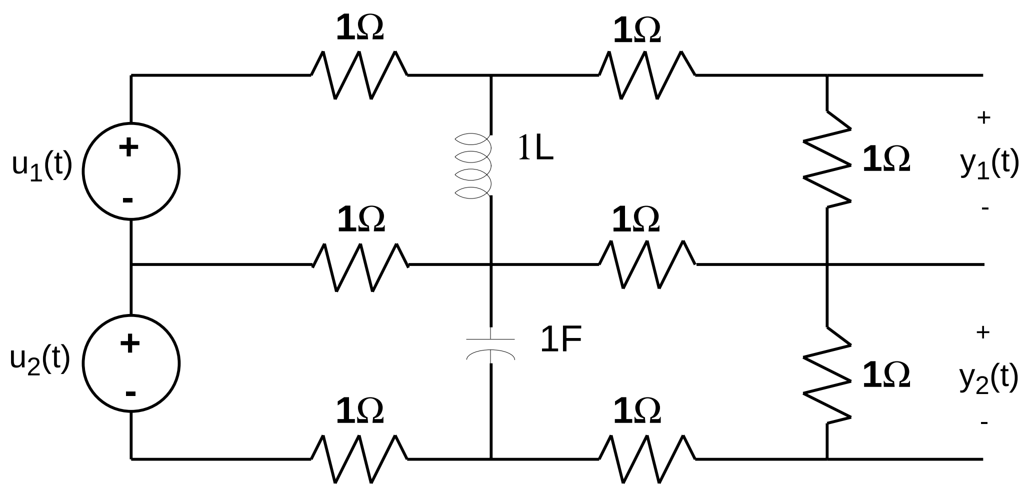
(c) Describe the effect β has on the system in terms of its frequency response, stability, and phase characteristics.

β primarily controls a zero in the spectrum. This will force attenuation of the spectrum at a specific frequency, or DC if β is real.

A negative value of β can force the system to be non-minimum phase; a positive value guarantees the system is minimum phase.

β doesn't influence stability.

Problem No. 2: For the circuit shown below (all components have a value of 1):



(a) Explain how many state variables you would use to analyze this circuit.

There are two memory elements, so two state variables are required.

(b) Do you expect the state transition matrix, $\phi(t)$, to be a diagonal matrix? Justify your answer (answers with out justifications are incorrect).

Each output is dependent on BOTH inputs. Therefore, the state transition matrix must be a 2x2 matrix. If it was diagonal, that would imply each output could be decoupled — and the system was really two single-input single-output systems.

(c) Find the state equations for this system. Begin by indicating the dimensions of each matrix in the state variables representation.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Using KVL:

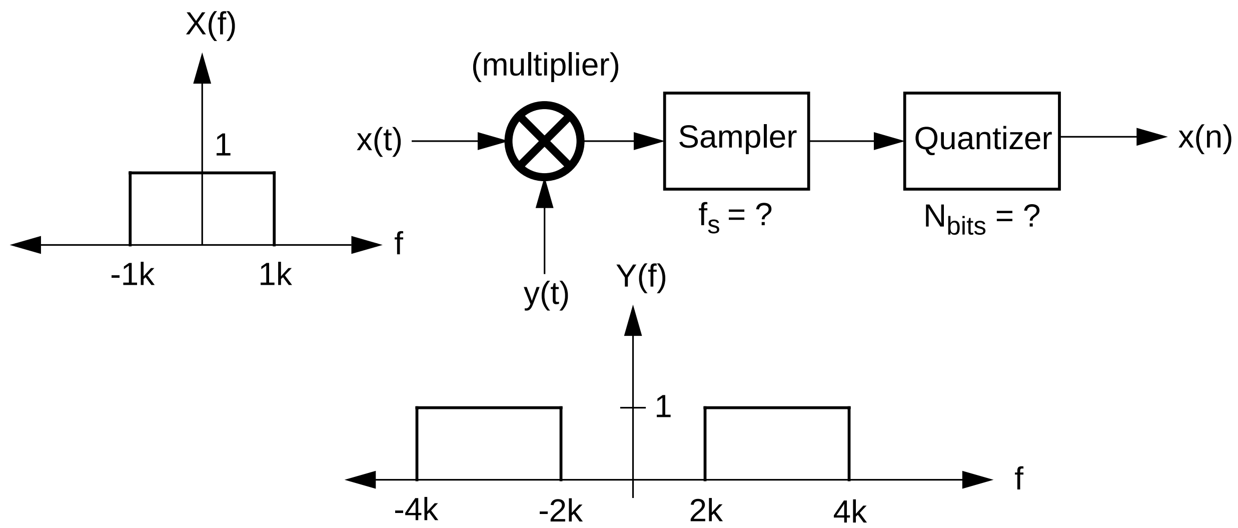
(d) Find the system impulse response matrix, $\boldsymbol{H}(t)$:

The impulse response matrix is of the form:

$$\boldsymbol{H}(t) = \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix}$$

where $h_{ij}(t) = Ae^{-\alpha t} + Be^{-\beta t}$.

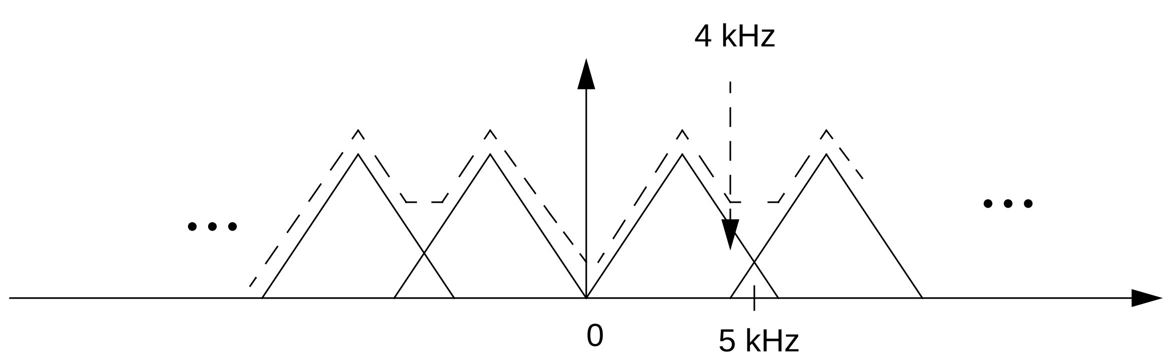
Problem No. 3: For the system shown below:



(a) What is the minimum required sample frequency to digitize the input signal without aliasing? Explain.

The spectrum of the input to the sampler is the convolution of X(f) and Y(f). This will result in a triangle-shaped spectrum of bandwidth 6 kHz. Hence, the required sample frequency is 12 kHz.

(b) Sketch the spectrum of x(n) if the signal is sampled at 10 kHz.



Editor's Note: This is the way I wanted the problem to work. Unfortunately, I blew the convolution. On the following pages, I have the correct solution. I guess this is why we have computer tools.

(c) If a 10 bit quantizer is used, what is the approximate signal-to-noise ratio for the system? Be sure to include any distortion due to aliasing during the sampling process.

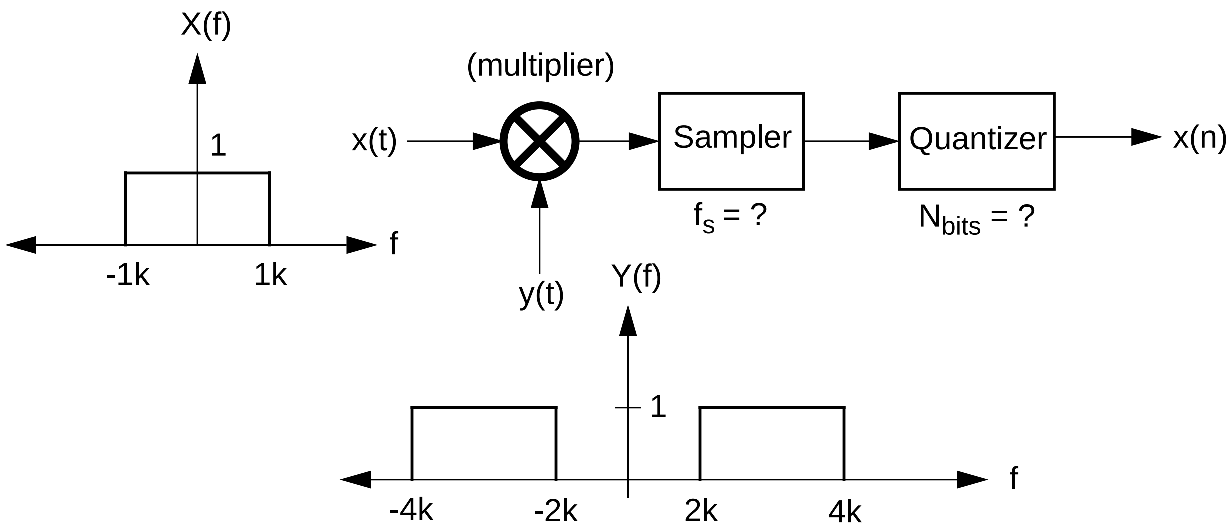
Using our standard formula, 10 bits is equvalent to an SNR of 6 dB x 10 = 60 dB

However, the aliasing introduces additional distortion in the spectrum. The signal in the region from 4 kHz to 5 kHz is corrupted. Hence, the actual SNR will be lower. We can approximate this by computing the ratio of the power in the region from 4 kHz to 6 kHz, and comparing that to the overall signal power:

$$\frac{P_{aliased}}{P_{total}} = \frac{\int_0^2 f^2 df}{\int_0^6 f^2 df} = \frac{2^3}{6^3} = 14 \text{ dB}$$

Hence, the overall SNR is approximately 46 dB.

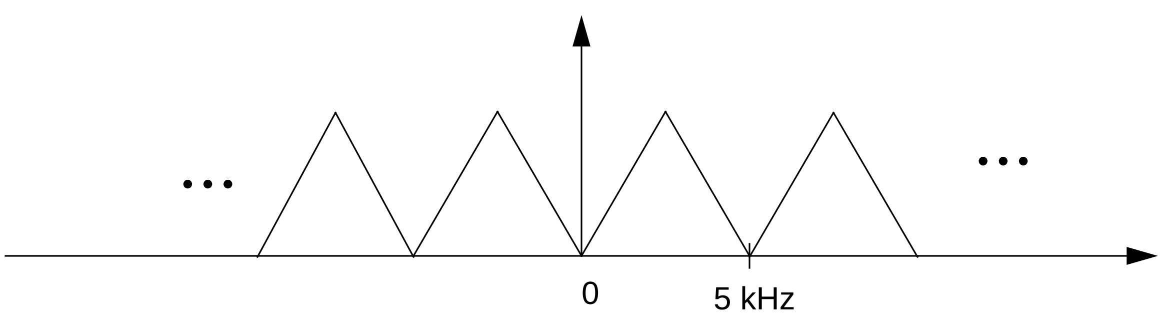
Problem No. 3: For the system shown below:



(a) What is the minimum required sample frequency to digitize the input signal without aliasing? Explain.

The spectrum of the input to the sampler is the convolution of X(f) and Y(f). This will result in a triangle-shaped spectrum centered at 3 kHz and of width 4 kHz. The required sample frequency is 2 x f_{max} = 2 x 5 kHz = 10 kHz. If we use the bandpass sampling theorem, we can actually sample the signal at 8 kHz.

b) Sketch the spectrum of x(n) if the signal is sampled at 10 kHz.



Revised solution!

(c) If a 10 bit quantizer is used, what is the approximate signal-to-noise ratio for the system? Be sure to include any distortion due to aliasing during the sampling process.

Using our standard formula, 10 bits is equvalent to an SNR of 6 dB x 10 = 60 dB

There is no distortion due to aliasing.