Name: Thomas Dodd

Problem	Points	Score
1a	10	
1b	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
3e	10	
Total	100	

Notes:

1.The exam is closed books/closed notes - except for one page of notes.

2.Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3.Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Show the following properties of the Fourier transform for **real** signals:

(a) $|X(f)| = |X(-f)|$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$ Using Euler's Theorem

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$$

Taking the magnitude

$$|X(f)| = \int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt$$

This is a real and even function of f. The magnitude's dependency on frequency it through the cosine function. Substituting -t for t yields:

$$|X(f)| = \int_{-\infty}^{\infty} x(-t)\cos(-2\pi ft)dt$$

Since cos(-t) = cos(t), from the definition of an even function, the conclusion

$|X(f)| = |X(-f)|$ can be drawn.

(b) $\theta(f) = -\theta(-f)$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$ Using Euler's Theorem

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$$

Taking the phase

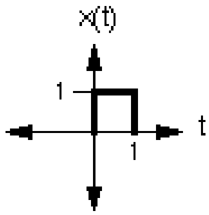
$$\theta(f) = -j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$$

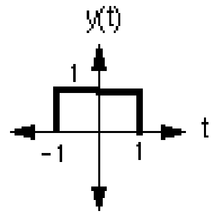
This is an off function of f due to the dependency through the sine. Subtituting -t for t yields

$\theta(-f) = -j\int_{-\infty}^{\infty} x(-t)\sin(-2\pi ft)dt$ and can conclude that

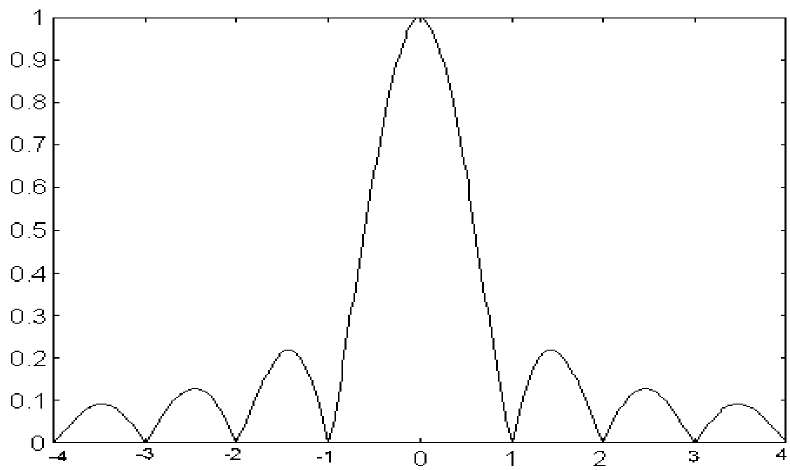
$$\theta(f) = -\theta(-f)$$

Problem No. 2: For the following two signals:





(a) Plot the magnitude spectrum of $x(t)$:

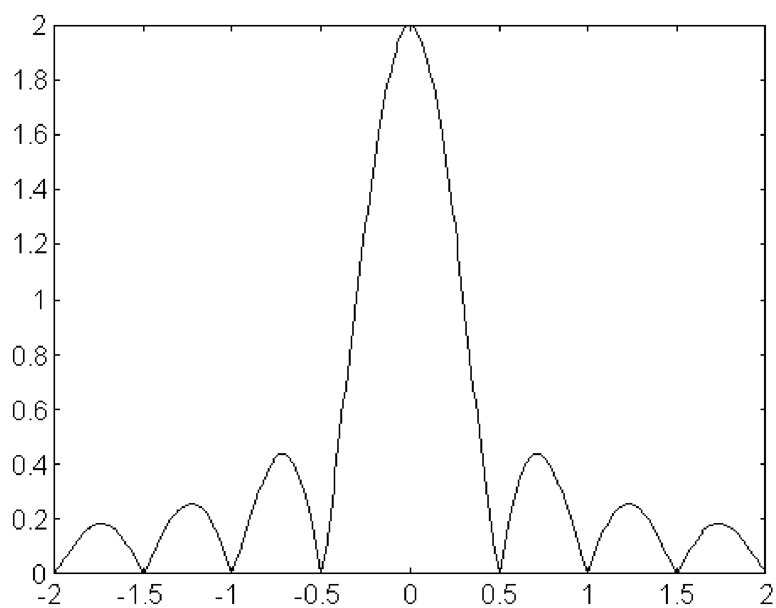


$$x(t) = \Pi(t - 1/2)$$

$$X(f) = \frac{\sin \pi f}{\pi f}$$

$$X(f) = \text{sinc(f)}$$

(b) Plot the magnitude spectrum of $y(t)$:



$$y(t) = \Pi(t/2)$$

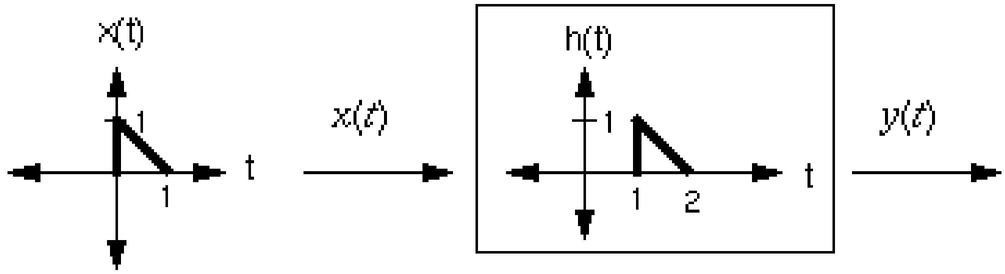
$$Y(f) = \frac{2\sin 2\pi f}{2\pi f}$$

$$Y(f) = \text{2sinc2f}$$

(c) Compare and contrast these plots (note that you can answer this question correctly without getting the correct answers for (a) and (b).

y(t) has twice the areaunder the curve as x(t) so the magnitude of Y(f) is ½ as wide as the magnitude of X(f). The time shift from x9t0 to y(t) has no effect on the magnitude spectrum, instead it causes a shift in the phase response).

Problem No. 3: For the following system,



(a) Find $X(s)$:

$$y(x) = ax + b$$
$$x(t) = (-t+1)\{0 < x < 1\}$$
$$X(S) = \int_0^{\infty} x(t)e^{-st}dt$$
$$X(S) = \int_0^{1} (1-t)e^{-st}dt$$
$$X(S) = \frac{1}{s} - \frac{1}{s^2}$$

Using the time-shift theorem

$$h(t) = (-t+2) = x(t-1)$$
$$H(S) = X(S)e^{-s}$$
$$H(s) = \left(\frac{1}{s} - \frac{1}{s^2}\right)e^{s}$$

(b) Find $Y(s)$:

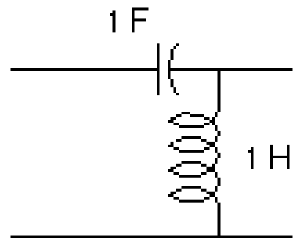
Using the Laplace transform of the convolution of two signals:

$$y(t) = x(t) * h(t)$$
$$Y(s) = X(s)H(s)$$

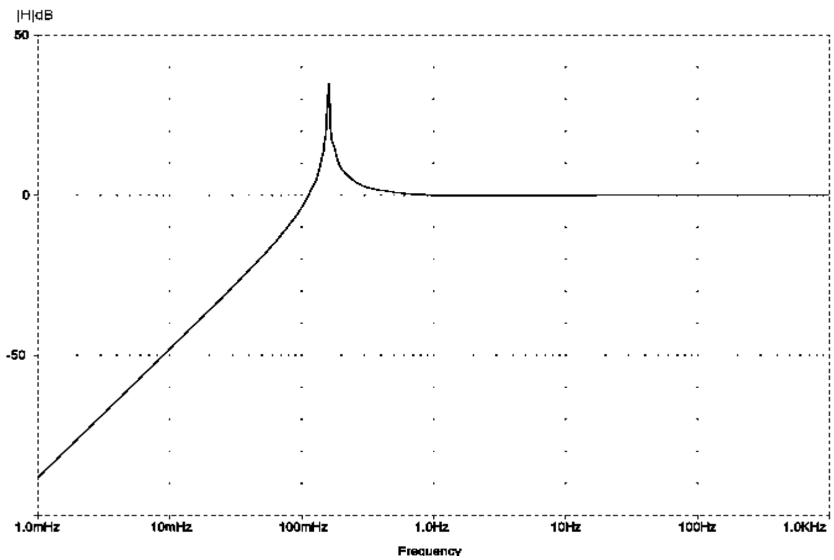
substituting the values for X(S) and H(S) gives:

$$Y(S) = \left(\frac{1}{s} - \frac{1}{s^2}\right)\left(\frac{1}{s} - \frac{1}{s^2}\right)e^{-s}$$
$$Y(S) = \left(\frac{1}{s} - \frac{1}{s^2}\right)^2 e^{-s}$$

(c) For the circuit shown, construct the Bode magnitude plot.



Start with slope of +40dB/dec.
Pole at $\omega=1$ rad/sec ($f=1/2\pi=.159$ Hz)
with $\zeta=0.5$



(d) How many poles and zeros does the system have?

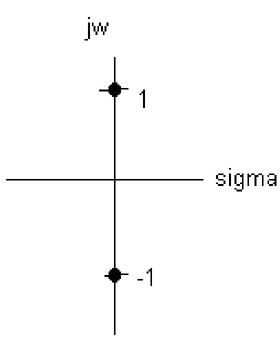
Poles: There are 2 poles. They are a complex conjugate pair.

$s^2+1=0$, or $s^2=-1$,or $s=j$ and$s= -j$

Zeros: There are 2 zeros. Both are at $s^2=0$

(e) Using concepts introduced in Chaps. 4 - 6, determine the stability of the system.

Zeros have no effect on the stability of the system.
To determine the stability, plot the poles, (s-1) and (s+1), shown below.



they are plotted on the imaginary axis,which makes the system marginally stable.