Name:Janna Shaffer

Problem	Points	Score
1a	10	
1b	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
3e	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Show the following properties of the Fourier transform for **real** signals:

(a) $|X(f)| = |X(-f)|$

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi f}dt$$

$$X(f) = \int_{-\infty}^{\infty} x(t)(\cos 2\pi ft - j\sin 2\pi ft)dt$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos 2\pi ftdt - j\int_{-\infty}^{\infty} x(t)\sin 2\pi ftdt$$

$$X(-f) = \int_{-\infty}^{\infty} x(t)\cos 2\pi(-f)tdt - j\int_{-\infty}^{\infty} x(t)\sin 2\pi(-f)tdt$$

$$X(f) = \int_{-\infty}^{\infty} x(t)\cos 2\pi ftdt + j\int_{-\infty}^{\infty} x(t)\sin 2\pi ftdt$$

$$|X(f)| = \int_{-\infty}^{\infty} x(t)\cos 2\pi ftdt = |X(-f)|$$

(b) $\theta(f) = -\theta(-f)$

$$\tan(\theta(f)) = \frac{-\int_{-\infty}^{\infty} x(t)\sin 2\pi ftdt}{\int_{-\infty}^{\infty} x(t)\cos 2\pi ftdt}$$

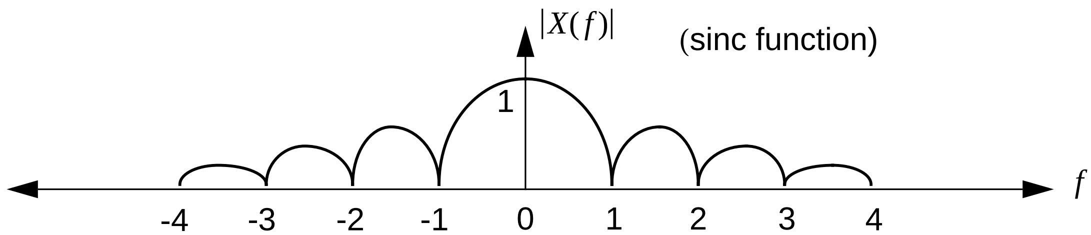
$$\tan(\theta(-f)) = \frac{\int_{-\infty}^{\infty} x(t)\sin 2\pi ftdt}{\int_{-\infty}^{\infty} x(t)\cos 2\pi ftdt}$$

Using the trig property: $\tan(\phi) = -\tan(-\phi)$

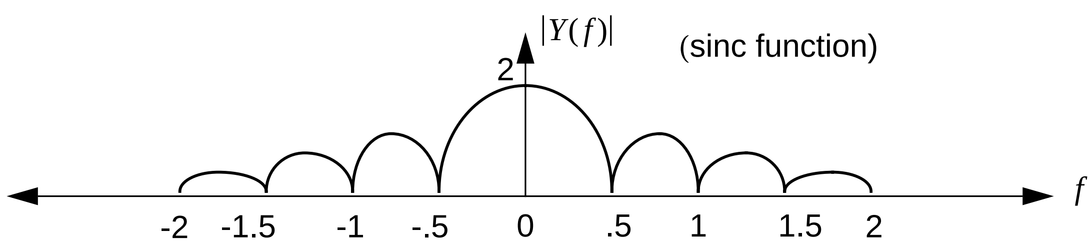
therefore: $\theta(f) = -\theta(-f)$

Problem No. 2: For the following two signals:

(a) Plot the magnitude spectrum of $x(t)$:



(b) Plot the magnitude spectrum of $y(t)$:



(c) Compare and contrast these plots (note that you can answer this question correctly without getting the correct answers for (a) and (b).

Differences:
The main lobe of X(f) twice as wide as that of Y(f).
Y(f) has a greater amplitude than X(f).

Similarities:
They are both sinc functions centered about the origin.

The two theorems that dictate the similarities/difference of these two magnitude plots:

Time Delay

$$x(t - t_0) \longrightarrow X(f)e^{-j2\pi f t_0}$$

y(t) starts at time t=-1 while x(t) starts at time t=0. However, time delay does not affect the magnitude response only the phase response. That it why that both magnitude responses are centered about the origin.

Scale Change:

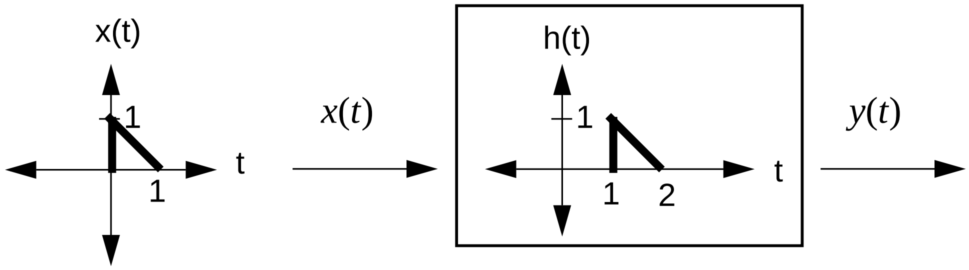
$$x(at) \longrightarrow |a|^{-1} X\left(\frac{f}{a}\right)$$

Because of this theorem, the width of the side lobes are narrower on y(t) and its maximum amplitude is twice that of x(t).

note:

(Both of these theorems can be tested by using the wonderful spectrum analysis tool available on the ISIP homepage: http://isip.msstate.edu/publications/1997/ee_4012/)

Problem No. 3: For the following system,



(a) Find $X(s)$:

$$x(t) = u(t) - r(t) + r(t-1) - u(t-1)$$

$$X(S) = \frac{1}{s} - \frac{1}{s^2} + \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} = \frac{1-e^{-s}}{s} + \frac{e^{-s}-1}{s^2}$$

note: used delay theorem to evaluate the (t-1) terms:

$$L\{x(t-t_0)u(t-t_0)\} = X(s)e^{-st_0}$$

$$L\{u(t-1)\} = \frac{1}{s} \times e^{-s}$$

(b) Find $Y(s)$:

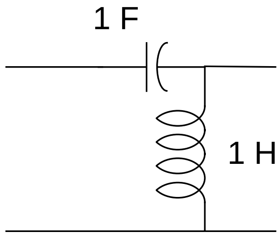
$$h(t) = u(t-1) - r(t-1) + r(t-2) - u(t-2)$$

$$H(S) = \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s} = \frac{e^{-s}-e^{-2s}}{s} + \frac{e^{-2s}-e^{-s}}{s^2}$$

$$y(t) = x(t) \otimes h(t) \longrightarrow Y(S) = X(S)H(S)$$

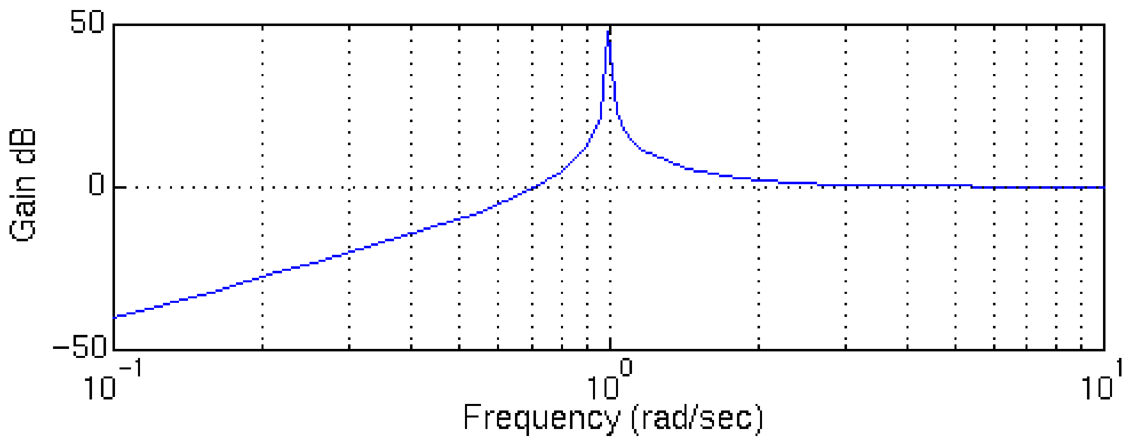
$$Y(S) = \left(\frac{1-e^{-s}}{s} + \frac{e^{-s}-1}{s^2}\right)\left(\frac{e^{-s}-e^{-2s}}{s} + \frac{e^{-2s}-e^{-s}}{s^2}\right)$$

(c) For the circuit shown, construct the Bode magnitude plot.



$$Y(S) = \frac{s}{s + \frac{1}{s}}$$

$$Y(S) = \frac{s^2}{s^2 + 1}$$



MATLAB code used to generate Bode Magnitude plot:

```
num[1 0 0]
den[1 0 1]
bode(num,den)
```

(d) How many poles and zeros does the system have?

Since there are two roots to s^2 there are 2 zeros.

Since there are two roots to $s^2 + 1$ there are 2 poles.

(e) Using concepts introduced in Chaps. 4 - 6, determine the stability of the system.

The system is marginally stable since the poles are on the imaginary axis and there are no poles in the RHP.