Name: **Stuart A. Shull**

Problem	Points	Score
1a	10	
1b	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
3e	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Show the following properties of the Fourier transform for **real** signals:

(a) $|X(f)| = |X(-f)|$

x(f) = $\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$ start with definition of Fourier transform

Using Eulers's Theorem: $\varepsilon^{\pm ju}$ = cos (u) ± j sin (u)

x(f) = $\int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$

Only the magnitude part is needed: $\int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt$

$$|X(f)| = \int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt$$

The magnitude (above equation) is a cosine function which is a real and even function of frequency. The dependence of the real part of X(f) is through cos2πft. By integrating the magnitude (the above equation), the following results occurs:

X(*f*) = $\int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt = \int_{-\infty}^{\infty} x(-t)\cos(-2\pi ft)dt$ sub -*t* for *t*

We know cos(t) = cos(-t), which is again an even function. Therefore, the conclusion that X(f) = X(-f) can be made.

(b) $\theta(f) = -\theta(-f)$

x(f) = $\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$ start with definition of Fourier transform

Using Eulers's Theorem: $\varepsilon^{\pm ju}$ = cos (u) ± j sin (u)

x(f) = $\int_{-\infty}^{\infty} x(t)\cos(2\pi ft)dt - j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$

Only the imaginary part is needed: $-j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$ assume x(t) is real

θ(f) = $-j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$

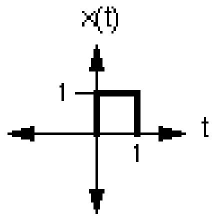
Like the previous problem, since the frequency dependence of the imaginary part of X(f) is through sin2πft, and we know that a sine function is an odd function, we know that the above equation is an odd function.

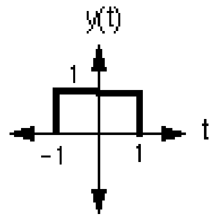
Therefore: θ(f) = $-j\int_{-\infty}^{\infty} x(t)\sin(2\pi ft)dt$ odd function

and θ(-f) = $j\int_{-\infty}^{\infty} x(-t)\sin(-2\pi ft)dt$ sub -*t* for *t*

We know sin(t) = -sin(-t), which is again an odd function. Therefore, the conclusion that
$\theta(f) = -\theta(-f)$ can be made.

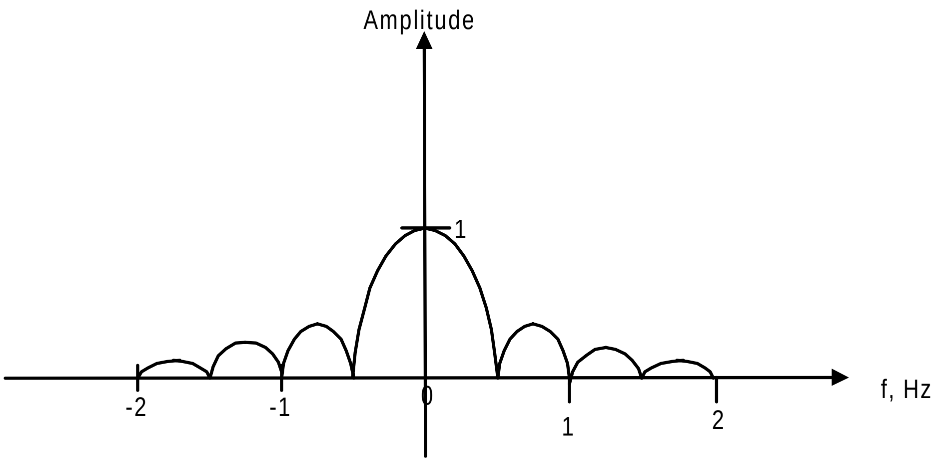
Problem No. 2: for the following two signals:





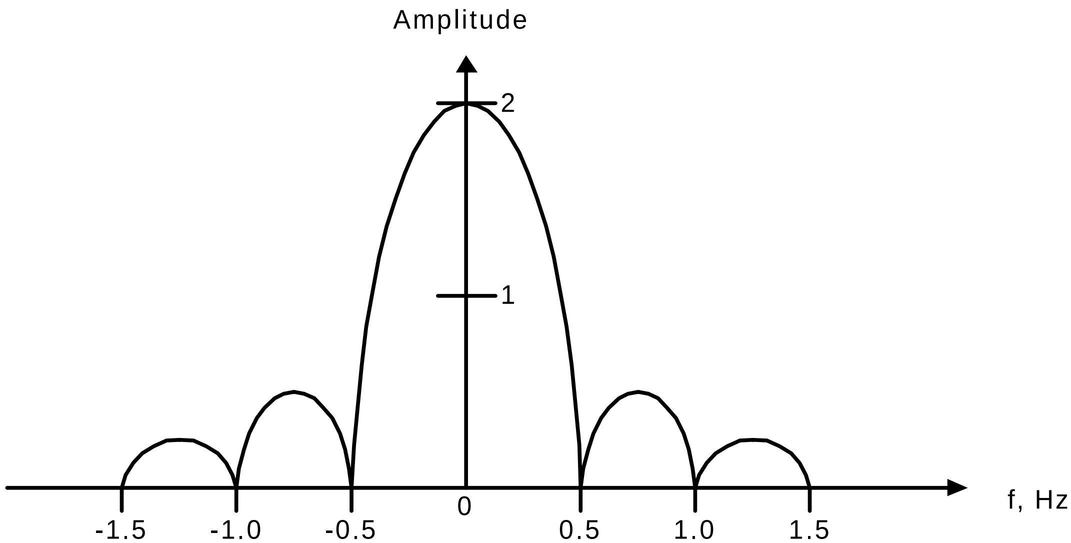
(a) Plot the magnitude spectrum of *x(t)*:

Π(t-) $\leftrightarrow$ ◊ sinc(f)$e^{-j\Pi f}$



(b) Plot the magnitude spectrum of *y(t)*:

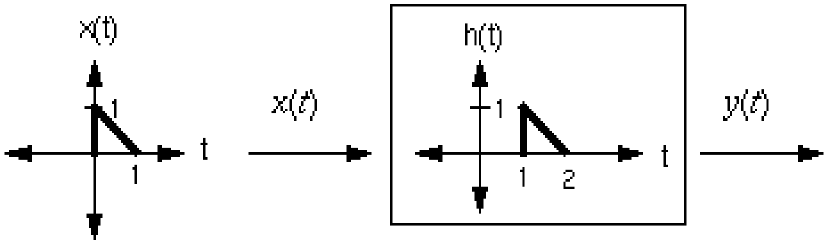
$$X_a(f) = \int_{-1}^{1} e^{-j2\pi ft}\,dt = -\frac{e^{-j2\pi ft}}{j2\pi f}\Big|_{-1}^{1} = \frac{2\sin 2\pi f}{2\pi f} = 2\ \mathrm{sinc}\ 2f$$



(c) Compare and contrast these plots (note that you can answer this question correctly without getting the correct answers for (a) and (b).

The first noticeable characteristic difference in the two above plots is the difference in the widths of the two wave forms. The magnitude spectrum of part (b) illustrates a narrow spectrum due to the initial wave having it's particular area. Compared to x(t), y(t) has twice the area under it's curve, therefore, the magnitude. wave of y(t) is half as wide as the magnitude. wave of x(t). One of the properties studied is that signals that have a wide spectrum in the time domain have a narrow spectrum in the frequency domain, and vise-versa. In contrast, the magnitude plot of x(t), part a, is wider by a multiple of 2 because is has half the area of y(t).
The magnitudes are affected in the same way as the width of the plots are in the above explanation.

Problem No. 3: For the following system,



(a) Find *X(s)*:

Using the time delay theorem we have:

$$L=[x(t-t_0)u(t-t_0)] = \int_{t_o}^{\infty} x(t-t_0)e^{-st}dt$$

$$X(s) = \int_0^1 (1-t)e^{-st}dt$$

$$X(s) = \int_0^1 e^{-st}dt - \int_0^1 te^{-st}dt$$

Now, Integrating by parts, using the following substitutions:

u=t　　　　　dv=e^{-st}dt
du=dt　　　　v=-e^{-st}/s

$$x(s) = --\frac{e^{-st}}{s}\Big|_0^1 - \left[-\frac{te^{-st}}{s}\Big|_0^1\right] - \int_0^1 -\frac{e^{-st}}{s}dt$$

$$X(s) = -\frac{e^{-st}}{s}\Big|_0^1 + \left[\frac{te^{-st}}{s}\Big|_0^1\right] - \frac{e^{-st}}{s^2}$$

$$X(s) = -\frac{e^{-st}}{s}\Big|_0^1 - \left(-\frac{1}{s}\right) + \frac{e^{-s}}{s} - 0 - \left[\frac{e^{-s}}{s^2} - \frac{1}{s^2}\right]$$

$$x(s) = -\frac{e^{-st}}{s} + \frac{1}{s} + \frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} + \frac{1}{s^2}$$

$$X(s) = \frac{1}{s} + \frac{1}{s^2} - \frac{e^{-s}}{s}$$

(b)

Find *Y(s)*:

First, we need to find H(s) and then multiply with X(s) in order to find Y(s) as dictated by the theorems below:

Convolution theorem for time domain: y(t) = x(t)*h(t)

Convolution theorem for frequency domain: Y(s) = X(s)H(s)

H(s) = $\int_1^2 (2-t)e^{-st}dt$

H(s) = $\int_1^2 2e^{-st}dt - \int_1^2 te^{-st}dt$

Now, using integration by parts and the following substitution:

u=t dv-e^{-st} dt
du=dt v=e^{-st} /sdt

H(s) = $-\frac{2e^{-st}}{s}\Big|_1^2 - \left[-\frac{te^{-st}}{s}\Big|_1^2\right] - \int_1^2 -\frac{e^{-st}}{s}dt$

H(s) = $-\frac{2e^{-st}}{s}\Big|_1^2 - \left[-\frac{te^{-st}}{s}\Big|_1^2\right] - \frac{e^{-st}}{s^2}\Big|_1^2$

H(s) = $-\frac{2e^{-2s}}{s} - \left[-\frac{2e^{-s}}{s}\right] - \left[-\frac{2e^{-2s}}{s} - \left[-\frac{e^{-s}}{s}\right] - \left[\frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s^2}\right]\right]$

H(s) = $-\frac{2e^{-2s}}{s} + \frac{2e^{-s}}{s} + \frac{2e^{-2s}}{s} - \frac{e^{-s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2}$

H(s) = $\frac{e^{-s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2}$

Now, we can Y(s) by using the convolution theorem stated above:

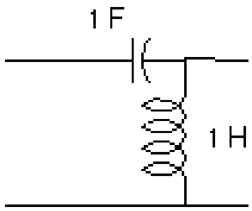
Y(S) = X(s)H(s)

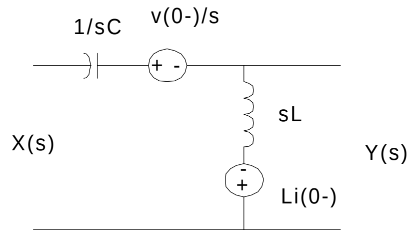
Y(S) = $\left[\frac{1}{s} + \frac{1}{s^2} - \frac{e^{-s}}{s}\right]\left[\frac{e^{-s}}{s} - \frac{e^{-2s}}{s^s} + \frac{e^{-s}}{s^2}\right]$

Y(S) = $\frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^3} + \frac{e^{-s}}{s^3} + \frac{e^{-s}}{s^3} - \frac{e^{-2s}}{s^4} + \frac{e^{-s}}{s^4} - \frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s^3} - \frac{e^{-2s}}{s^3}$

$$Y(S) = \frac{1}{s^2}\left[e^{-s} - e^{-2s}\right] + \frac{1}{s^3}\left[2e^{-s} - 2e^{-2s} + e^{-3s}\right] + \frac{1}{s^4}\left[e^{-s} - e^{-2s}\right]$$

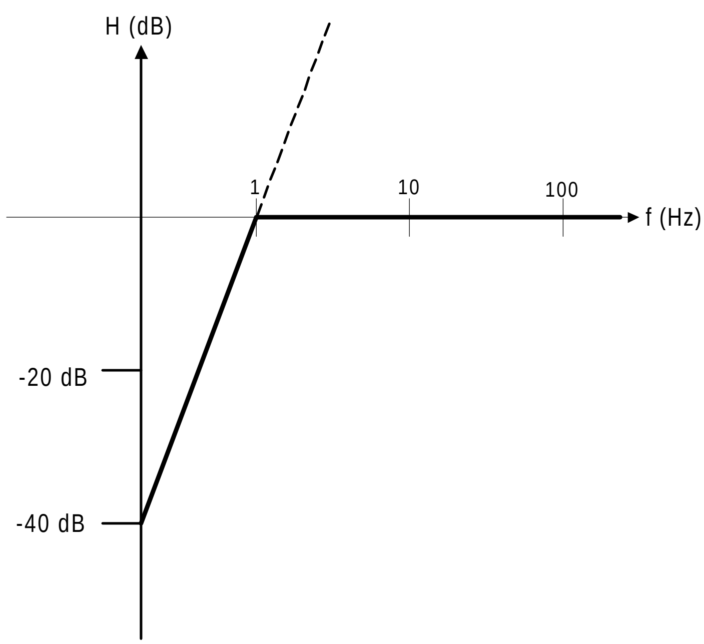
(c) For the circuit shown, construct the Bode magnitude plot.





$$Y(s) = \left(\frac{sL}{\frac{1}{sL} + sL}\right)(x(s))$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{sL}{\frac{1}{sC} + sL} = \frac{s^2}{s^2 + 1}$$



(d) How many poles and zeros does the system have.

Poles: 1 Complex Conjugate Pair at $s^2 + 1 = 0$, or at $(s - j)(s + j)$.

Zeros: 2 zeros at $s^2 = 0$.

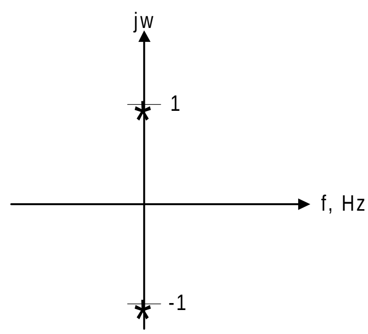
(e) Using concepts introduced in Chaps. 4 - 6, determine the stability of the system.

In order to determine the stability of the system, the poles of the system must be analyzed. For this particular system, the poles are found by finding the zeros in the following equation:

$$s^2 + 1 = 0$$

$$(s - j)\,(s + j) \text{ which yields: } \pm j$$

The zeros for the above equation are at $s = \pm 1$ on the imaginary axis. The following is a plot of the zeros:



Since the poles are on the $j\omega$ axis, and not completely in the right-half plane or the left-half plane, therefore, the system is marginally stable.