Problem No. 1: For the linear time-invariant system: $H(s) = \frac{s+2}{(s^2-2s-3)}$

(a) Find the state variable description of the system.

First expand H(s) using partial fractions.

$H(s) = Y(s)/U(s) = A/(s-3) + B/(s+1)$

$A = 5/4, \; B = -1/4$

Y(s) can now be rewritten as

$Y(s) = (5/4U(s))/(s-3) + (-1/4U(s))/(s+1)$

we define

$X1(s) = U(s)/(s-3)$ And $X2(s) = U(s)/(s+1)$

rearranging and inverse transforming gives

$\dot{x}1 = 3x1 + u \qquad \dot{x}2 = -x2 + u$

(b) Compute the state transition matrix, Θ(t).

$$\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = [5/4 \quad -1/4] \begin{bmatrix} x1 \\ x2 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} \qquad SI - A = \begin{vmatrix} (s-3) & 0 \\ 0 & (s+1) \end{vmatrix}$$

$$(sI - A)^{-1} = \begin{vmatrix} \frac{(s+1)}{(S-3)(s+1)} & 0 \\ 0 & \frac{(s-3)}{(s-3)(s+1)} \end{vmatrix}$$

$e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}] = (e^{3t} + e^{-t})\, u(t)$

(c) Using the state variable representation, implement this as an RLC circuit.

The system is non-stable therefore it can not be implemented as an RLC circuit.

Problem No. 2

(a) Derive the expression for the Z-Transform of $x(n) = na^{-n}\,u(n)$.

The Z - transform of x(n) is defined as

$X(z) = \Sigma\, a^n\,(n)z^{-n}$ (n = 0 to ∞)

The solution can be derived from $\Sigma\, a^n\, z^{-n}$ (n = 0 to ∞) = $1/(\,1/a\, z^{-1}\,)$

Differentiating with respect to gives

$\Sigma\, a^n\,(-n)z^{-n-1} \;=\; a\, z^{-2} /(\,1/a\, z^{-1}\,)^2$

Multiplying by -z gives puts the summation equation and thus the solution in the form needed.

$\Sigma\, a^n\,(n)z^{-n} \;=\; -a\, z^{-1} /(\,1/a\, z^{-1}\,)^2$

(b) For the transfer function, $H(z) = \dfrac{1 - (½)z^{-1}}{1 - (3/2)z^{-1} - z^{-2}}$, find the closed form expression for h(n).

Expanding $H(z) = a/(½ - z^{-1}) + b/(2 + z^{-1})$

$a = \lim z\text{->}1 \;\; (1 - ½\, z^{-1})/(2 + z^{-1}) = 1/6$

$b = \lim z\text{->}1\; (1 - ½\, z^{-1})/(½ - z^{-1}) = -1$

$H(z) = 1/6/(½ - z^{-1}) + -1/(2 + z^{-1})$

$H(z) = (1/3) / (\,1 - 2\, z^{-1}) + (-½) / (1 + ½z^{-1})$ $\qquad 1/(1\text{-}a\, z^{-1}) = a^n u(n)$

$H(n) = [1/3\,(2)^n - ½(-½\,)^n]u(n)$

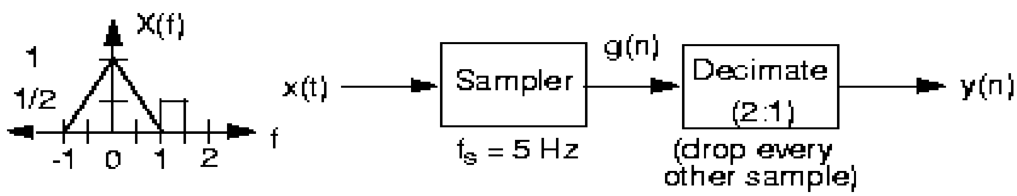
(c) Is the system stable?

For $H(n) = [1/3\,(2)^n - ½(-½\,)^n]u(n)$ As n goes to infinity so does $(2)^n$ therefore the system is non-stable.

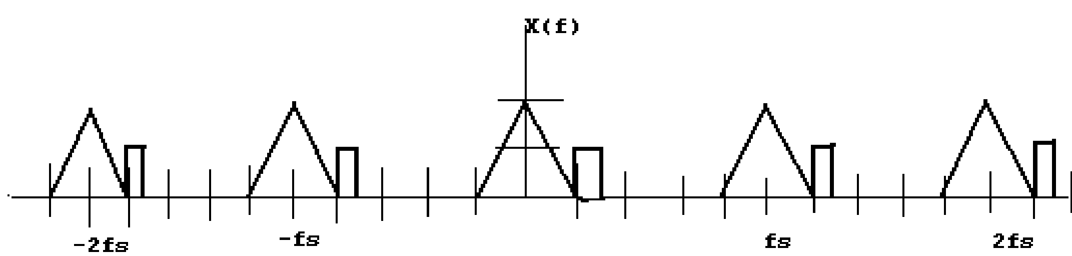
(d) Is the system causal? Explain.

The system is causal. H(n) is defined as being zero for n < 0 there for the system is causal.

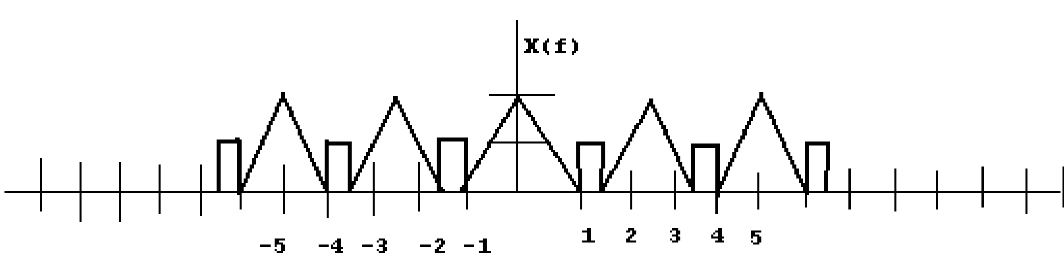
Problem No. 3: For the system shown:



(a) Plot the spectrum of g(n):



(b) Plot the spectrum of y(n).



(c) How do you explain the fact that the sample frequency of y(n) is less than the Nyquist rate, yet there is no distortion?

The spectrum of g(n) has a period of 1/2.5 and a frequency of 5 Hz. Dropping every other sample of g(n) results in a spectrum in where the sample frequency is 2.5Hz. Because the period of the spectrum of the original signal is only 1/2.5, overlapping or aliasing does not occur.