Name:

| Problem | Points | Score |
|---|---|---|
| 1a | 10 | |
| 1b | 10 | |
| 1c | 10 | |
| 1d | 10 | |
| 2a | 10 | |
| 2b | 10 | |
| 2c | 10 | |
| 2d | 10 | |
| 3a | 10 | |
| 3b | 10 | |
| Total | 100 | |

Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

**Problem No. 1**: Modeling Concepts

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy signal or power signal.

The definition states that $x(t) = te^{-\alpha t}u(t)$ is $\int_0^\infty |te^{-\alpha t}|^2 dt = \int_0^\infty t^2 e^{-2\alpha t} dt$

And, by using the identity: $\int_0^\infty t^m e^{-\alpha t} = \frac{m!}{\alpha^{n+1}}$

it can be clearly seen that:

$$E = \frac{1}{4\alpha^3}$$

The energy of this signal is clearly finite. Moreover, the power of the signal is zero,
Therefore, this signal is an ENERGY SIGNAL.

(b) Is the signal $x(t) = \sin^2 \omega_0 t$ periodic? If so, what is its period? If not, explain.

Foremost, a signal is periodic if it satsifies the following conditions:

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = (\sin\omega_0(t + T_0))^2$$

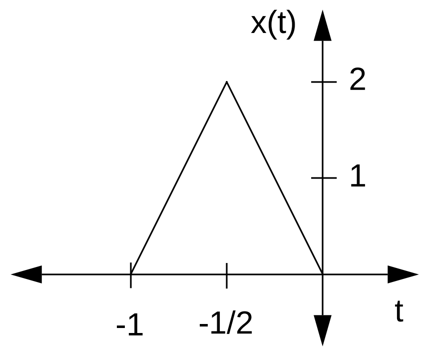
$$= \left(\sin\omega_0\left(t + 2\frac{\pi}{\omega_0}\right)\right)^2$$

$$= (\sin\omega_0 t)^2$$

The condition holds for this function, proving that this signal is periodic. The period is found by the followinng statement:

$$(\sin\omega_0 t)^2 = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t \text{ so } T_0 = \frac{\pi}{\omega_0} = \frac{1}{2f_0}$$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: $\delta(t), u(t), r(t)$.



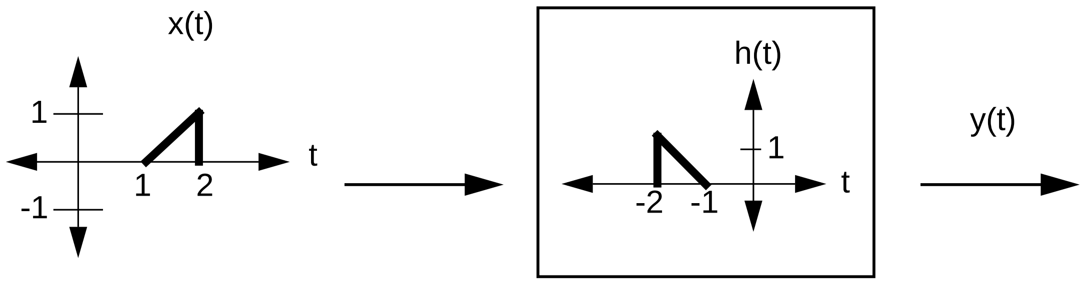

$$x(t) = 4r(t+1) - 8r\left(t + \frac{1}{2}\right) + 4r(t)$$

(d) Compute the energy value of the signal in (c).

$$E = 2\int_0^{0.5} (4t)^2 dt \quad = \quad 2\left(\left(\frac{16}{3}\right)t^3\Big|_0^{0.5}\right) \quad = 1.33 \text{ Joules}$$

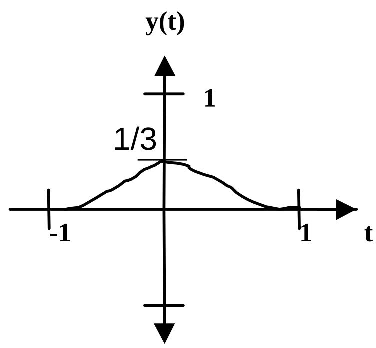
**Problem No. 2**: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):




(a) Compute and plot output, $y(t)$, for the system shown above.

The lower time limit that the output signal starts, y(t), is




(A rough representation of the function symmetric about the y axis)

equal to the lower time limit of the input, x(t), plus the lower limit of the impulse response, h(t). Since 1+(-2)=-1, -1 is where the output starts and 1 is where the output ends. The maximum amplitude of the output response is equal to the maximum area of both x(t) and a convolved h(t). The output is computed as follows:

$$y(t) = (0, (t < -1), (t > 1))$$

$$x(t) = (t-1), 1 \le t \le 2$$

$$h(t) = (t+2), -2 \le t \le -1$$

$$\int_{-2}^{(t-1)} (t-\lambda-1)d\lambda = -\frac{t^2}{6} + \frac{t}{2} + \frac{1}{3}, -1 \le t \le 0$$

$$-\int_{(t-2)}^{-1} (t-\lambda-1)(\lambda+1)d\lambda = \frac{t^2}{6} - \frac{t}{2} + \frac{1}{3}, 0 \le t \le 1$$

(b) Without using the answer to part (a), explain whether the system is causal.

The system is clearly non-causal because h(t) is not 0 for negative values of time. The impulse response's initial position is before t=0 and the input starts after t=0, and both signals are alike when the impulse response is flipped.

(c) Use your answer to part (a) to support your reasoning given in (b).

Looking at the graphs, output y(t) anticipates input x(t). As seen from the convolving of both signals, the output begins at t=-1, while the input starts at t=1. Therefore, the convolving of both the input and the impulse response reinforces the definition of noncausality.

(d) Describe the system using as many system modeling concepts as possible (for example, linearity). Justify your answers.

Time invariant - the output is the same regardless of ohen time starts

Aperiodic - the output experiences no periodic behavior

Instantaneous - there are no memory elements; the system does not depend on past values of the input

Linear - superposition holds

Non-causal - explained above

Continuous-it is defined at every point

**Problem No. 3**: Fourier Series

Given the signal $x(t) = 3\sin(1.5\omega_1 t) + 5\cos(1.75\omega_1 t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}). Be careful and be precise :)

One can see that the signal has no constant, or DC value. The signal moves about zero with no offset, so this signal has an average value of zero. Therefore, $a_0 = 0$ , and all other coefficients are nonzero. This is holds true by the following statements:

If x(-t) = x(t), then the function is even, and if x(-t) = -x(t), then the function is odd.

$$x(t) = (3\sin(1.5\omega_1 t)) + (5\cos(1.75\omega_1 t))$$
$$x(-t) = (3\sin(-1.5\omega_1 t)) + (5\cos(-1.75\omega_1 t))$$
$$-x(t) = -(3\sin(1.5\omega_1 t)) - (5\cos(1.75\omega_1 t))$$

x(-t) does not equal x(t), and x(-t) does not equal -x(t); therefore, neither $a_n$ nor $b_n$ will be zero.

(b) Compute the Fourier series coefficients.

The Trigonometric Fourier Series for this signal is:

$$x(t) = a_0 + \sum_{n=1}^{\infty}((a_n \cos n\omega_0 t) + (b_n \sin n\omega_0 t))$$

$$x(t) = (3\sin 1.5\omega_1 t) + (5\cos 1.75\omega_1 t)$$

The trigonometric fourier series is periodic with a fundamental frequency, $f_0 = \frac{f_1}{4}$, which can also be written as $\omega_1 = 4\omega_0$. One can then determine the Fourier coefficients of the series. From different integration techniques:

$$I_1 = \int_{T_0}(\sin m\omega_0 t)\sin n\omega_0 t dt = 0, m \neq n$$

and

$$I_1 = \frac{T_0}{2}, m = n \neq 0$$

$$I_2 = \int_{T_0} (\cos m\omega_0 t)(\cos n\omega_0 t) dt = 0, m \neq n$$

and

$$I_2 = \frac{T_0}{2}, m = n \neq 0$$

one knows that $1.75\omega_1 t = n\omega_0 t$, and so the following is true:

$$1.75\omega_1 = n\omega_0$$

$$n = 1.75\frac{\omega_1}{\omega_0}$$

$$n = 1.75\frac{(4\omega_0)}{\omega_0}$$

$$n = 7$$

Therefore, $a_7 = \frac{T_0}{2}$.

As for $b_n$, we see that:

$$1.5\omega_1 = n\omega_0$$

$$n = 1.5\frac{\omega_1}{\omega_0}$$

$$n = 1.5\frac{(4\omega_0)}{\omega_0}$$

$$n = 6$$

Therefore, $b_6 = \frac{T_0}{2}$. This clearly states onlytwo coefficients for this series, $a_7$ and $b_6$. All the other coefficients are zero.