Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

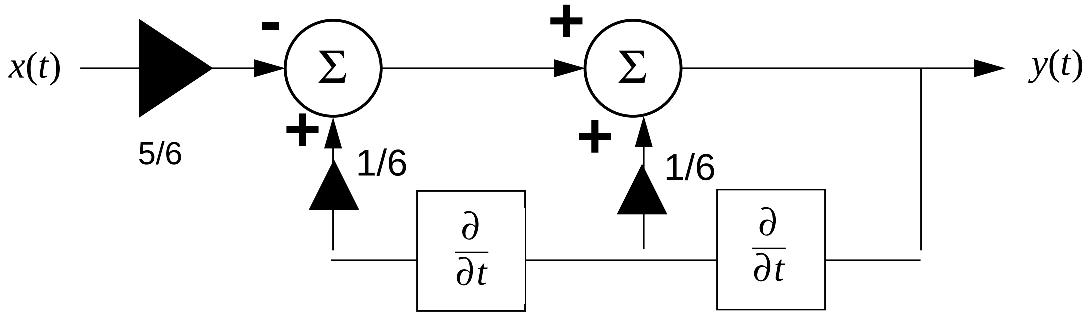
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Find the transfer function of this system using Laplace transforms.

$$y(t) = -\frac{5}{6}x(t) + \frac{1}{6}\frac{d}{dt}y(t) + \frac{1}{6}\frac{d^2}{dt^2}y(t)$$

By taking the Laplace transform:

$$Y(s) = -\frac{5}{6}X(s) + \frac{1}{6}sY(s) + \frac{1}{6}s^2Y(s)$$

$$Y(s) - \frac{1}{6}sY(s) - \frac{1}{6}s^2Y(s) = -\frac{5}{6}X(s)$$

$$Y(s)\left(1 - \frac{1}{6}s - s^2\right) = -\frac{5}{6}X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \left(-\frac{\frac{5}{6}}{1 - \frac{1}{6}s - s^2}\right) = \frac{5}{s^2 + s - 6} = \frac{5}{(s+3)(s-2)}$$

(b) Find the impulse response.

$$H(s) = \frac{5}{((s+3)(s-2))}$$

Using Partial Fraction Expansion:

$$\frac{5}{((s+3)(s-2))} = \frac{A}{s+3} + \frac{B}{s-2}$$

$$\frac{5}{(s-2)} = A + B\frac{(s+3)}{(s-2)}\bigg|_{s=-3} \text{ so } A = -1$$

$$\frac{5}{s+3} = A\frac{(s-2)}{s+3} + B\bigg|_{s=2} \text{ so } B = 1$$

$$H(s) = \frac{1}{s-2} - \frac{1}{s+3} \text{ which yields: } h(t) = (e^{2t} - e^{-3t})u(t)$$

(c) Determine whether the system is stable or unstable. Show ALL work — the correct answer with no supporting work gets no points. Be as detailed as possible.

$$H(s) = \frac{5}{((s+3)(s-2))}$$

A system is stable iff none of the poles of the transfer function are in the right half plane (RHP). By solving for the roots of s in the denominator, we see that there exists a root, s=2, in the RHP. Therefore, the system is unstable.

(d) Sketch the frequency response of the system using Bode plots.

Analyzing the transfer function, H(s), we can calculate the poles and zeros associated with it which are necessary to sketch the frequency response of the system using a Bode plot:

Zeros:

There is a constant, K, value:	K = 5/6
Therefore, to find the dc offset in dB:	20 log(5/6) = -1.584 dB

Poles:

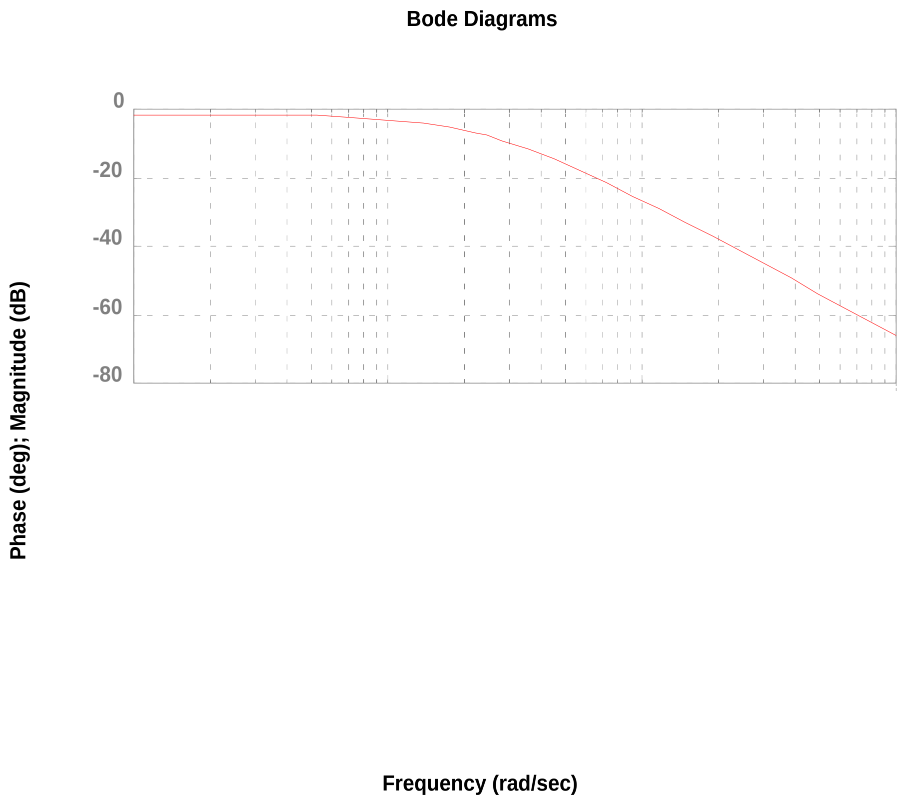
There are two simple poles: $\omega\tau = 2, -3$

The negative break frequency is simply mirrored about the zero frequency axis.

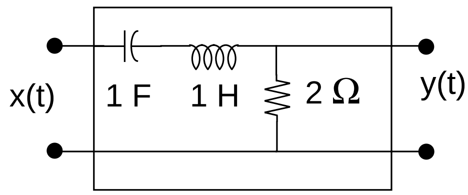
The frequency response for both poles is -20 dB/decade.

* $\omega\tau$ is the break frequency

The two poles essentially compound to form an asymptote at 2 with a slope of -20 dB/decade, and an asymptote at 3, with a slope of -20 dB/decade. The asymptote at 3 adds to the asymptote from 2 to form a combined asymptote with a slope of -40 dB/decade beginning at 3. The resulting magnitude of the frequency response is going to be a curve that approximates the asymptotes mentioned above. The largest errors between the approximated asymptotes and the actual curve of the frequency response of the system will occur at 2, where there is a -3 dB difference between the asymptote and the actual curve; and at 3, where there is a -6 dB difference between the asymptote and the actual curve of the frequency response. A Bode plot was generated using Matlab to clearly demonstrate the frequency response of this system and is included on the next page.


Bode Diagrams
0
-20
-40
-60
-80
Phase (deg); Magnitude (dB)
Frequency (rad/sec)

Problem No. 2: Circuit analysis using Fourier transforms.



For the circuit shown above:

(a) State all the Fourier transform theorems that are invoked when you compute the transfer function of this circuit. You must give specific evidence to support each theorem described (and I must be able to understand your logic!).

Linearity: this is shown by $ax(t) + bh(t) = aX(f) + bH(f)$

Differentation: This is shown by $Z = \frac{V}{I} = \frac{\left(\frac{d}{dt}i(t)\right)}{i(t)} = j\omega L$

Integration: This is shown by $Z = \frac{V}{I} = \frac{\left(\frac{1}{C}\int i(t)dt\right)}{\left(C\frac{dv}{dt}\right)} = \frac{1}{(j\omega C)}$

(b) State and prove the Frequency Translation Theorem.

$$F\{x(t)e^{j\omega_0 t}\} = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} [x(t)e^{j\omega_0 t}]e^{-j\omega t}dt = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} xt)e^{-j(\omega - \omega_0)}dt = X(\omega - \omega_0)$$

Therefore, the righthandsideof the equation is simply the Fourier Transform of x(t) with ω replaced by $(\omega - \omega_0)$.

(c) Find the impulse response of the circuit using Fourier Transforms.

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{R}{\left(R + \left(j\omega L + \frac{1}{j\omega C}\right)\right)}$$

$$\frac{R}{\left(\frac{(j\omega RC + (j\omega)^2 LC + 1)}{j\omega C}\right)} = H(j\omega)$$

$$H(j\omega) = \frac{(j\omega RC)}{(j\omega)^2 LC + j\omega RC + 1}$$

Inputing values for the components,

$$H(j\omega) = \frac{(2j\omega)}{(j\omega)^2 + 2j\omega + 1} = \frac{(2j\omega)}{(j\omega + 1)^2}$$

Now using Partial Fraction Expansion,

$$H(j\omega) = \left(\frac{A}{j\omega + 1} + \frac{B}{(j\omega + 1)^2}\right)$$

Finding the coefficients:

$$2j\omega = A(j\omega + 1) + B$$

$A = 2$ and $B = -A = -2$

Therefore,

$$H(j\omega) = \left(\frac{2}{j\omega + 1} - \frac{2}{(j\omega + 1)^2}\right)$$

Of course, the impulse response of the system is found by taking the inverse Fourier Transform of the transfer function:

$$h(t) = F^{-1}\{H(j\omega)\} = F^{-1}\left\{\frac{2}{j\omega + 1}\right\} - F^{-1}\left\{\frac{2}{(j\omega + 1)^2}\right\} = [2e^{-t} - 2te^{-t}]u(t)$$

Problem No. 3: The Dreaded Thought Problem

Signal to Noise (SNR) ratio is defined as the ratio of the power of a signal and the power of the noise in a system, computed on a log scale and measured in dB:

$$SNR|_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Assume the signal is given by $x(t) = \sin\omega_o t$, and the noise is given by $w(t) = e^{-\alpha|t-n|}$.

(a) Compute SNR in the time domain.

For A=1,

$$P[x(t)] = \frac{A^2}{2} = \frac{1}{2}$$

$$P[w(t)] = \int_0^1 e^{-2\alpha|t-n|}dt$$

Let n=0

$$P[w(t)] = -\frac{1}{(2\alpha)}e^{-2\alpha t}\Big|_0^1 = -\frac{1}{(2\alpha)} + \frac{1}{(2\alpha)}e^{-2\alpha}$$

$$SNR|_{dB} = 10\log\left(\frac{(P[x(t)])}{(P[w(t)])}\right) = \frac{\left(\frac{1}{2}\right)}{\left(\left(-\frac{1}{(2\alpha)}\right) + \left(\frac{1}{2\alpha}\right)e^{-2\alpha}\right)}$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(-\frac{1}{\alpha} + \frac{1}{\alpha}e^{-2\alpha}\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{1}{\left(\left(-\frac{1}{\alpha}\right)(1 - e^{-2\alpha})\right)}\right)$$

$$SNR|_{dB} = 10\log\left(\frac{\alpha}{(1-e^{-2\alpha})}\right)$$

(b) Compute SNR in the frequency domain and prove it is equivalent to the time domain calculation.

$$x(t) = \sin\omega_o t$$

$$X(s) = \frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)$$

$$\int_{-\infty}^{\infty}\left|\frac{1}{2j}\delta(f-fo) - \frac{1}{2j}\delta(f+fo)\right|^2 d$$

Since the area of a delta function is unity, we can easily evaluate this integral:

$$P(X(s)) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Now, we look at the noise signal:

$$e^{-jm\omega t}\left(\frac{1}{To}\right)\int_0^{To} e^{-\alpha t - jm\omega t}$$

We let $\omega = 2\pi fo$ where $fo = 1$ and $To = 1$ to get the equation in the form of:

$$\int_0^1 e^{-\alpha t - jm2\pi t}$$

$$\int_0^1 e^{t(-\alpha - jm2\pi)}$$

$$\frac{1}{-\alpha - jm2\pi}e^{t(-\alpha - jm2\pi)}\Big|_0^1$$

$$\frac{1}{-\alpha - jm2\pi}(1 - e^{-\alpha - jm2\pi})$$

According to Parseval's theorem, power in the frequency domain will be equal to power in the time domain. By taking the sum of the above equation squared from negative infinity to positive infinity we found this to be true.

(c) Explain how the SNR varies with ω_0 and α.

As alpha increases, the SNR also increases as shown in the graph below. Because the power of a sine wave is independent of ω_0, the SNR will not vary with change in ω_0. Therefore the signal to noise ratio is proportional to alpha but is independent of ω_0.