Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Modeling Concepts

(a) Prove whether the signal $x(t) = te^{-\alpha t}u(t)$ is an energy signal or power signal.

By definition, the Energy in $x(t) = te^{-\alpha t}u(t)$ is $\int_0^{\infty}|te^{-\alpha t}|^2 dt = \int_0^{\infty} t^2 e^{-2\alpha t}dt$

By using the identity $\int_0^{\infty} t^m e^{-\alpha t} = \frac{m!}{\alpha^{n+1}}$

we can see that:

$$E = \frac{1}{4\alpha^3}$$

It is evident that there is zero power in this signal and that the energy is finite, as shown above. Therefore, this is an energy signal.

(b) Is the signal $x(t) = \sin^2 \omega_0 t$ periodic? If so, what is its period? If not, explain.

This signal is periodic. To show this the following definition was used:

$$x(t) = x(t + T_0)$$

$$x(t + T_0) = (\sin \omega_0 (t + T_0))^2$$

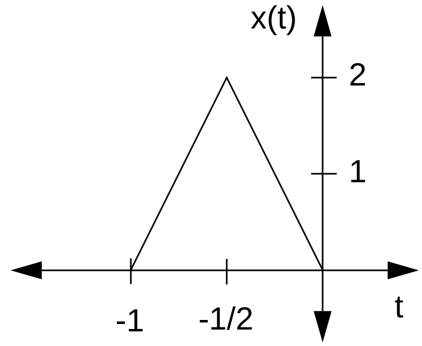
$$= \left(\sin \omega_0 \left(t + 2\frac{\pi}{\omega_0}\right)\right)^2$$

$$= (\sin \omega_0 t)^2$$

The period is found by using the following equation:

$$(\sin \omega_0 t)^2 = \frac{1}{2} - \frac{1}{2}\cos 2\omega_0 t \text{ so } T_0 = \frac{\pi}{\omega_0} = \frac{1}{2f_0}$$

(c) Write the following signal in terms of a weighted sum of a combination of one or all of the following functions: $\delta(t), u(t), r(t)$.



$$x(t) = 4r(t+1) - 8r\left(t+\frac{1}{2}\right) + 4r(t)$$

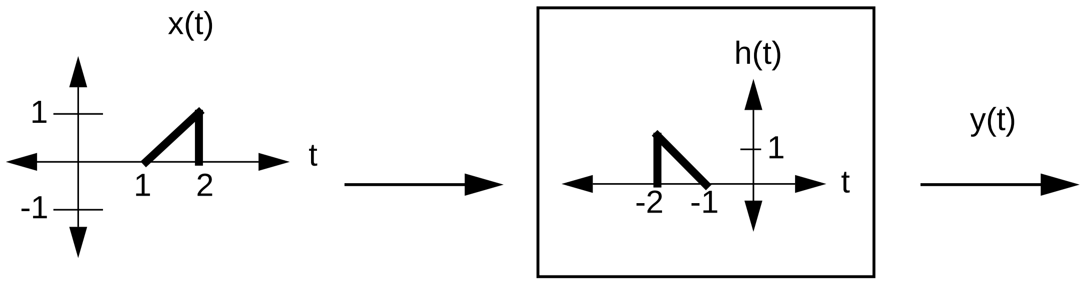
(d) Compute the energy value of the signal in (c).

The following equation is used to compute the energy in the signal:

$$E = 2\int_0^{0.5} (4t)^2 dt \quad = \quad 2\left(\left(\frac{16}{3}\right)t^3\Big|_0^{0.5}\right) \quad = 1.33 \text{ Joules}$$

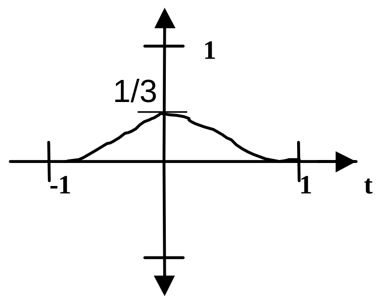
Problem No. 2: Time-Domain Solutions

Consider the signal and system (completely described by its impulse response):



(a) Compute and plot output, $y(t)$, for the system shown above.

The following is a graphical representation of the convolution of the two signals above:



This is a slightly innacurate representaion of the convolution of the above signals.

According to the definition (-2) + (1) = (-1) which is where the two signals begin to overlap so that is the lower limit of the convolution. The two signals stop overlapping at time = 1 which is shown by using the equation above. The maximum amplitude is equal to the maximum area shared between the convolution x(t) and h(t). The output is computed as shown below:

$$y(t) = (0, (t < -1), (t > 1))$$
$$x(t) = (t-1), 1 \le t \le 2$$
$$h(t) = (t+2), -2 \le t \le -1$$
$$\int_{-2}^{(t-1)} (t - \lambda - 1) d\lambda = -\frac{t^2}{6} + \frac{t}{2} + \frac{1}{3}, -1 \le t \le 0$$
$$-\int_{(t-2)}^{-1} (t - \lambda - 1)(\lambda + 1) d\lambda = \frac{t^2}{6} - \frac{t}{2} + \frac{1}{3}, 0 \le t \le 1$$

(b) Without using the answer to part (a), explain whether the system is causal.

The system is non-causal because h(t) has a non-zero value before x(t)[the input] is applied to the system.

(c) Use your answer to part (a) to support your reasoning given in (b).

From the plot y(t) in part (a) it is obvious that the output begins at t=0 and the input x(t) begins at t=1.
Hence, from the statement in part(b) both the definition and my computation agree that the system is non-causal,
or, in other words, it is anticpatory.

(d) Describe the system using as many system modeling concepts as possible (for example, linearity). Justify your answers.

Time-Invariant - no matter when time starts, the output will always be the same

Aperiodic - no period

Instantaneous - no memory elements

Linear - superposition holds

Non-causal - explained above

Problem No. 3: Fourier Series

Given the signal $x(t) = 3\sin(1.5\omega_1 t) + 5\cos(1.75\omega_1 t)$,

(a) Using symmetry arguments, explain which Fourier coefficients of the trigonometric Fourier Series will be zero ({an} and {bn}). Be careful and be precise :)

It can be seen that the signal has no constant, or DC value. The signal propagates about zero with no offset, so this signal has an average value of zero. Therefore, $a_0 = 0$, and all other coefficients are nonzero. This hypothesis is supported mathematically as shown below:

By definiton, if x(-t) = x(t), then the function is even, and if x(-t) = -x(t), then the function is odd.

$$x(t) = (3\sin(1.5\omega_1 t)) + (5\cos(1.75\omega_1 t))$$
$$x(-t) = (3\sin(-1.5\omega_1 t)) + (5\cos(-1.75\omega_1 t))$$
$$-x(t) = -(3\sin(1.5\omega_1 t)) - (5\cos(1.75\omega_1 t))$$

since x(-t) does not equal x(t), and x(-t) does not equal -x(t), neither a_n nor b_n will be zero.

(b) Compute the Fourier series coefficients.

The signal x(t) is already in standard form for a Trigonometric Fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty}((a_n \cos n\omega_0 t) + (b_n \sin n\omega_0 t))$$

$$x(t) = (3\sin 1.5\omega_1 t) + (5\cos 1.75\omega_1 t)$$

It is evident that the above series is periodic with a fundamental frequency, $f_0 = \frac{f_1}{4}$, which can be transformed into $\omega_1 = 4\omega_0$. Using this relationship, we can determine the Fourier coefficients of the series. By exploiting the properties of integrals involving the products of sines and cosines:

$$I_1 = \int_{T_0}(\sin m\omega_0 t)\sin n\omega_0 t dt = 0, m \neq n$$

and

$$I_1 = \frac{T_0}{2}, m = n \neq 0$$

$$I_2 = \int_{T_0} (\cos m\omega_0 t)(\cos n\omega_0 t)dt = 0, m \neq n$$

and

$$I_2 = \frac{T_0}{2}, m = n \neq 0$$

we know that $1.75\omega_1 t = n\omega_0 t$, so the following is true:

$$1.75\omega_1 = n\omega_0$$

$$n = 1.75\frac{\omega_1}{\omega_0}$$

$$n = 1.75\frac{(4\omega_0)}{\omega_0}$$

$$n = 7$$

Therefore, $a_7 = \frac{T_0}{2}$.

Using the same argument for b_n, we see that:

$$1.5\omega_1 = n\omega_0$$

$$n = 1.5\frac{\omega_1}{\omega_0}$$

$$n = 1.5\frac{(4\omega_0)}{\omega_0}$$

$$n = 6$$

Therefore, $b_6 = \frac{T_0}{2}$. So there are only two coefficients for this series, a_7 and b_6. All other coefficients are zero.