Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem.

Problem No. 1: For the linear time-invariant system: $H(s) = \dfrac{s+2}{(s^2-2s-3)}$

(a) Find the state variable description of the system.

$$H(s) = \frac{s+2}{s^2-2s-3}$$

$$\frac{Y(s)}{U(s)} = \frac{s+2}{(s-3)(s+1)}$$

$$= \frac{A}{s-3} + \frac{B}{S+1}$$

By using partial fractions we found that A=(5/4) and B=(-1/4)

$$\frac{Y(s)}{U(s)} = \frac{5/4}{s-3} - \frac{1/4}{s+1}$$

$$X1(s) = \frac{U(s)}{s-3}$$

$$X2(s) = \frac{U(s)}{s+1}$$

$$\begin{bmatrix} \dot{X}1 \\ \dot{X}2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U$$

$$y = \begin{bmatrix} \frac{5}{4} & -\frac{1}{4} \end{bmatrix} \begin{bmatrix} X1 \\ X2 \end{bmatrix}$$

(b) Compute the state transition matrix, $\Phi(t)$.

$$\bar{\Phi}(t) = e^{\bar{A}t}$$

To find this, we can use the relationship

$$\bar{\Phi}(s) = (s\bar{I} - A)^{-1}$$

and take the inverse Laplace Transform to find $\bar{\Phi}(t)$.

$$(s\bar{I} - A)^{-1} = \begin{bmatrix} (s-3) & 0 \\ 0 & (s+1) \end{bmatrix}^{-1}$$

$$= \frac{1}{(s-3)(s+1)} \begin{bmatrix} (s+1) & 0 \\ 0 & (s-3) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+1}{(s-3)(s+1)} & 0 \\ 0 & \frac{s-3}{(s-3)(s+1)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s-3} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

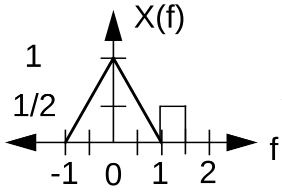
We can now take the inverse Laplace transform to obtain:

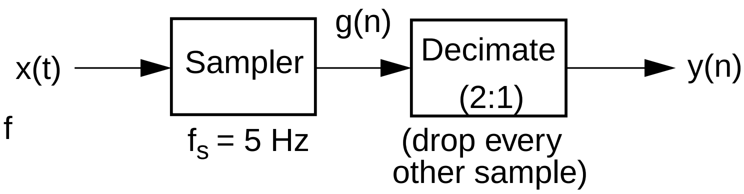
$$\Phi(t) = \begin{bmatrix} e^{3t} & 0 \\ 0 & e^{-t} \end{bmatrix}$$

(c) Using the state variable representation, implement this as an RLC circuit.

This cannot physically be accomplished because the system is unstable!

Problem No. 2: This problem deals with various aspects of Z-Transforms.





(a) **Derive** the expression for the Z-Transform of $x(n) = na^{-n}u(n)$.

$$X(z) = \sum_{n=0}^{\infty} x(nT)z^{-n}$$

Differentiating with respect to z gives

$$\frac{d}{dz}X(z) = \sum_{n=0}^{\infty} x(nT)(-n)z^{-n-1}$$

Multiplying through by -z gives us

$$-z\frac{d}{dz}X(z) = \sum_{n=0}^{\infty} [nx(nT)]z^{-n}$$

By using entry 3 in Table 8-1

$$\sum_{n=0}^{\infty} e^{-anT}z^{-n} = \frac{1}{1-e^{-aT}z^{-1}}$$

Differentiating with respect to z gives

$$\sum_{n=0}^{\infty} e^{-anT}(-n)z^{-n-1} = \frac{-e^{-aT}z^{-2}}{(1-e^{-aT}z^{-1})^2}$$

Multiplying by -z yields

$$\sum_{n=o}^{\infty} ne^{-anT}z^{-n} = \frac{e^{aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$$

Multiplying through by T and letting a=0 gives

$$\sum_{n=0}^{\infty}(nT)z^{-n} = \frac{Tz^{-1}}{(1-z^{-1})^2}$$

(b) For the transfer function, $H(z) = \frac{1-(1/2)z^{-1}}{1-(3/2)z^{-1}-z^{-2}}$, find a closed-form expression for $h(n)$ (don't use long division).

$$\frac{h(z)}{z} = \frac{z-\left(\frac{1}{2}\right)}{(z-2)\left(z-\frac{1}{2}\right)} = \frac{A}{z-2}+\frac{B}{z+\frac{1}{2}}$$

Using partial fractions and solving for A and B we get

$$\frac{H(z)}{z} = \frac{\frac{3}{5}}{z-2}+\frac{\frac{2}{5}}{z+\frac{1}{2}}$$

$$H(z) = \frac{3}{5}\left(\frac{1}{1-2z^{-1}}\right) + \frac{2}{5}\left(\frac{1}{1+\frac{1}{2}z^{-1}}\right)$$

$$h(n) = z^{-1}\{H(z)\}$$

$$h(n) = \left[\frac{3}{5}(2)^2 + \frac{2}{5}\left(-\frac{1}{2}\right)^n\right]u(n)$$

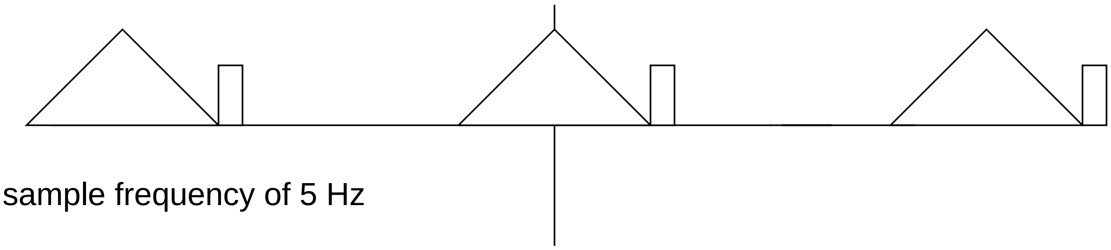
(c) Is the system stable?

This system is not stable. There exists a pole at z=2 which lies outside the unit circle in the z-plane. The poles of a system must be inside the unit circle for the system to be stable.

(d) Is the system causal? Explain.
This system is causal. It is multiplied by a unit step function which means that the output does not start until t=0. Therefore the system is non-anticipatory or causal.

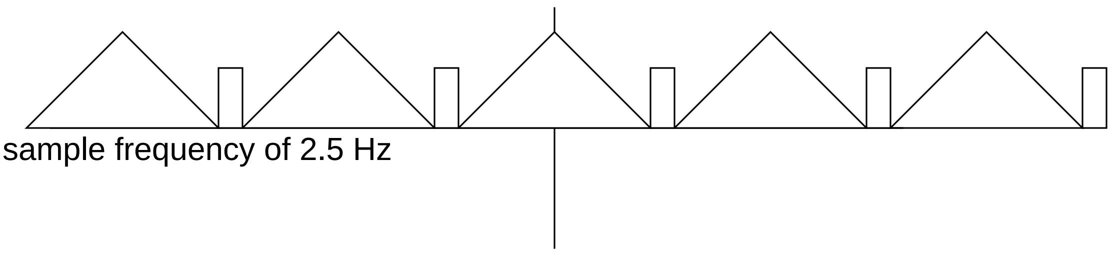
Problem No. 3: For the system shown:

(a) Plot the spectrum of g(n):



sample frequency of 5 Hz

(b) Plot the spectrum of y(n):



sample frequency of 2.5 Hz

(c) How do you explain the fact that the sample frequency of y(n) is less than the Nyquist rate, yet there is no distortion?

The sampling thereom states that for a signal to be perfectly reconstructed from its sampled values, the bandwidth of the signal must be less than or equal to one-half the sampling frequency. The Nyquist Rate is $2f_c$, and in this case f_c=2.5 Hz and f_s=2.5 Hz. Because the bandwidth of the signal is 2.5 Hz and the sampling frequency is 2.5 Hz, there will be no aliasing of the signal and therefore no distortion of the signal. This means the Nyquist Rate is not the final determining factor as to whether or not the signal can be reproduced from its sampled values.