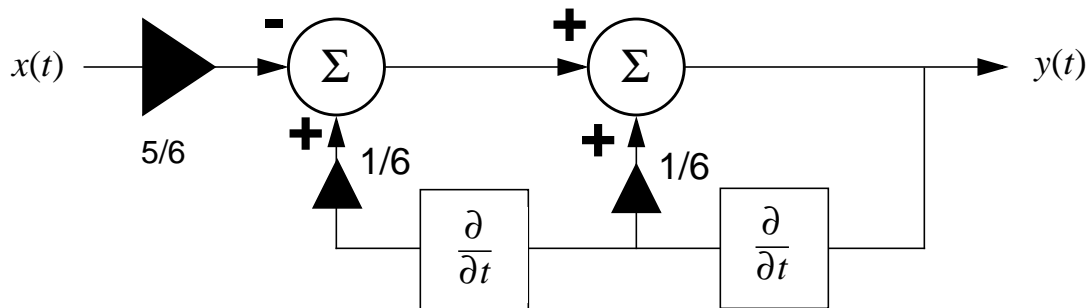


Name: Thomas Pitts

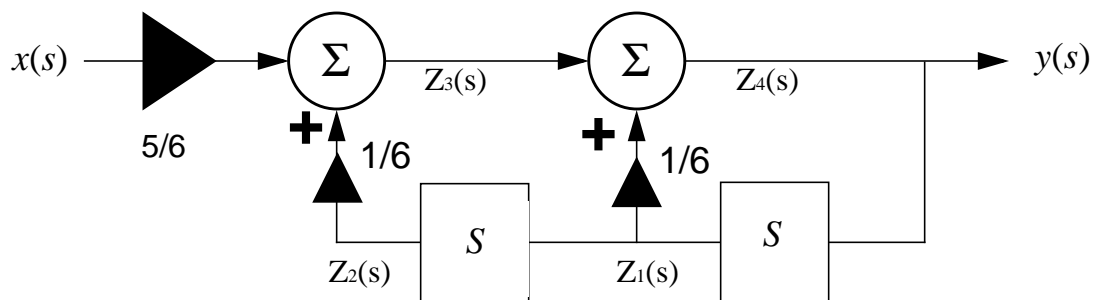
Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams

(a) Find the transfer function of this system using Laplace transforms.



$$Y(s) = Ay(s)$$

$$A_1(s) = Y(s)S$$

$$A_2(s) = S^2Y(s)$$

$$A_3(s) = -\frac{S}{6}X(s) + \frac{S^2}{6}Y(s)$$

$$A_4(s) = A_3(s) + \frac{(A_1(s))}{6} = -\frac{S}{6}X(s) + \frac{S^2}{6}Y(s) + \frac{S}{6}Y(s)$$

$$Y(s) = \frac{S^2}{6}Y(s) + \frac{S}{6}Y(s) - \frac{S}{6}X(s)$$

Solving for the transfer function:

$$1 = \frac{S^2}{6} + \frac{S}{6} - \frac{(5X(s))}{6Y(s)}$$

From this we get:

$$\frac{5(X(s))}{6 Y(s)} = \frac{S^2}{6} + \frac{S}{6} - 1$$

Taking the reciprocal of both sides, gives us.

$$\frac{5(Y(s))}{6 X(s)} = \frac{6}{s^2} + \frac{6}{s} - 1$$

This gives us a transfer function of:

$$H(s) = \frac{(Y(s))}{X(s)} = \frac{5}{s^2 + s - 6}$$

(b) Find the impulse response.

$$H(s) = \frac{5}{(s+3)(s-2)} = \frac{(As+B)}{(s+3)(s-2)}$$

After using partial fractions, we find that A = -1 and B = 1.

Therefore, we have a transfer function of:

$$H(s) = -\frac{1}{s+3} + \frac{1}{s-2}$$

Taking the Laplace transform we get:

$$L\{H(s)\} = H(t) = [(-e^{-3t}) + (e^{2t})]u(t)$$

(c) Determine whether the system is stable or unstable. Show ALL work.

The correct answer with no supporting work gets no points. Be as detailed as possible.

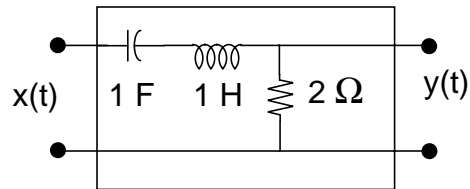
After examining this system, we find that it has poles of -3 and 2. Therefore, the system is unstable because of the 2 being in the r.h.p.

(d) Sketch the frequency response (magnitude only) of the system using Bode plots.

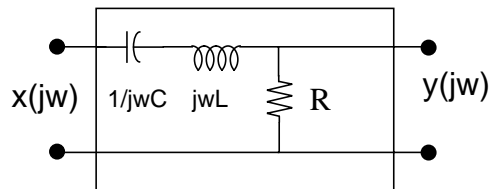
Wasn't sure about this one.

Problem No. 2: Circuit analysis using Fourier transforms.

For the circuit shown below:



- (a) State all the Fourier transform theorems that are invoked when you compute the transfer function of this circuit. You must give specific evidence to support each theorem described (and I must be able to understand your logic!).



Superposition: Used when adding Inductance and Capacitance.

$$\frac{1}{j\omega C} + j\omega L = \frac{(1 - LC\omega^2)}{j\omega C}$$

Differentiation:

$$\frac{\partial}{\partial t} i(t) = j\omega I(j\omega)$$

Integration:

$$\left(-\frac{1}{C}\right) \int_{-\infty}^t i(t) dt = I \frac{j\omega}{j(\omega)} + \frac{1}{2} x(0) \zeta(j\omega)$$

- (b) State and prove the Frequency Translation Theorem.

$$\mathfrak{F}\{x(t)e^{j\omega_0 t}\} = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} [x(t)e^{j\omega_0 t}] e^{-j\omega t} dt = X(\omega - \omega_0)$$

$$\int_{-\infty}^{\infty} x(t) e^{-j(\omega - \omega_0)t} dt = X(\omega - \omega_0)$$

The right hand side is the Fourier transform of $x(t)$ with w replaced by $(w-w_0)$

(c) Find the impulse response of the circuit using Fourier Transforms.

$$I(j\omega) = Y \frac{(j\omega)}{X(j\omega)} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{(j\omega RC)}{((j\omega)^2)LC + j\omega RC + 1}$$

After we input our values:

$$I(j\omega) = \frac{(2j\omega)}{((j\omega)^2) + 2j\omega + 1} = \frac{(2j\omega)}{(j\omega + 1)^2} = \frac{A}{j\omega + 1} + \frac{B}{(j\omega + 1)^2}$$

Now, find coefficients using partial fractions.

$$2j\omega = A(j\omega + 1) + B, A = 2, B = -2$$

This gives us:

$$H(j\omega) = \frac{2}{j\omega + 1} - \frac{2}{(j\omega + 1)^2}$$

Performing the inverse Fourier gives us:

$$h(t) = \mathcal{F}^{-1}\{H(j\omega)\} = \mathcal{F}^{-1}\left\{\frac{2}{j\omega + 1}\right\} - \mathcal{F}^{-1}\left\{\frac{2}{(j\omega + 1)^2}\right\} = (2e^{-t} - 2te^{-t})u(t)$$

Problem No. 3: The Dreaded Thought Problem

Signal to Noise (SNR) ratio is defined as the ratio of the power of a signal and the power of the noise in a system, computed on a log scale and measured in dB:

$$SNR|_{dB} = 10\log_{10}\left(\frac{P_{signal}}{P_{noise}}\right)$$

Assume the signal is given by $x(t) = \sin\omega_0 t$, and the noise is given by $w(t) = e^{-\alpha|t|}$.

(a) Compute SNR in the time domain.

$$P_{signal} = \text{Power of Sine Wave} = \frac{A^2}{2}$$

$$P_{noise} = \frac{1}{T} \int_{-\infty}^{\infty} e^{-\alpha|t-n|} dt = \frac{1}{T} \int_{-\infty}^{\infty} e^{-\alpha|t-n|} dt$$

This signal is periodic, so we can take the integral over one period. $T=1$.

Therefore:

$$P_{noise} = \int_0^1 e^{-2\alpha|t-n|} dt$$

Letting $n=0$, we get:

$$P = -\frac{1}{2\alpha} e^{-2\alpha 1} + \frac{1}{2\alpha} e^{-2\alpha 0} = \frac{1}{2\alpha} ((e^{-\alpha}) - 1)$$

$$SNR = 10\log\left(\frac{\left(\frac{A^2}{2}\right)}{\frac{((e^{-2\alpha}) - 1)}{\alpha}}\right) = 10\log\frac{\alpha}{((e^{-2\alpha}) - 1)} \text{ Where } t=1.$$

(b) Compute SNR in the frequency domain and prove it is equivalent to the time domain calculation.

Compute using Parseval's Theorem

(c) Explain how the SNR varies with ω_0 and α .

The SNR is affected by alpha in that as alpha gets larger, the SNR gets large and complex. As alpha gets smaller (more negative), the SNR gets much smaller. Omega does not affect the SNR.