Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

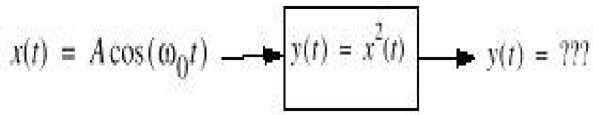
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series



$$a0 := \frac{1}{t0}\cdot\int_0^{t0} a^2\cdot\cos\left(\frac{2\cdot\pi}{t0}\cdot t\right)^2 dt$$

$$\frac{1}{t0}\cdot\int_0^{t0} a^2\cdot\cos\left[\left(\frac{2\cdot\pi}{t0}\right)\cdot t\right]^2 dt$$

$$\frac{1}{(4\cdot t0)}\cdot\left(\cos\left(2\cdot\pi\cdot\frac{t0}{t0}\right)\cdot\sin\left(2\cdot\pi\cdot\frac{t0}{t0}\right)\cdot t0 + 2\cdot\pi\cdot t0\right)\cdot\frac{a^2}{\pi}$$

$$\frac{a^2}{2}$$

In this case complex math is not needed to find an or bn. It should be noted that the original signal is a sinusoidal signal and therefore can be completely represented as sinusoid. We must only extrapolate the fourier coeficients by inspection.

We need an *a* coefficient to handle the cosine. The cosine component will occur at an angular frequency of 2*ω_0. Therefore, we need an *a2* term. There is no sine term so we do not need a *b* coefficient.

The final fourier series is an exact representation of the original signal:

$$y(t) := \frac{a^2}{2}\cdot(1 + \cos(2\cdot w0\cdot t))$$

Using trigonometric identities, this equation can easily be transformed back into the original problem statement.

$$y(t) := a^2\cdot\cos(wo\,t)^2$$

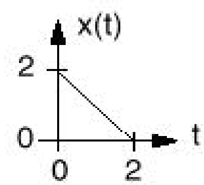
(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

$$P := \lim_{t_0 \to \infty} \frac{1}{2 \cdot t_0} \cdot \int_{-t_0}^{t_0} \left(\left| a^2 \cdot \cos\left(\frac{2 \cdot \pi}{t_0} \cdot t \right) \right| \right)^2 dt \qquad E := \lim_{t_0 \to \infty} \int_{-t_0}^{t_0} \left(\left| a^2 \cdot \cos\left(\frac{2 \cdot \pi}{t_0} \cdot t \right) \right| \right)^2 dt$$

$$P := \lim_{t_0 \to \infty} \frac{1}{2 \cdot t_0} \cdot \int_{-t_0}^{t_0} a^4 \cdot \cos\left(2 \cdot \frac{\pi}{t_0} \cdot t \right)^4 dt \qquad E := \infty$$

$$P := \frac{3}{8} \cdot a^4$$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).



$$x(t) := (-t + 2)$$

$$y(t) := (-t + 2)^2$$

Energy = E =

$$\lim_{t_0 \to \infty} \int_0^2 (|(-t + 2)|)^4 dt$$

$$E := \frac{32}{5}$$

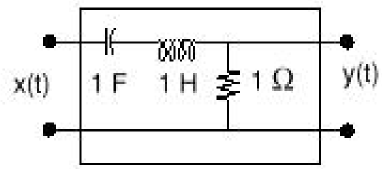
Power = P =

$$\lim_{t_0 \to \infty} \frac{1}{2 \cdot t_0} \cdot \int_0^2 (|(-t + 2)|)^4 dt$$

$$P := 0$$

Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, if x(t) is the unit step function. Explain.



<u>Answer:</u>

The output signal of this circuit is characterized by a second order differential equation. Therefore the output signal will resemble a damped sinusoid. The loop equation for this circuit is :

$$\frac{d}{dt}x(t) := L\cdot\frac{d^2}{dt^2}i + R\cdot\frac{d}{dt}i + \frac{i}{C}$$

Transferring everything to the Laplace domain the loop equation becomes:

$$x(s) := I\cdot\left(\frac{1}{s} + s + 1\right)$$

$$I(s) := \frac{u(t)}{\frac{1}{s} + s + 1}$$

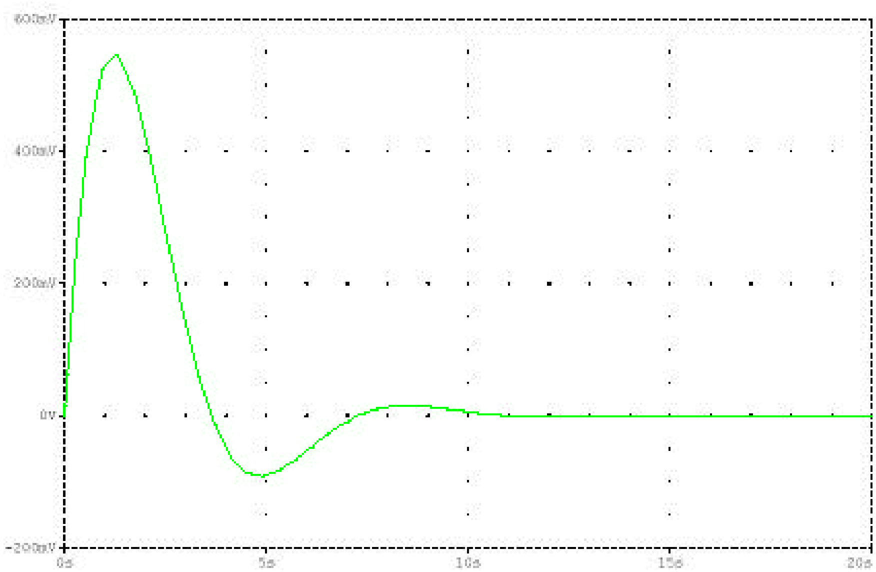
$$I(s) := \frac{s}{s^2 + s + 1} \text{ invlaplace }, s \rightarrow \frac{-1}{3}\cdot\exp\left(\frac{-1}{2}\cdot t\right)\cdot\sin\left(\frac{1}{2}\cdot\sqrt{3}\cdot t\right)\cdot\sqrt{3} + \exp\left(\frac{-1}{2}\cdot t\right)\cdot\cos\left(\frac{1}{2}\cdot\sqrt{3}\cdot t\right)$$

$$y(s) := I(s)\cdot R$$

$$y(s) := I(s)\cdot 1$$

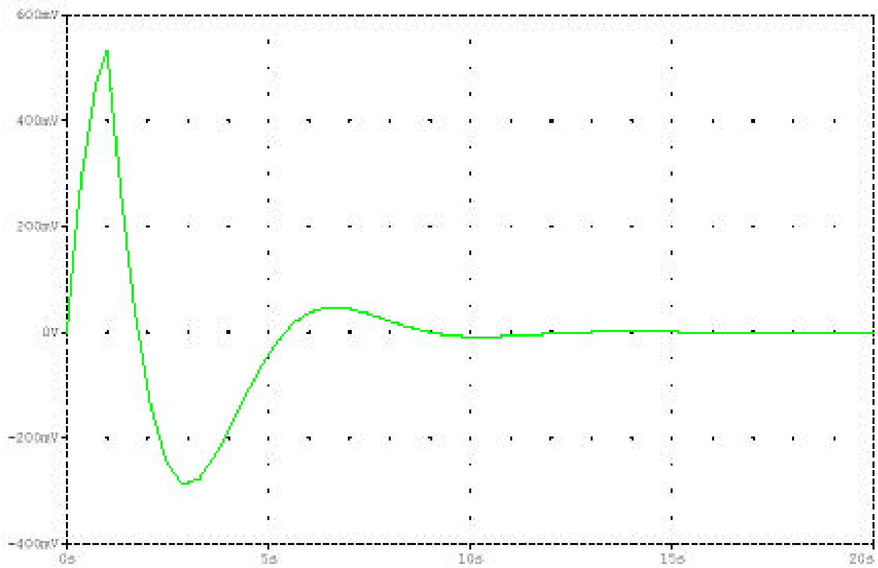
$$y(t) := I(t)$$

The output waveform should look like the following Pspice simulation.

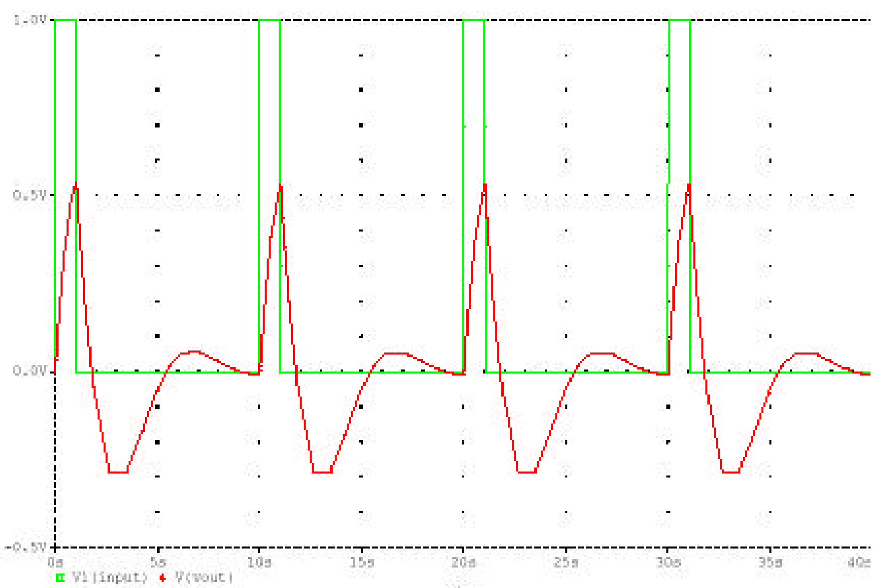


(b)Sketch the output if x(t) is a unit pulse. Explain.

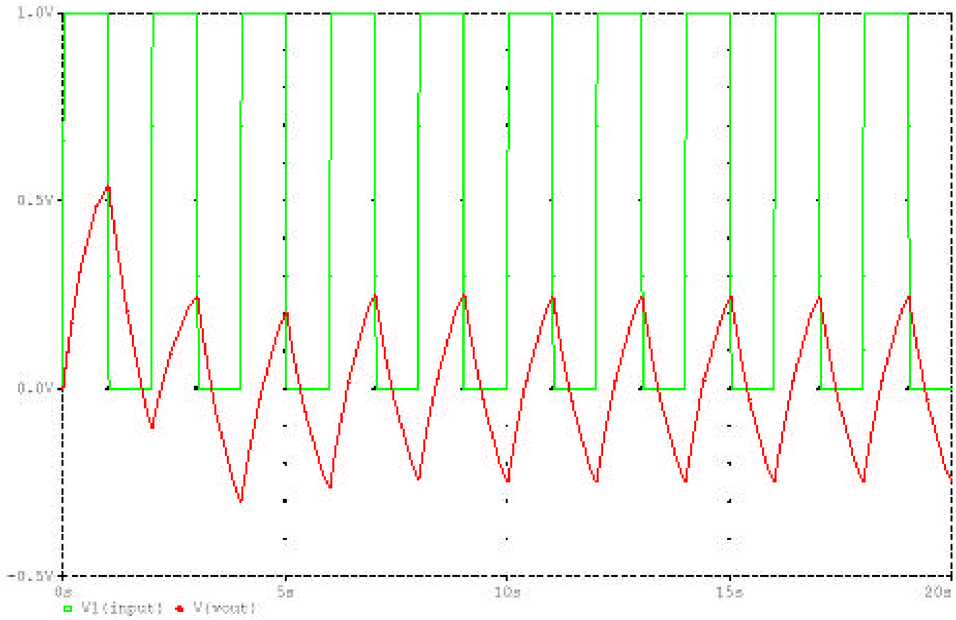
The shape of the output signal will be somewhat dependant on the width of the pulse, but for the most part the signal should still resemble a damped sinusoid.
Here is a plot of the output with x(t) = 1sec wide pulse.



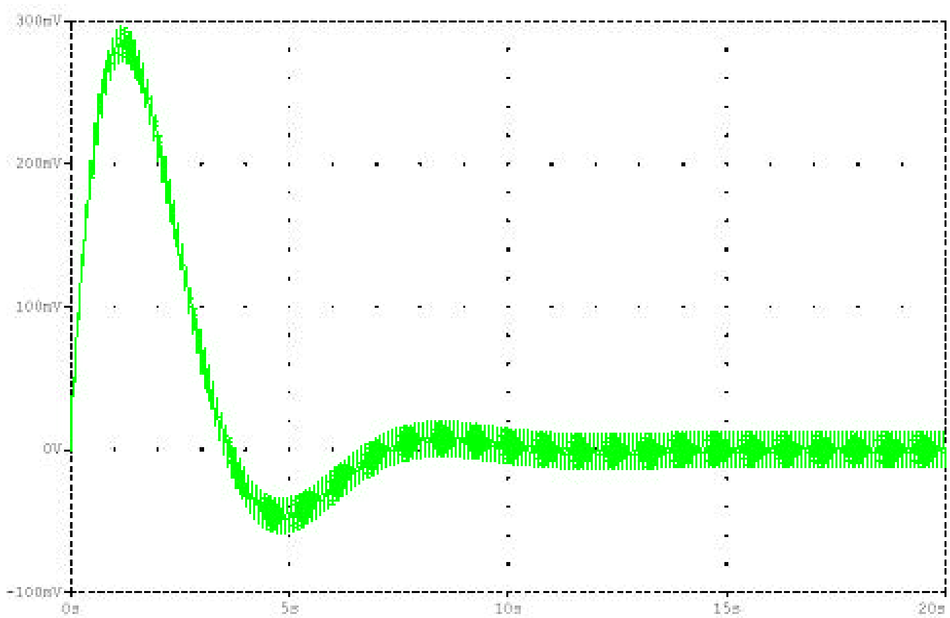
(c) Sketch the output if x(t) is a periodic sequence of unit pulses with a period of 10 seconds.
<u>Answer:</u>
The output will be a periodic signal with each pulse resembling the previous plot.



(d) Suppose the period in (c) is decreased to 2 seconds. How does this effect the shape of the output?
Plot width a period of 2 seconds and a pulse width of 1 sec. The circuit doesnt have adequet time to respond so the response is similar to what I have below.



As the pulse width decreases, the output will gradually start to look like the result of a unit step or dc input. Here is a plot of the output with a pulse width of .05 and a period of .1 seconds:



Problem No. 3: The dreaded thought problem...
(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.
Causal – the system cannot predict changes in the road, it can only react to them.

Dynamic—the output depends on previous and present values of the values of the input.

Time delayed – the system responds to an input after a time delay, not instantaneously.

Linear – the outout over time is the superposition of all the little inputs, road changes.

Continuous time- the signals processed by the system are continuous time signals

Time Invariant – the input-output relationship does not change with time.

(b) If this system were a linear time-invariant system, design the impulse response of a good" system. Explain how you might implement this in a circuit.

<u>Answer:</u>
The impulse response should resemble a low pass filter. We wouldn't want the circuit to react to every bump and pothole in the road; therefore, we want the high frequency inputs to be ignored.

In a circuit we would build a low pass filter to filter out the high frequency inputs and to process the slow changing or low frequency inputs.

(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

<u>Answer:</u>
It is not optimal because as the changes in the road become more frequent and more severe, the cruise control system will begin to react to quickly. Gas is wasted as it tries to speed up and slow down too frequently.

<u>Modifications:</u>

Build a low pass filter to filter more of the high frequency inputs and to let only the low frequency inputs through.

Make τ of h(t) longer to give some momentum, making it more dynamic.
We can try also to imitate a noncausal system by buffering the input and delaying the output. The impulse response could then be engineered to be a function of what came before and what is to come, so that the system could have a better understanding of the road, and thus make more informed decisions. Sort of like anti-skip on a portable cd-player.