Name: Matthew Phillip Gunter

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos 2\, w_o t$

identity: $\cos^2 u = ½ (1 + \cos 2u)$
applying: identity you get

$y(t) = A^2/2 + A^2 /2 \cos (2w_o t)$

knowing that cosine is an even function you get $b_n = 0$ & a_n ¹ 0
therefore you only have a_n terms

knowing that the fourier series of a sine or cosine is its own fourier series we obtain

$a_0 = A^2 /2$ $a_2 = A^2 / 2$

so the fourier series is $y(t) = a_o + a^2 \cos 2w_o t$

the answer is $y(t) = A^2 / 2 + A^2 / 2 \cos 2w_o t$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

output is $y(t) = A^2 / 2 + A^2 / 2 \cos^2 w_o t$
because the output is a periodic function with infinite area under the curve its energy is infinite.

therefore E = ¥
P = lim $_{T ® ¥}$ $1 / 2T \int_{-T}^{T} | y(t) |^2 dt$
$P = 1 /2T \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)]\, dt$

substituting wo = $2\pi / T$ the first cosine term goes to zero we then have
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)]\, dt$

using trigonometric identity from 1A on the $\cos^2$ term we get
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4/8 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos(4w_o T)]\, dt$

after substituting wo = $2\pi / T$ the cosine term goes to zero we have
$1 / 2T_o \int_{-T}^{T} (A^4 / 4 + A^4/8)$

integrating $1 / 2T_o (A^4 / 4 + A^4/8)\, 2T_o$; the $2T_o$ cancelled leaving a power of $A^4 / 4 + A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

x(t) = -t + 2 0 £ t £ 2

y(t) = x^2(t) = $(-t + 2)^2$

F = lim T ® ¥ $\int_{-T}^{T} (-t + 2)^4$ dt

using U substitution u = -t + 2
du = -dt

We have E = - $\int u^4$ du

-1/5 $(-t + 2)^5$ evaluated from [0,2]

0-(-32 / 5) = 32/5 Joules = 6.4 Joules

P = lim T ® ¥ 1 / 2T $\int_{-T}^{T}$ | t^2 – 4t + 4) $|^2$ dt

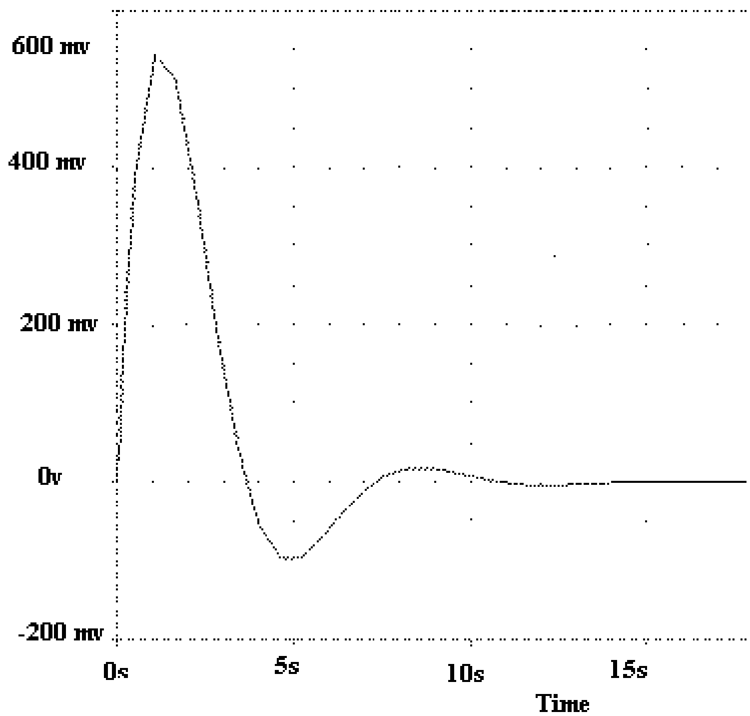
P = 0

The power is zero because when the integral is evaluted and the limit taken there will be a value of infinty in the denominator and thus the power is equal to zero.

Problem No. 2: Time-Domain Solutions

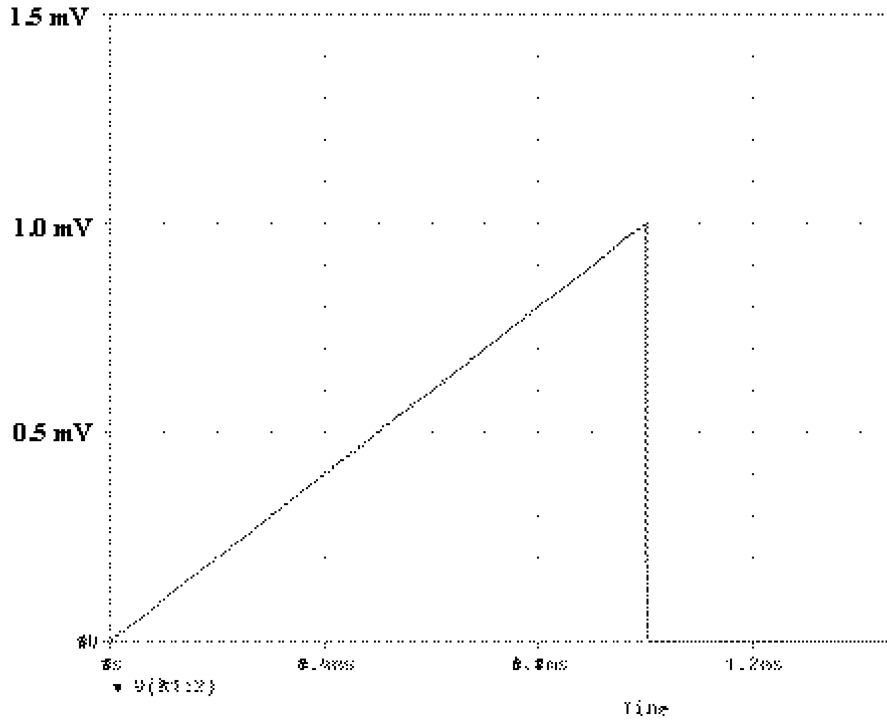
(a) For the circuit shown, using concepts developed in this class, sketch the output signal, , if x(t) is the unit step function. Explain.

$$v(t) = e^{(-a*t(b1*\cos(w_d t) + b1*\sin(w_d t))}$$



The reason y(t) looks as it does when a unit step is the input is because this signal is the natural response of the circuit. It is known that there is no voltage across an inductor if the current through it is not time varying. Also, it is impossible to change the current through an inductor by any amount instantaneously. Because the voltage across the capacitor is not time varying, there will be no current flow across it. In this case, the capacitor is an open circuit at DC. It is not possible to instantaneously change the voltage across a capacitor by a finite amount. Therefore, the steady state of this circuit at infinite time will have zero current flow through the resistor and therefore the voltage across the resistor will also be zero at infinte time. The only current that will flow through the resitor is during the transient response. This circuit is an underdamped series RLC circuit. Therefore the output on the resistor is a damped sinusoidal response.

(b) Sketch the output if x(t) is a unit pulse with width of 1 millsecond. Explain.



Assuming the pulse to have a width of 1 ms: At t= - 0.5 ms x(t) becomes 1V and remains 1V until t= 0.5ms. When x(t) becomes 1V the voltage across y(t) starts to increase as the resistance across the inductor decreases and allows current to pass through the circuit, but at t=0.5 ms. x(t) becomes 0V. PSpice says that the graph will look like the one above when voltage is no longer supplied to the circuit, but, in actuality, I believe that the y(t) voltage drop would have a slight curve as it dropped back to zero since the capacitor and inductor will have charged up and will discharge when the voltage source x(t) becomes 0V.

Note: The graphs shown have been shifted because PSpice will not take negative values for time.

(c) Sketch the output if x(t) is a train of pulses of width 1 millsecond and a period of 10 seconds.

This graph would like the one in part b, but now the output would become periodic. An output identical to part b would occur every 10 seconds. Very little overlapping of the output would occur in the T_{off} part of the pulse period.

(d) Suppose the period is dropped to 0.1 seconds. How does this affect the shape of the output?

The output would essentially become more frequent, meaning that more pulses generate more output response in less time. Because the T_{off} part of the pulse period is shortened greatly, the output would have more overlapping voltage levels after the pulse goes to zero voltage. This slight overlap in response would either add or subtract to the next pulse response depending on the polarity. To help visualize this output there would be 10 pulse responses similar to that in part 2-B over 1 second of time but probably with a slight ripple from the previous pulse response.

EXTRA CREDIT FROM E-MAIL

In digital communications a train of weighted pulse are transmitted over communication lines. If these communication lines are in the form of wires such as that used by a modem a problem may arise. This problem is an occurrence due to the baud rate and the type of wire used to relay the message. The baud rate is related to the pulse width and the period of between pulses. Transmission wire can be viewed as an RLC circuit. The inductance is usually neglected so the transmission line can be modeled as a low pass filter. This low pass filter has a response similar to that of 2-C, and 2-D. It can be seen that when the period is wide in time one does not have to worry much about the transient signals remaining after the pulse is sent. If the period is shortened then the designer has to worry about transients interfering with the next pulse in the train and distorting the signal. So to conclude this, as the baud rate increases, the period between pulses is shortened, and thus the problem of overlapping signals over the transmission line occurs. To compensate for this, the pulse widths can be shortened and the highest quality cable with the correct filtering characteristics be chosen.

Problem No. 3: The dreaded thought problem...

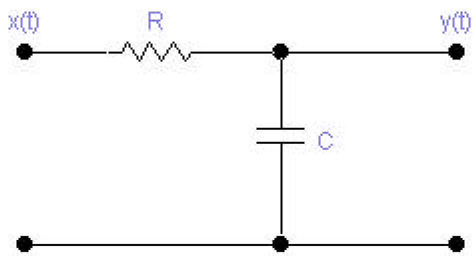
(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

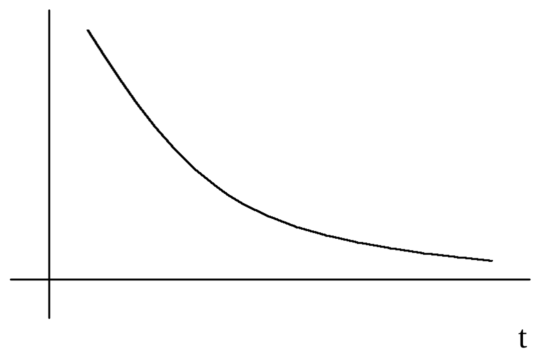
The system is causal because it does not anticipate future input. The system is dynamic because it would have to remember the set speed to be able to adjust acceleration given change in terrain. The system is linear because when you increase the speed of the car or when it receives an input you get the output to be the speed of the car plus the increase. Therefore, this system is linear due to the fact that superposition holds. The system is also fixed because the input/output relationship does not change with time. The system is also a continuous - time system because is always receiving an input signal.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

For a good cruise control system, the impulse response for the system would be an integrator where it smoothes out the input discontinuities and gives a smooth output curve.

To implement this in a circuit the signal is run through a series RC circuit as seen below. X(t) is the input Y(t) is the output. This circuit acts as a low pass filter, thus cutting off the high frequencies which can be viewed as quickly changing terrain.





smoothing impulse repsonse

(c) It is well-known that cruise controls are not optimal for terrain that is mountains (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

The mountainous terrian is too varying (i.e. it has too many ups and downs). A cruise control system is not optimal for a mountainous terrian because there are too many discontinuities over a small period of time.

To correct the problem of dynamic terrain, a circuit could be constructed that could remember the previous terrain (i.e. going up a hill) and be able to predict the upcoming terrain (i.e. going downhill). This would make this circuit a non-causal system. Another solution to this problem would be the use of a global positioning system to determine the altitude with respect to the road and then be able to calculate the necessary speed adjustments for the upcoming terrain. This also would be semi-non-causal, if this approach were used.