Name: Brad Lowe

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos^2 w_o t$

Using the identity $\cos^2 u = ½ (1 + \cos 2u)$, we get

$y(t) = A^2/2 + A^2/2 [\cos (2w_o t)]$

Since cosine is an even function we know that all sine coefficients or b_n's = 0. Further, since this function is already in the form of a trigonometric Fourier series, we can conclude that $A^2/2$ is a_0 as well as a_2, and there are no more nonzero coefficients in the series.

Thus we have the Fourier series $y(t) = a_0 + a_2 \cos 2w_o t$

where $a_0 = A^2/2$, and $a_2 = A^2/2$.

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

Since the ouput is a periodic function, we know that it is a power signal. Thus, energy is infinite.

For power:

$P = \lim_{T \to \infty} ½ T \int_{-T}^{T} | y(t) |^2 dt$
$P = ½ T \int_{-T}^{T} [A^4/4 + A^4/2 \cos(2w_0T) + A^4/4 \cos^2(2w_0T)] dt$

By using the same identity as in part A we obtain
$½ T_0 \int_{-T}^{T} [A^4/4 + A^4/8 + A^4/2 \cos (2w_0T) + A^4/4 \cos(4w_0T)] dt$

Substituting $w_0 = 2\pi/T$ the integral of the cosine terms will go to zero, and we have
$½ T_0 \int_{-T}^{T} (A^4 / 4 + A^4/8) = ½ T_0 * 2 T_0 * (A^4/4 + A^4/8)$

$= A^4/4 + A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

$x(t) = -t + 2$ $\qquad 0 \le t \le 2$

$y(t) = x^2(t) \quad = (-t + 2)^2$

$P = \lim_{T \to \infty} \int_{-T}^{T} (-t + 2)^4 \, dt$

Using a substitution, $u = -t + 2$ and $du = -dt$, we have

$P = -\int u^4 \, du$

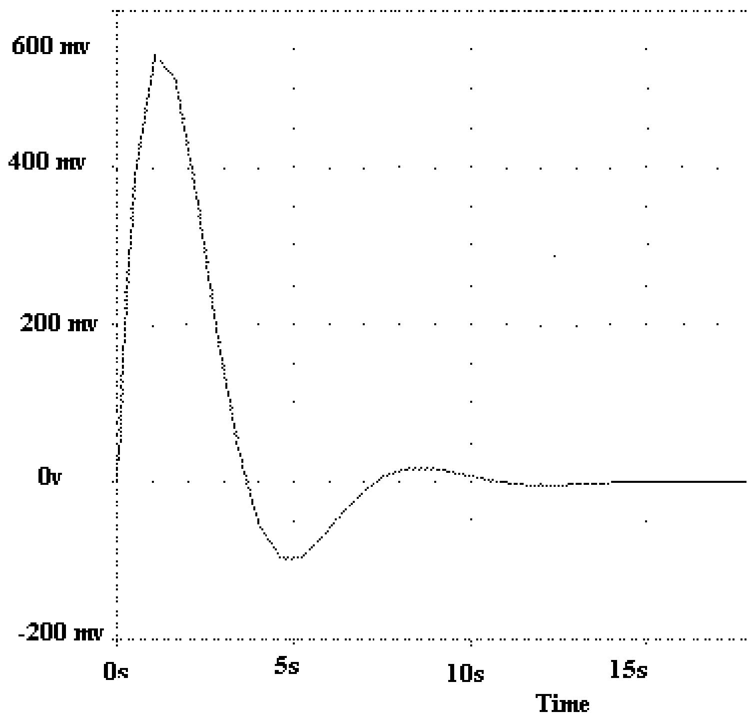
$= -1/5 \, (-t + 2)^5 \; |_0^2$

$= 0-(-32 / 5) = 32/5$ Joules $= 6.4$ Joules

Problem No. 2: Time-Domain Solutions

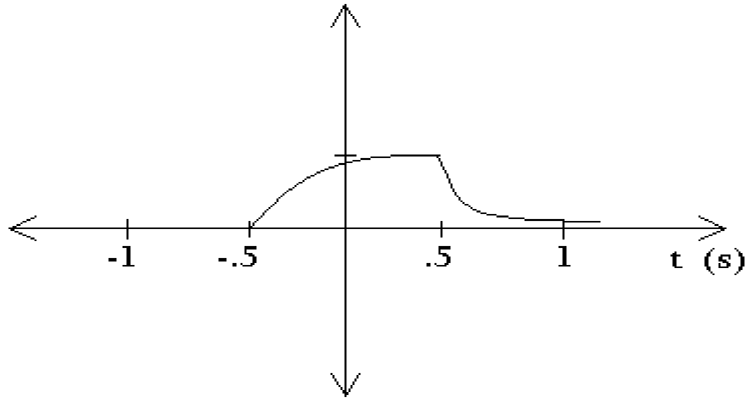
(a) For the circuit shown, using concepts developed in this class, sketch the output signal, , if x(t) is the unit step function. Explain.

$v(t) = e^{(-a*t(b1*\cos(w_d t) + b1*\sin(w_d t))}$



The reason that y(t) looks as it does when a unit step function is the input is because this signal is the natural response of the circuit. There is no voltage across an inductor if the current through it is not changing with time. Also, it is impossible to change the current through an inductor by a finite amount in zero time. There is no current passing through the capacitor if the voltage across it is not changing with time. A capacitor is an open circuit at DC. Also it is impossible to change the voltage across a capacitor by a finite amount in zero time. So, for infinite time, there will be no current flow or voltage across the resistor. The only time current will flow through the resitor is during the transient response caused by the discontinuity at t=0. This circuit is also an underdamped series RLC circuit. Therefore, a damped sinusoid is the output.

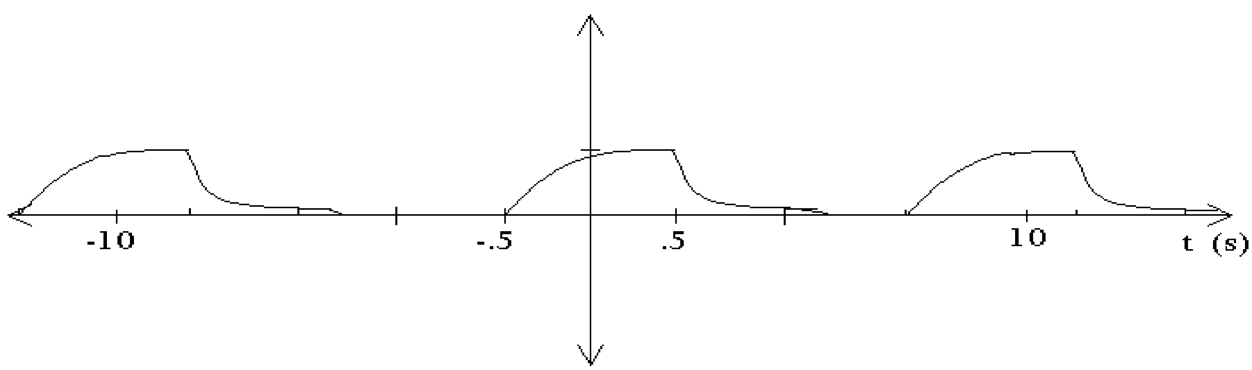
(b) Sketch the output if x(t) is a unit pulse with width of 1 millsecond. Explain.



Assuming that the pulse has a width of 1 ms, at t= - 0.5 ms, x(t) becomes 1V and remains 1V until t= 0.5ms. When x(t) becomes 1V, the voltage across y(t) increases as the resistance across the inductor decreases and lets current pass through the circuit. But at t=0.5ms, x(t) becomes 0V. The output voltage drop has a slight curve as it goes back to zero since the capacitor and inductor discharge when the voltage source x(t) becomes 0V.

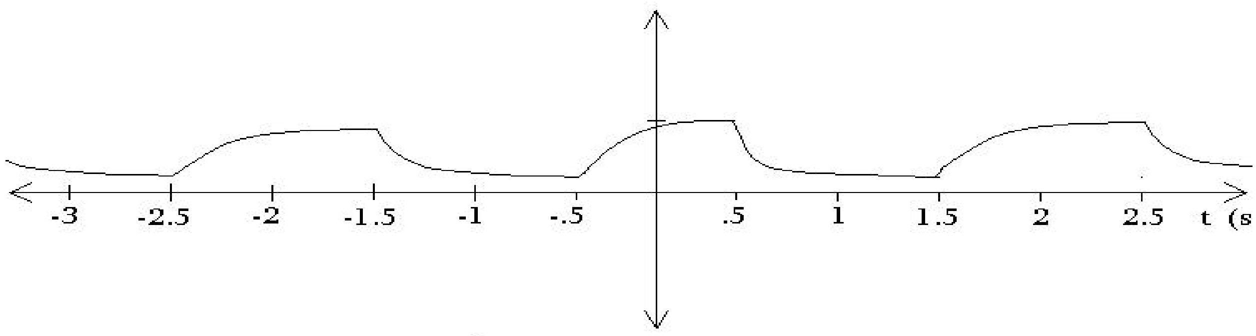
(c) Sketch the output if x(t) is a train of pulses of width 1 millsecond and a period of 10 seconds.

This graph would be like the one in part b, except the output would become periodic. An output identical to that of part b would occur every 10 seconds.



(d) Suppose the period is dropped to 0.1 seconds. How does this affect the shape of the output?

The graph would be like the one in part b, except now it is periodic, with the output in part b occuring every 2 seconds. There would also be a slight overlap.



Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

The cruise control of a car is a low pass filter because it cuts off all high-frequency inputs and creates a smooth, steady change of speed.

The system is causal because it does not anticipate future input. It is impossible for the system to anticipate the terrain it may encounter in the future.

The system is instantaneous because it does not use past values of the input to create an output.

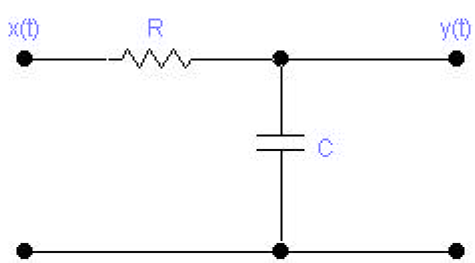
The system is also nonlinear because of fuel consumption. It takes more fuel and power to get over one large hill than to get over many small hills.

The system is fixed because the output does not change with time.

The system is a continuous-time system because it always has an input.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

For a good cruise control system, the impulse response for the sysem would be an integrator that smoothes out the discontinuities in the input and gives a smooth curve output.
To implement this in a circuit, you can put the signal through a series RC circuit as seen below.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

A cruise control system is not optimal for mountainous terrain because there is a discontinuity at the peak of a mountain. The circuit integrates the input signal to give a steady curve, but the curve will not decrease rapidly as you go over the peak, so the accelerator will still be accelerating the car for a short time after you are actually going downhill.

The circuit would have to be modified to be non-causal, so it could see if the road ahead was uphill or downhill.