Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos^2 w_o t$

Using the identity $\cos^2 u = ½ (1 + \cos 2u)$, we get

$y(t) = A^2/2 + A^2/2 [\cos (2w_o t)]$

Since cosine is an even function we know that all sine coefficients or b_n's = 0. Further, since this function is already in the form of a trigonometric Fourier series, we can conclude that $A^2/2$ is a_0 as well as a_2, and there are no more nonzero coefficients in the series.

Thus we have the Fourier series $y(t) = a_0 + a_2 \cos 2w_o t$

where $a_0 = A^2/2$, and $a_2 = A^2/2$.

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

Since the ouput is a periodic function, we know that it is a power signal. Thus, energy is infinite.

For power:

$P = \lim_{T \to \infty} ½ T \int_{-T}^{T} | y(t) |^2 dt$
$P = ½ T \int_{-T}^{T} [A^4/4 + A^4/2 \cos(2w_0T) + A^4/4 \cos^2(2w_0T)] dt$

By using the same identity as in part A we obtain
$½ T_0 \int_{-T}^{T} [A^4/4 + A^4/8 + A^4/2 \cos (2w_0T) + A^4/4 \cos(4w_0T)] dt$

Substituting $w_0 = 2\pi/T$ the integral of the cosine terms will go to zero, and we have
$½ T_0 \int_{-T}^{T} (A^4 / 4 + A^4/8) = ½ T_0 * 2 T_0 * (A^4/4 + A^4/8)$

$= A^4/4 + A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

$x(t) = -t + 2 \qquad\qquad 0 \le t \le 2$

$y(t) = x^2(t) \quad = (-t + 2)^2$

$P = \lim_{T \to \infty} \int_{-T}^{T} (-t + 2)^4 \, dt$

Using a substitution, u = -t + 2 and du = -dt, we have

$P = -\int u^4 \, du$

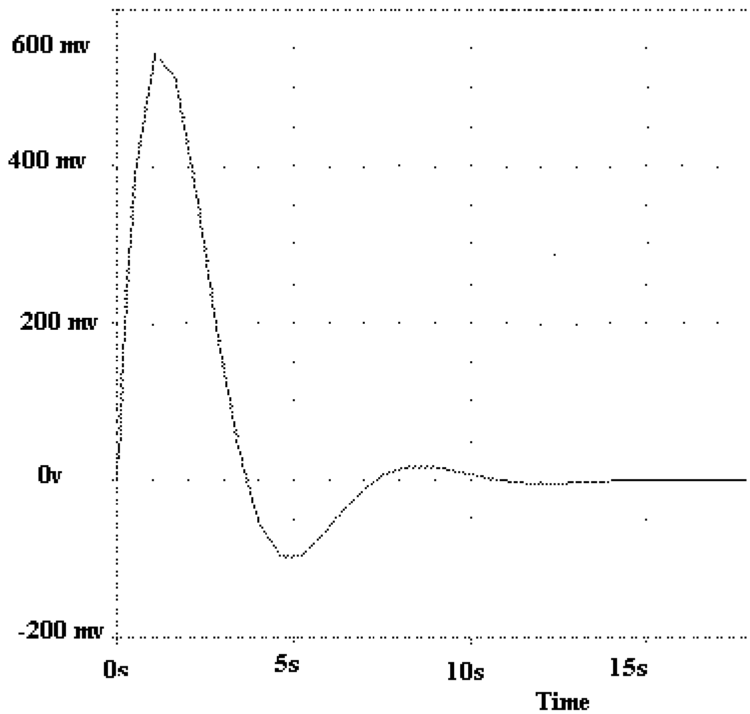
$= -1/5 \, (-t + 2)^5 \; |_0^2$

$= 0-(-32 / 5) = 32/5$ Joules = 6.4 Joules

Problem No. 2: Time-Domain Solutions

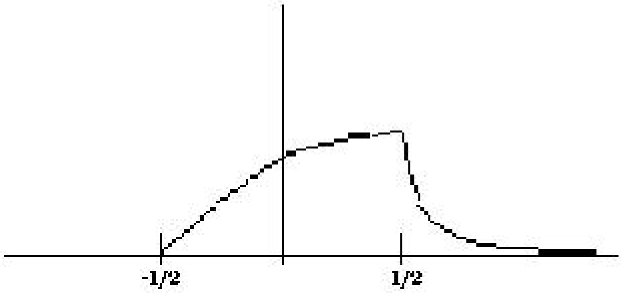
(a) For the circuit shown, using concepts developed in this class, sketch the output signal if x(t) is the unit step function. Explain.

$v(t) = e^{(-a*t(b1*\cos(w_d t) + b1*\sin(w_d t))}$



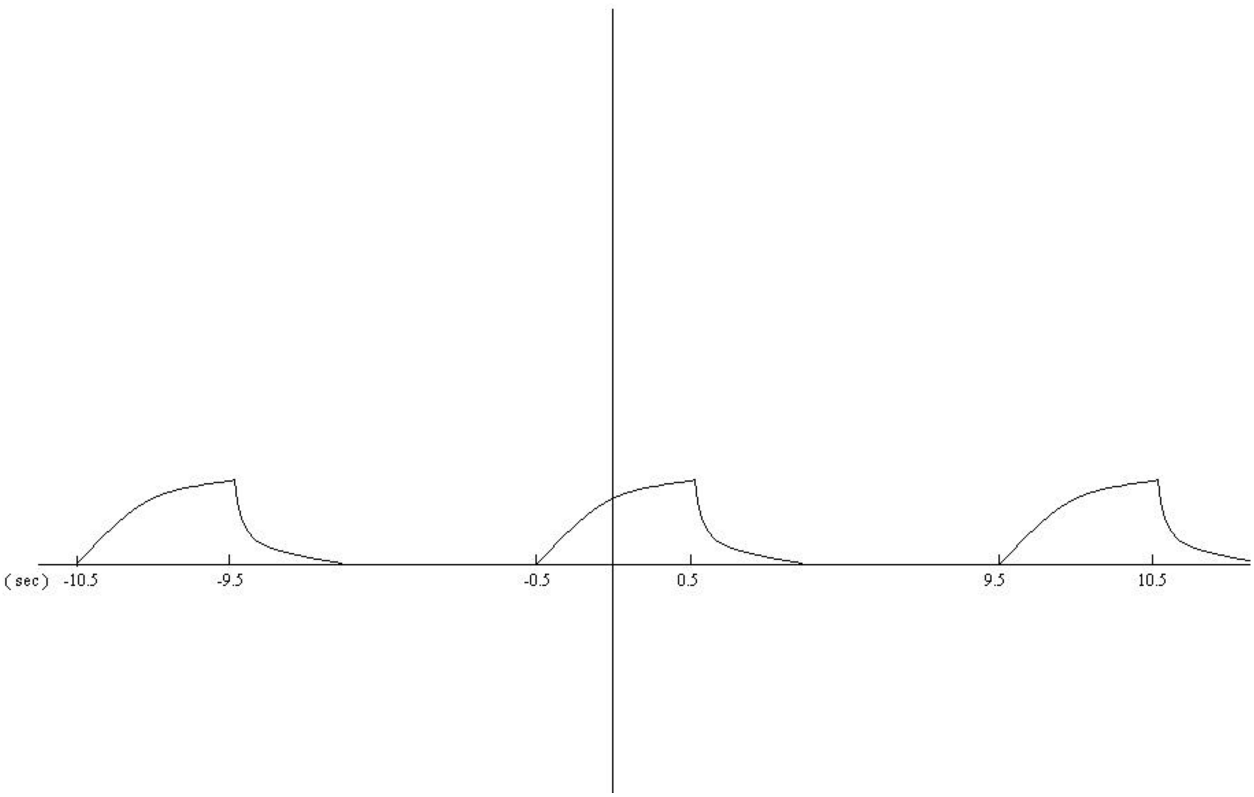
Using P-Spice the response of the circuit to a unit step is shown in the above diagram. This diagram makes sense because of some things we know about an RLC circuit. First, we know that the application of a DC voltage at time 0 will create a quick but not instantaneous jump in current. Second, the current at infinite time will go to zero since the capacitor begins to appear more and more like an open circuit. The voltage across the resistor, being equal to the current through the circuit in this case, will oscillate some between the initial peak and infinity due to the properties of capacitors and inductors in a circuit. Using P-Spice we are able to determine exactly what the oscillation will look like.

(b) Sketch the output if x(t) is a unit pulse with width of 1 second. Explain.



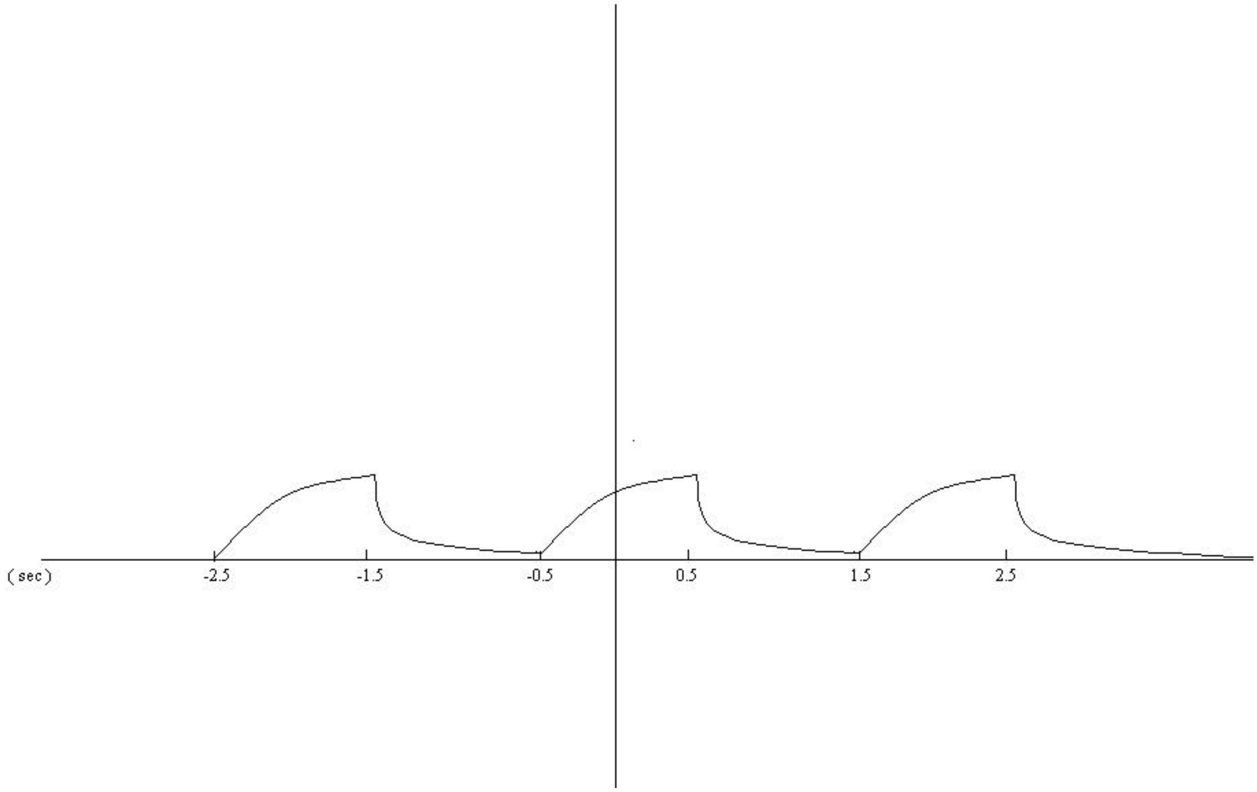
This is the graph of the ouput with the x-axis in seconds. The graph makes sense because with the given circuit, a pulse will cause the circuit to begin to charge like it did with the unit step input. However, when the pulse cuts off after 1 second, the capacitor and inductor will both begin to discharge through the resistor to create the fall-off effect seen above.

(c) Sketch the output if x(t) is a train of pulses of width 1 second and a period of 10 seconds.



With the period at 10 seconds there is no overlap of the signals. Thus we just have a train of the signal from part b.

(d) Suppose the period is dropped to 2 seconds. How does this affect the shape of the output?



Due to the drop in period for this, the output signals would overlap resulting in a signal like the one shown above.

Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

Causal: The system does not predict future inputs.

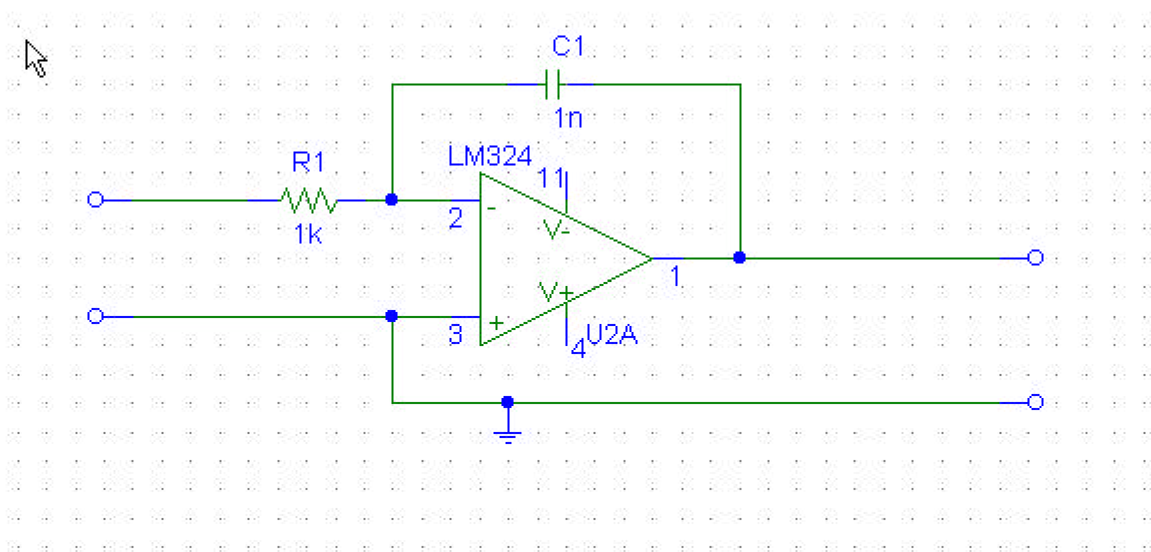
Dynamic: For a relatively good system, we would want some kind of memory so that we don't have immediate responses to small hills.

Fixed: The system output will not vary as a result of time because the response of the system to certain conditions is the same at any given time.

Non-linear: With the simple case of two inputs, current velocity and acceleration, differences in the inputs will not cause proportional differences in the output for several reasons, one of which is the difference between being in 3^{rd} gear and 4^{th}.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

For a good system we would want an impulse response that was smooth in nature. We would want more gradual changes in acceleration instead of a jumpy response. We could use the circuit below to do this. Not only will this circuit act as a low pass filter because of the resistor/capacitor combination, but it will also act as an integrator because of its configuration with the op-amp. In this way, any high frequency inputs will be filtered out which will smooth the output of the circuit. The output will be further smoothed by the integration of the input signal. Thus, for terrain with frequently occurring steep hills, the acceleration of the car will adjust much more smoothly with little or no response to a quick rise and fall of the road.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

Cruise control cannot see into the future of the road. It is a causal system that depends solely on the current speed of the vehicle. If we could use sensory equipment such as radar, or even image processing, we could in some sense develop a non-causal system where the cruise control adjusts according to both current speed and future road conditions such as a steep incline. Thus, instead of dropping a gear on every mountain, the cruise would adjust speed gradually to prepare for the incline.