Name:LaMont Parson

| Problem | Points | Score |
|---|---|---|
| 1a | 10 | |
| 1b | 10 | |
| 1c | 10 | |
| 2a | 10 | |
| 2b | 10 | |
| 2c | 10 | |
| 2d | 10 | |
| 3a | 10 | |
| 3b | 10 | |
| 3c | 10 | |
| Total | 100 | |

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

**Problem No. 1**: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos 2\, w_o t$

identity: $\cos^2 u = ½ (1 + \cos 2u)$
applying: identity you get

$y(t) = A^2/2 + A^2 /2 \cos (2w_o t)$

Cosine is an even function ,so $b_n = 0$ & $a_n \neq 0$
therefore there are only $a_n$ terms

The fourier series of a sine or cosine is its own fourier series therefore
$a_0 = A^2 /2$ $a_2 = A^2 / 2$

so the fourier series is $y(t) = a_o + a^2 \cos 2w_o t$

The answer is $y(t) = A^2 / 2 + A^2 / 2 \cos 2w_o t$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

The output is $y(t) = A^2 / 2 + A^2 / 2 \cos^2 w_o t$
Since the output is a periodic function with infinite area under the curve, its energy is will also be infinite.

therefore $E = \infty$
$P = \lim_{T \to \infty} 1 / 2T \int_{-T}^{T} 6\, y(t)\, 6^2\, dt$
$P = 1 /2T \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)]\, dt$

substituting wo = 2 $\pi$ / T the first cosine term goes to zero and
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)]\, dt$

using the trigonometric identity from 1A on the $\cos^2$ term results in
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4/8 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos(4w_o T)]\, dt$

after substituting wo = 2$\pi$ / T the cosine term goes to zero and the result is
$1 / 2T_o \int_{-T}^{T} (A^4 / 4 + A^4/8)$
Integrating $1 / 2T_o (A^4 / 4 + A^4/8)\, 2T_o$ ; the $2T_o$ cancelles leaving a power of $A^4/4+A^4/8= 3A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

$x(t) = -t + 2 \qquad 0 \leq t \leq 2$

$y(t) = x^2(t) = (-t + 2)^2$

$E = \lim_{T \to \infty} \int_{-T}^{T} (-t + 2)^4 \, dt$

using U substitution $u = -t + 2$
$du = -dt$

Therefore $E = -\int u^4 \, du$

$-1/5 \, (-t + 2)^5$ evaluated from [0,2]

$0-(-32 / 5) = 32/5$ Joules $= 6.4$ Joules

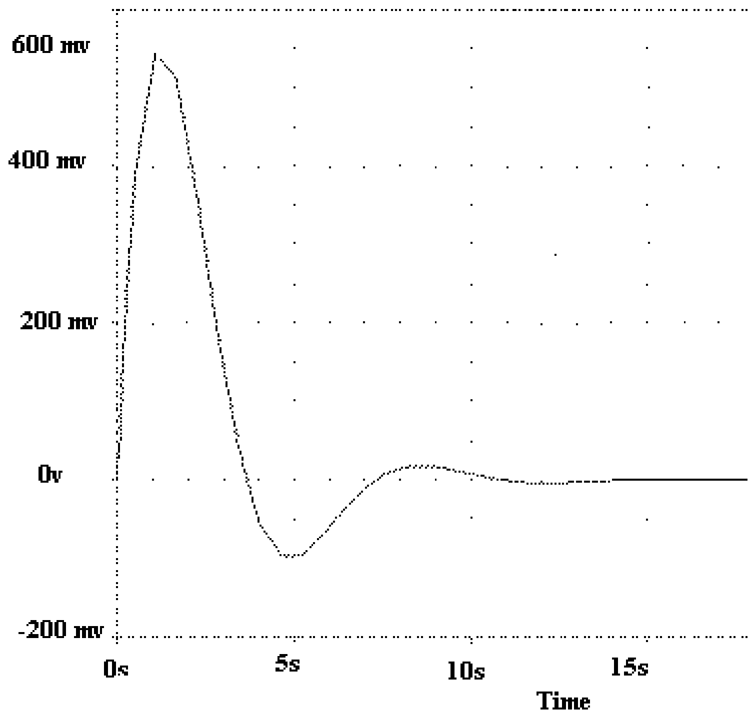
$P = \lim_{T \to \infty} 1 / 2T \int_{-T}^{T} 6 \, t^2 - 4t + 4) \, 6^2 \, dt$

$P = 0$

**Problem No. 2**: Time-Domain Solutions

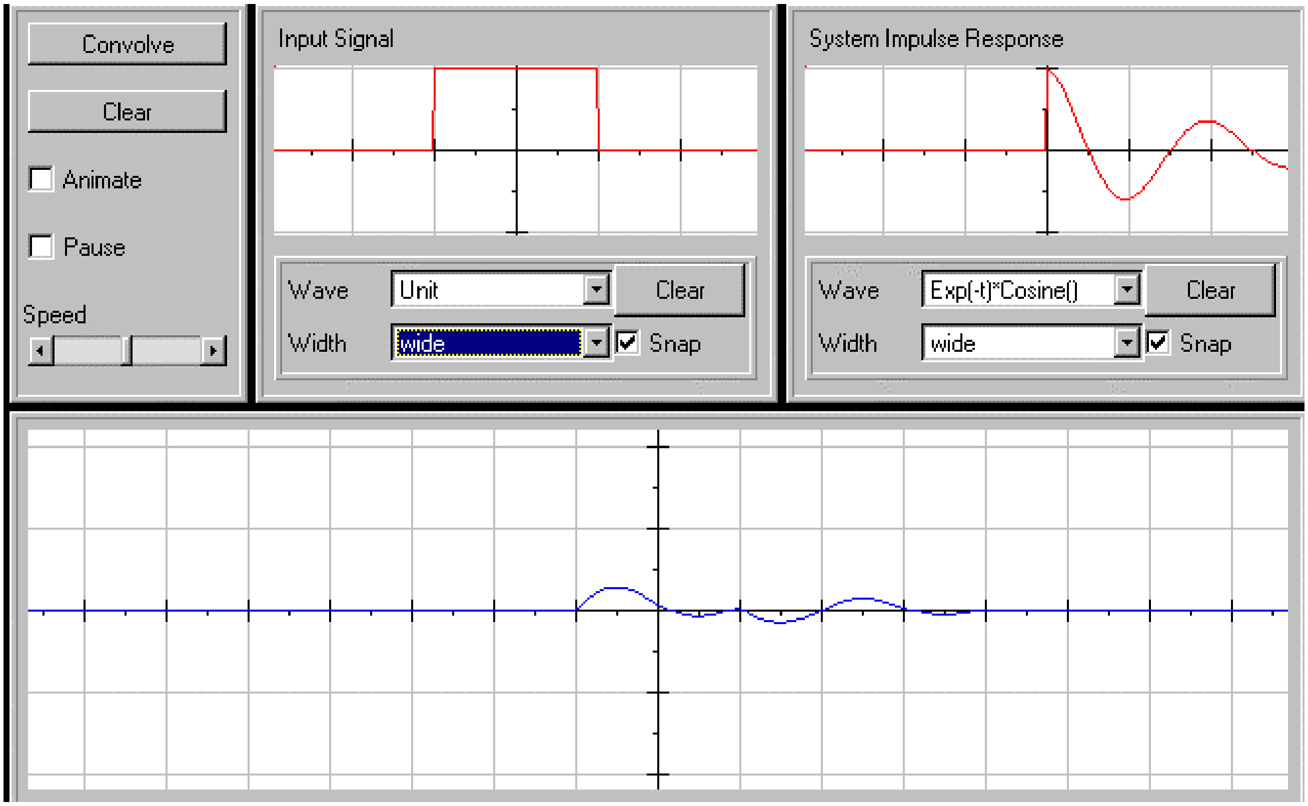
(a) For the circuit shown, using concepts developed in this class, sketch the output signal,y(t) , if x(t) is the unit step function. Explain.

$$v(t) = e^{(-a*t(b1*\cos(w_d t) + b1*\sin(w_d t))}$$




For an underdamped RLC circuit with a unit step input the natural response is an exponentially damped sinusoid. There is no voltage across an inductor if the current through it is not changing with time. The current and voltage relationships with inductors and capacitors are defined as the differential equations *(v = L * di /dt and I = C * dv /dt)* . The current through an inductor cannot change instaneously. There is no current passing through the capacitor because it acts as an open circuit at DC. It is also impossible to change the voltage across a capacitor in zero time by a finite amount. Therefore, the steady state respone of the circuit for infinite time will have no current flow across the resistor and therefore the voltage across the resistor will be zero for infinite time. The only time current will flow through the resitor is during the transient response cause by the discontinuity at 't' equal to zero. Since this circuit is an underdamped series RLC circuit the output on the resistor is a damped sinusoidal response.

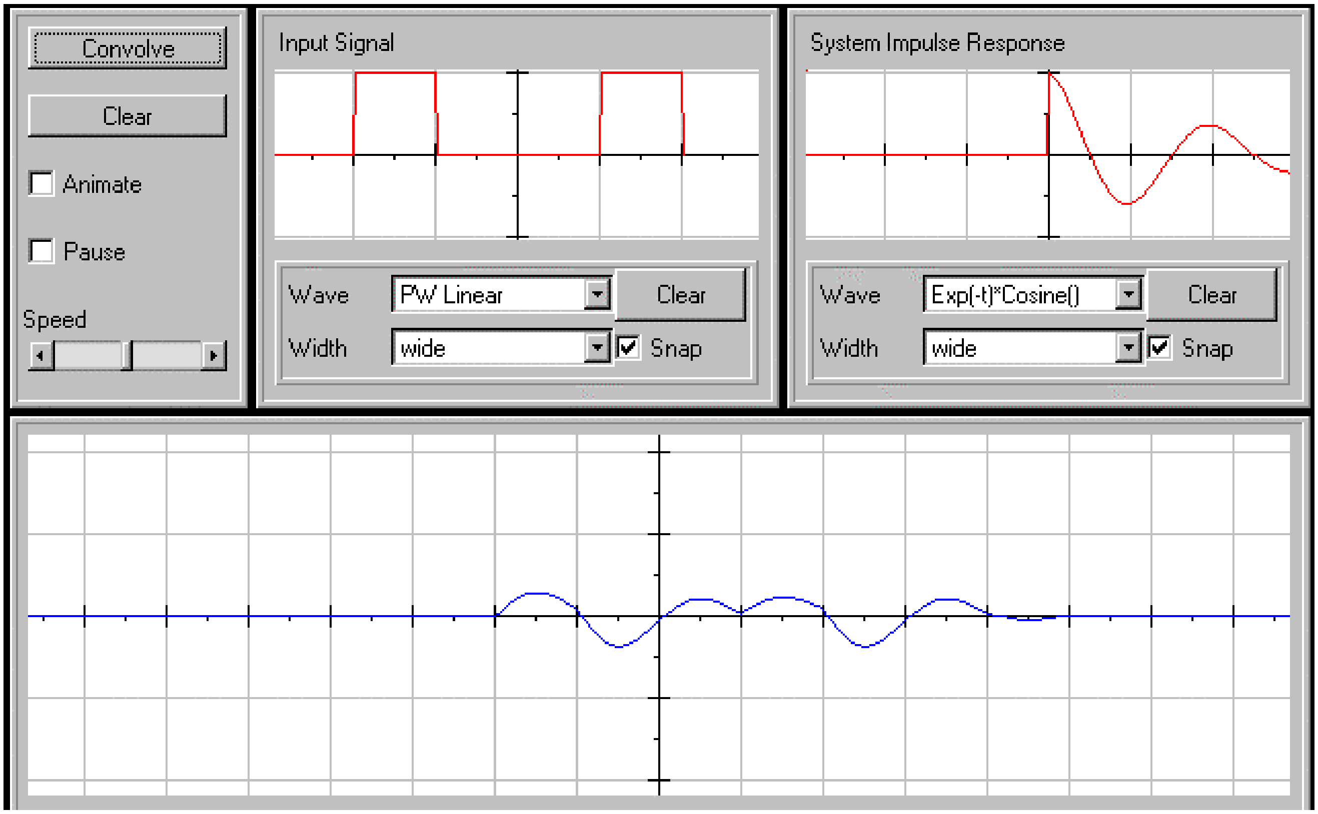
(b) Sketch the output if x(t) is a unit pulse. Explain.




The output graph is a unit step function convolved with a decaying exponintial sinusoid. The pulse was assumed to have a width of 1 ms: x(t) becomes 1V at t= - 0.5 ms and remains 1V until t= 0.5ms. When x(t) becomes 1V the voltage across y(t) starts to increase as the resistance across the inductor decreases and allows current to pass through the circuit, x(t) becomes 0V at t=0.5 ms.. The y(t) voltage drop would have a slight curve as it dropped back to zero since the capacitor and inductor will be charged and they will discharge when the voltage source x(t) becomes 0V.

**Note:** The Java Convolution tool was used to generate the graph, I was not able to extend the periodic output for all time due to limitations and time constraints.

(c) Sketch the output if x(t) is a periodic sepuence of unit pulses with a period of 10 seconds.




This graph above looks like the graph in part b, but now the output is periodic. An output identical to the output in part b would occur every 10 seconds.

**Note:** The Java Convolution tool was used to generate the graph, I was not able to extend the periodic output for all time due to limitations and time constraints.

(d) Suppose the period is dropped to 2 seconds. How does this affect the shape of the output?

Decreasing the period brings the output pulses together. This Creates a sharper amplitude and a narrower bandwidth.

**Problem No. 3**: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

The cruise control of a car is a low pass filter because it cuts off all high frequency inputs and creates a steady change of speed.The system is causal because it does not anticipate it’s future input. It is not possible for the system to anticipate the terrain it may encounter because there is not a completely described path. The system is fixed because the input/output relationship does not change with time. The system is a continuous - time system because it is always inputing a signal. The system is also linear because when the speed of the car is increased or when it receives an input, the output is the speed of the car plus the increase in speed. Therefore, this is a linear relationship and superposition holds. The system is also instantaneous because it does not use past values of the input to create an output.

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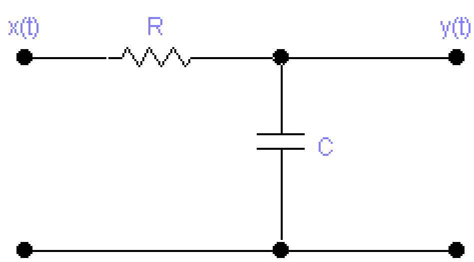
(b) If this system were a linear time-invariant system, design the impulse response of a “good” system. Explain how you might implement this in a circuit.

For a good cruise control system the impulse responses for the sysem would be an integrator where it smoothes out the discontinuities at the input and gives a smooth curve at the output.

To implement this in a circuit, run the signal through a series RC low-pass filter as seen below. X(t) is the input Y(t) is the output.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

A cruise control system is not optimal for a mountainous terrian because at the peak of each mountain there is a discontinuity. The elevation changes so frequently that the car's speed will vary significantly, thus defeating the purpose of cruise control, which is to maintain a constant velocity.

The circuit is integrating the input signal to give a steady curve so the curve will not decrease rapidly as it goes over the peak. Therefore the accelerator will still be accelerating the car for a short time as it goes down hill. The triangular signal beolow would be the same as the signal for a mountain and the output signal is how much acceleration the system is producing. Using a GPS satellite system that will identify the road ahead to indicate the elevation , sensors would be on the automobile to keep track of the level of the car.