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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_0 t)^2$

$y(t) = A^2 \cos^2 w_0 t$

identity : $\cos^2 u = ½(1 + \cos 2u)$
applying : identity you get

$$y(t) = \frac{A^2}{2} + \frac{A^2}{2}\cos(2 w_0 t)$$

knowing that cosine is an even function you get $b_n = 0$ & $a_n \neq 0$
therefore you only have a_n terms

$\cos(2 w_0 t)$ denotes the second harmonic
in the Fourier series . Therefore the cofeccient of $\cos(2 w_0 t)$ is a_2

$$a_0 = \frac{A^2}{2} \quad a_2 = \frac{A^2}{2}$$

so the fourier series is $y(t) = a_0 + a_2 \cos 2 w_0 t$

the answer is $y(t) = \dfrac{A^2}{2} + \dfrac{A^2}{2}\cos 2 w_0 t$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

output is $y(t) = \frac{A^2}{2} + \frac{A^2}{2}\cos 2 w_o t$

The signal is not an energy signal because the area bounded by the curve of the signal

The signal has a finite average over a period, therefore it is a power signal

$$P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |y(t)|^2\,dt$$

$$P = \left(\frac{1}{2T}\right) * \int_{-T}^{T} \left[\frac{A^4}{4} + \frac{A^4}{4}\cos(2 w_o T) + \frac{A^4}{8}\cos^2(2 w_o T)\right] dt$$

substituting $w_o = \frac{2\pi}{T}$ and using the same trig identity from part a we get :

$$P = \left(\frac{1}{2T}\right) * \int_{-T}^{T} \left[\frac{A^4}{4} + \frac{A^4}{4}\cos\left(2\frac{2\pi}{T}T\right) + \frac{A^4}{8}\left(1 + \cos\left(2\frac{2\pi}{T}T\right)\right)\right] dt$$

$$P = \left(\frac{1}{2T}\right) * \int_{-T}^{T} \left[\frac{A^4}{4} + \frac{A^4}{4}\cos(4\pi) + \frac{A^4}{8} + \frac{A^4}{8}\cos(4\pi)\right)\Big] dt$$

The T's cancel and leave $\cos(4\pi)$.

The $\int \cos(4\pi) = \sin(4\pi) = 0$

Therefore the terms containing $\cos(4\pi)$ go to zero, leaving :

$$P = \left(\frac{1}{2T}\right) * \int_{-T}^{T} \left[\frac{A^4}{4} + \frac{A^4}{8}\right] dt = \left(\frac{1}{2T}\right)\left(\frac{A^4}{4} + \frac{A^4}{8}\right)(T - (-T)) =$$

$$P = \frac{A^4}{4} + \frac{A^4}{8}$$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

This is an energy signal because it has a finite area over an inifinite interval.

$x(t) = -t + 2 \quad 0 \le t \le 2$

$x(t) = 0 \quad \text{for } t > 2$

This graph has the area bounded by the graph

$x(t) = t$

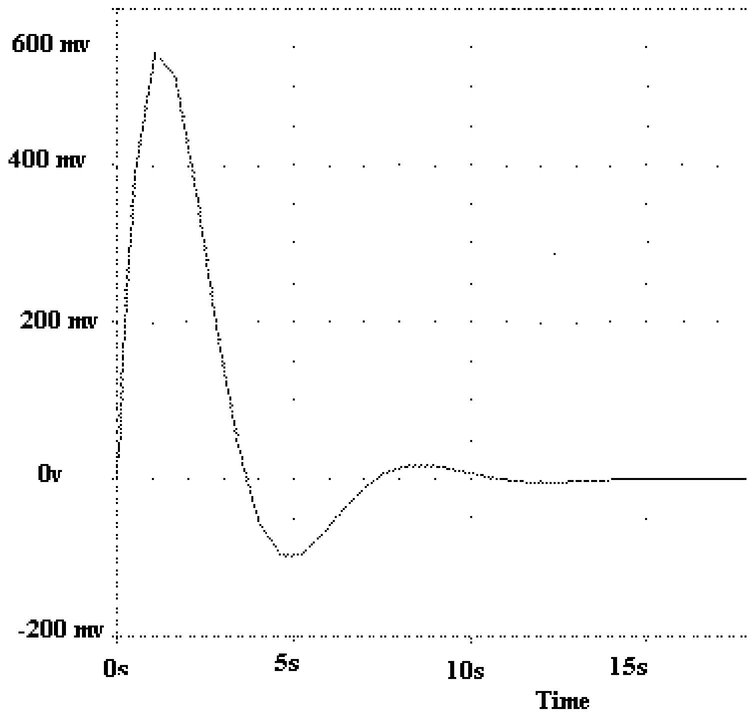
$$E = \lim_{T \to \infty} \int_{-T}^{T} |y(t)|^2 dt$$

$$E = \int_0^2 t^4 \, dt = \frac{1}{5} t^5 = \frac{1}{5}(2^5 - 0^5) = \frac{32}{5} \text{ J}$$

$$E = \frac{32}{5} \text{ J}$$

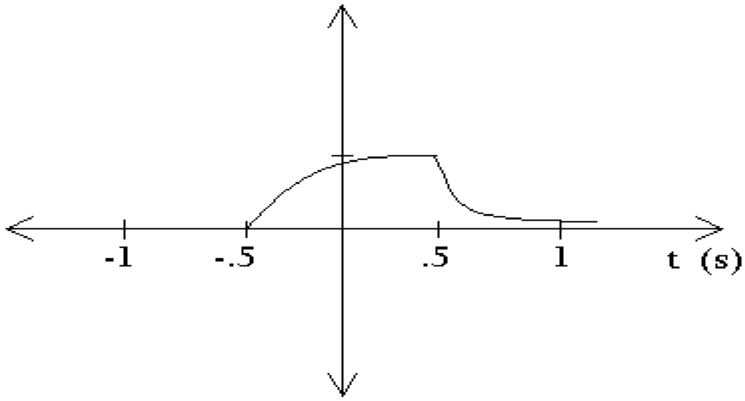
Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, , if x(t) is the unit step function. Explain.



This is what the graph looks like when the circuit is entered into PSpice. The Voltage rises as the resistance induced by the inductor drops from infinity and approaches zero. The resistance induced by the capacitor slowly increases. When the sum of these induced resistances is at its smallest value, then the output is at its maximum value. We know that as time approaches inifity then the output voltage will approach zero, because the capacitance will act as an open circuit to the DC source. In between the maximum voltage and the final voltage, there is an oscillation due to the fact that the capacitor and inductor are charging and discharging energy into the circuit—which is eventually absorbed by the resistor.

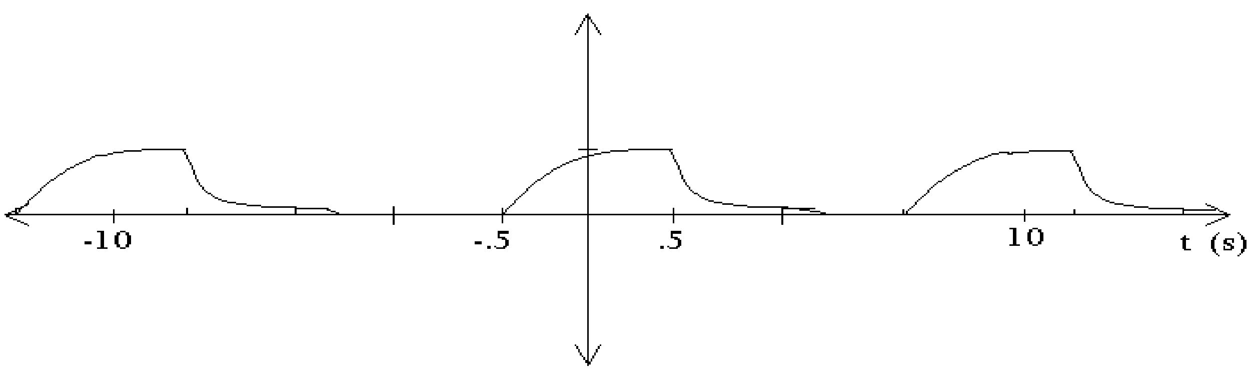
(b) Sketch the output if x(t) is a unit pulse with width of 1. Explain.



Assuming the pulse to have a width of 1 ms: At t= - 0.5 ms x(t) becomes 1V and remains 1V until t= 0.5ms. When x(t) becomes 1V the voltage across y(t) starts to increase as the resistance across the inductor decreases and allows current to pass through the circuit, but at t=0.5 ms. x(t) becomes 0V. The curve as y(t) slopes back to zero is caused by the charged up capacitor and inductor slowly discharging when the voltage source x(t) becomes 0V.

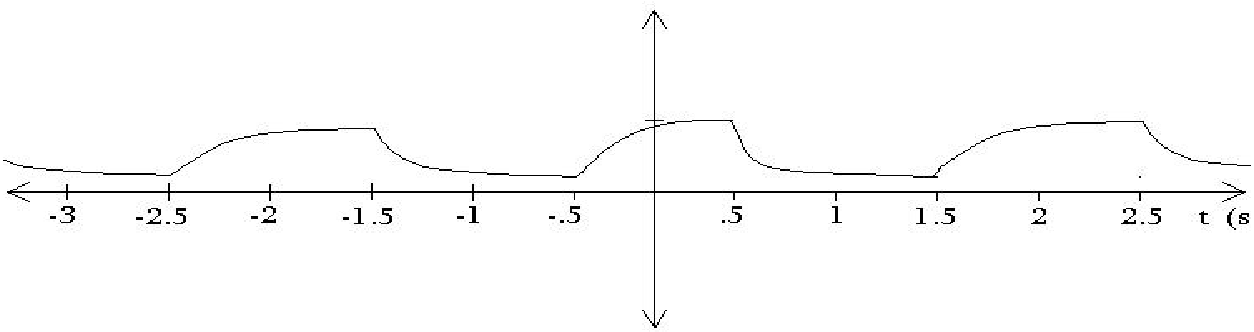
(c) Sketch the output if x(t) is a train of pulses of width 1 and a period of 10 seconds.

This graph would like the one in part b, but now the output would become periodic. An output identical to part b would occur every 10 seconds Therefore the graph would look something like this:



(d) Suppose the period is dropped to 2 seconds. How does this affect the shape of the output?

This graph would like the one in part b, but now the output would become periodic. An output identical to part b would occur every 2 seconds. This also causes a slight overlap since the circuit may still have some power being supplied by the capacitor and/or inductor. Therefore the graph would look something like this:



Notice how the y(t) never becomes zero on the graph (i.e. the graph never touches the time axis). This is because the outputs from the unit pulses overlap before the circuit's output becomes zero.

Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

Dynamic – The circuit has to remember what previos speeds were so that the system can accelerate, decelerate, and react to maintain a constant velocity.

Time-InVariant – If you went over the same road at a different time of the day, then you would still get the same output from the cruise control.

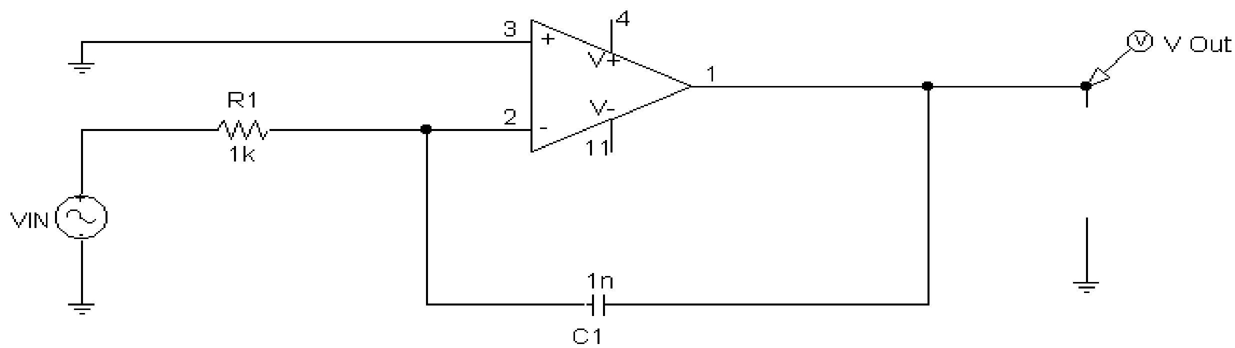
Causal –The car can not truly predict when hills are coming up or you would have the perfect cruise controls.

Non-Linear – The force to get up a hill twice as steep as two smaller hills, would not be the same as the combined force needed to get up the two smaller hills

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

A good cruise control would not respond to every single change in amplitude. It would only respond to changes in amplitude that would effect the car for a long time (relatively long time compared to a bump in the road).

Since bumps are very frequent, a low pass filter could be used to filter out bumps in the road because they occur very often (or at high frequencies). Integrators are also use to smooth output from a signal. This is because an integrator has a capacitor that allows higher frequencies to flow around the Op-Amp in the circuit, but the lower frequencies enter the Op-Amp and experience gain. The final signal is the combination of the two, but the lower frequency will have more of an impact of the signal's output because of the gain.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

The problem is that the system is causal. You can do a better job of simulating non-causality if your car already knew the slope in terrain it was about to encounter. If the car could store maps that contained the specified terrain information—such as slope and distances—and had a way to self-determine the weight of the car, then the car could begin to act for a terrain even though the car was not there yet.