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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$$x(t) = A\cos(w_o t) \rightarrow y(t) = x^2(t) \rightarrow y(t) = ???$$

Given:

$$x(t) = A\cos(w_o t) \qquad y(t) = x^2(t)$$

Substitute x(t)'s value into y(t):

$$y(t) = (A\cos(w_o t))^2 = A^2\cos^2(w_o t)$$

From Trigonometric Identity,

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Using Identity

$$y(t) = A^2\left(\frac{1}{2} + \frac{\cos(2w_o t)}{2}\right) = \frac{A^2}{2} + \frac{A^2\cos(2w_o t)}{2}$$

Because the Fourier Series of the cosine series is an even function

$$a_o = \frac{1}{T_o}\int_{T_o} y(t)dt \qquad a_n = \frac{2}{T_o}\int_{T_o} y(t)\cos(nw_o t)dt \qquad b_n = 0$$

Substitute y(t) and solve for the Fourier coefficients

$$a_o = \frac{A^2}{2} \qquad a_n = \frac{A^2}{2} \qquad b_n = 0$$

Basic equation of Fourier Series can be expressed as

$$y(t) = a_o + a_n\cos(2w_o t) + b_n\sin(2w_o t)$$

Substitute the coefficient into the basic equation, we get

$$y(t) = \frac{A^2}{2} + \frac{A^2\cos(2w_o t)}{2}$$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

From part a, the output is

$$y(t) = \frac{A^2}{2} + \frac{A^2 \cos(2\omega_o t)}{2}$$

The output of part a is a periodic function; therefore, its area under the curve is infinite. This means Energy is infinite.

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |y(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \left(\frac{A^2}{2} + \frac{A^2 \cos(2\omega_o t)}{2} \right) \right|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \left(\frac{A^4}{4} + \frac{A^2}{2}\cos(2\omega_o t) + \frac{A^4 \cos^2(2\omega_o t)}{4} \right) \right| dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \left(\frac{A^4}{4} + \frac{A^2}{2}\cos(2\omega_o t) + \frac{A^4}{8} + \frac{A^4}{8}\cos(4\omega_o t) \right) \right| dt$$

Let $\omega_o = 2\pi/T$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \left(\frac{A^4}{4} + \frac{A^2}{2}\cos\left(\frac{4\pi}{T} t\right) + \frac{A^4}{8} + \frac{A^4}{8}\cos\left(\frac{8\pi}{T} t\right) \right) \right| dt$$

The integral of cosine function will evaluate to zero; this leaves us with

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left| \left(\frac{A^4}{4} + \frac{A^4}{8} \right) \right| dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{0}^{T} \frac{3A^4}{8} dt = \lim_{T\to\infty} \frac{1}{T} \times \frac{3A^4}{8} T = \frac{3A^4}{8}$$

$$\therefore Power = \frac{3A^4}{8}, Energy = \infty$$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

Given that

$$x(t) = -t + 2,\ 0 \le x \le 2 \qquad \text{and} \qquad y(t) = x^2(t)$$

Substitute x(t)'s value into y(t), we get,

$$y(t) = x^2(t) = (-t+2)^2 = [(-1)(t-2)]^2 = (t-2)^2$$

$$Energy, E = \lim_{T\to\infty} \int_{-T}^{T} [y(t)]^2 dt$$

$$= \int_0^2 \left[(t-2)^2\right]^2 dt = \int_0^2 (t-2)^4 dt$$

Let u = (t – 2) and du = dt

$$= \int_0^2 u^4 du = \left.\frac{u^5}{5}\right|_{0\ where\ u=(t-2)}^{2}$$

$$= \left.\frac{(t-2)^5}{5}\right|_0^2$$

$$= 0 - \frac{-2^5}{5} = \frac{32}{5} = 6.4 Joules$$

$$Power, P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} [y(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (t-2)^4 dt$$

From previous integration of "u substitution", we can write the expression as

$$= \lim_{T\to\infty} \frac{1}{T} \times \left.\frac{(t-2)^5}{5}\right|_0^2$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\frac{32}{5}\right] = 0\ watt$$

$$\therefore Energy = 6.4\ Joules, and\ Power = 0\ watt$$

Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, , if x(t) is the unit step function. Explain.

The circuit shown is a RLC band-pass filter

$$V_s(t) = V_R(t) + V_C(t) + V_L(t)$$
$$= R_i + \frac{1}{C}\int_{t_o}^{t} i(x)dx + L\frac{di}{dt}$$
$$= L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = \frac{dV_s}{dt}$$

The characteristics equation of the network is

$$s^2 + s + 1 = 0$$
$$s = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm 3i}{2} = -0.5 \pm 1.5i$$

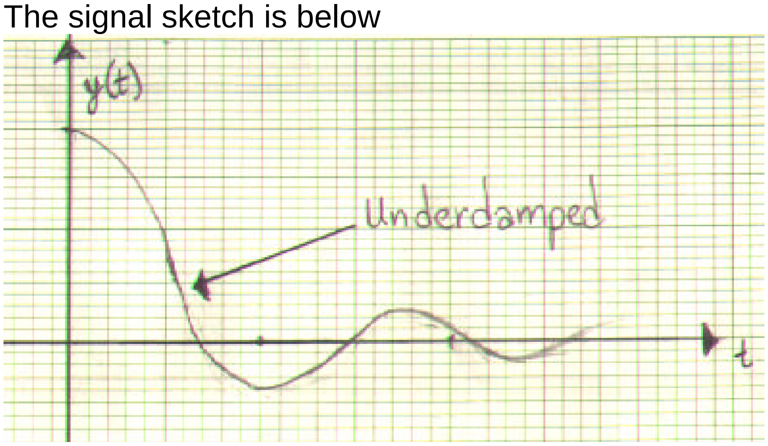
The circuit is underdamped

$$y(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$$
$$= K_1 e^{(-0.5+1.5i)t} + K_2 e^{(-0.5-1.5i)t}$$

Using Euler's identity :

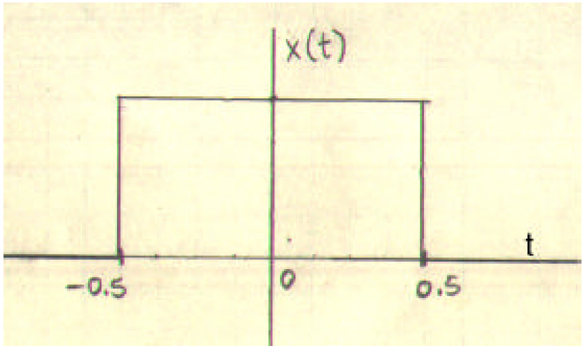
$$y(t) = e^{-at}(A_1 \cos w_d t + A_2 \sin w_d t)$$

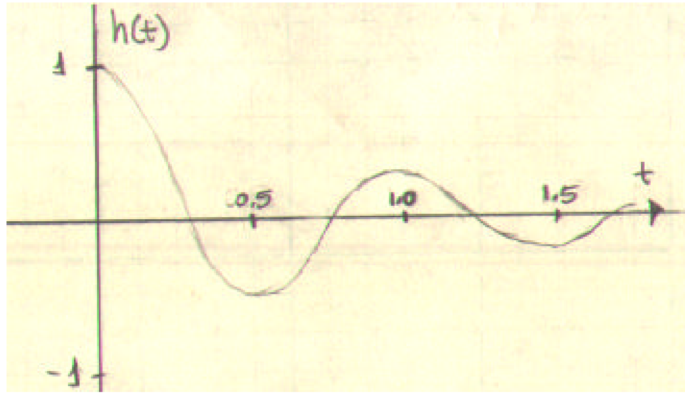
The signal sketch is below



If the current through an inductor is not changing with time, voltage across an inductor is zero. The current across the capacitor is zero because it is an open circuit at DC. The voltage across the capacitor cannot be changed by a finite amount in zero time. There will be no current flow through the circuit in infinite time, so the voltage across the resistor will be zero for infinite time. The current will flow across the resistor during the transient response caused by the discontinuity at t equal to zero.

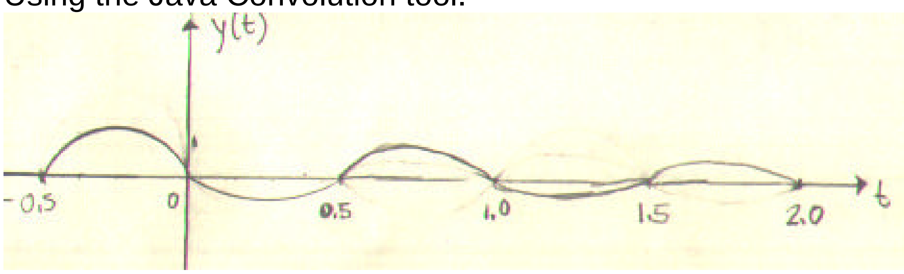
(b) Sketch the output if x(t) is a unit pulse. Explain.





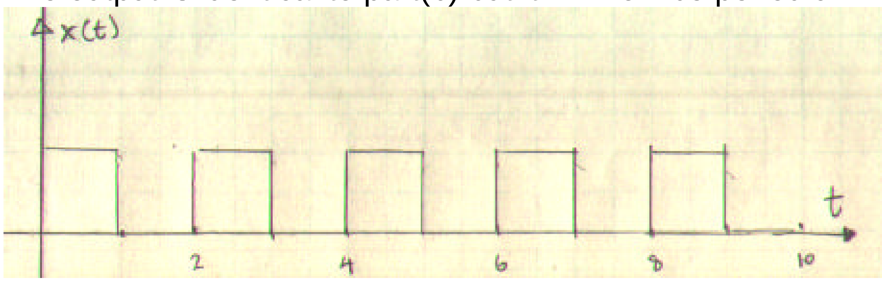
y(t) = x(t) * h(t)

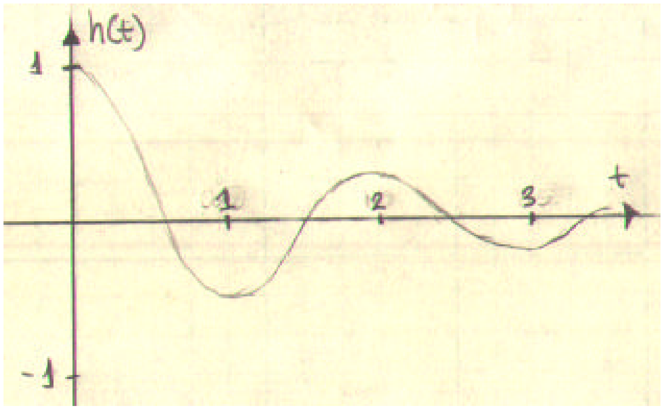
Using the Java Convolution tool.



(c) Sketch the output if x(t) is a periodic sequence of unit pulses with a period of 10 secs.

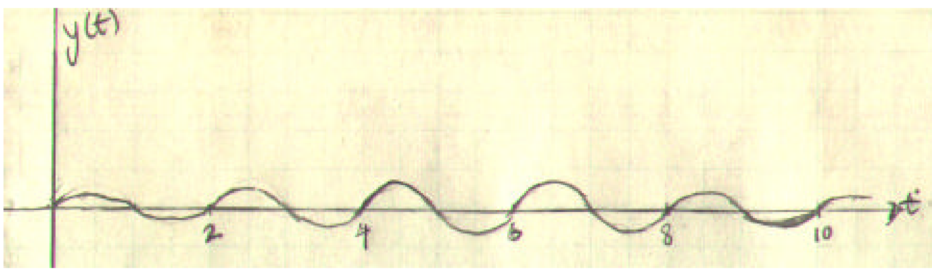
The output is identical to part(b) but it will now be periodic.





y(t) = x(t) * h(t)

The output is periodic, similar output as part b.



(d) Suppose the period in (c) is decreased to 2 secs. How does this affect the shape of the output?

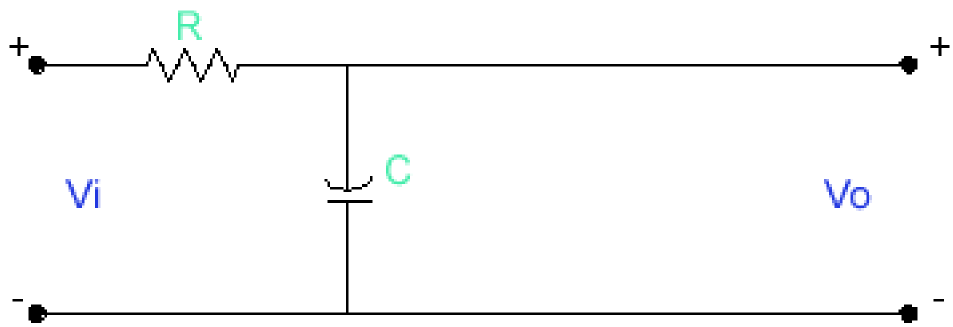
The output will have a narrower bandwidth and a shaper amplitude.

Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

1) The system is causal, which the output does not depend on future values of that input, because the system cannot predict what kind of the ground is ahead.
2) The system is instantaneous, which the output is the function of the input at the present time only, because the cruising speed is set by the driver’s input speed at the time of setting.
3) The system is linear, which means the superposition holds, because when an input speed is applied, the speed of the car increases and adjusts accordingly to the terrain.
4) The system is fixed, which means the input-output relationship does not change with time, because the cruising speed is set at one fixed time, and output speed is just adjust according the the setting.
5) The system is a continous-time, because the system is constantly inputting signal to retain the inputted speed.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.



This circuit is a low-pass filter. Filtering the high frequencies, the cruise control will guarantee the steadiness and smoothness of the vehicle's velocity. Speed will automatically increase while vechicle is advancing to a high slope, and decrease while during downhills.in order to make up for the change in velocity that had set by the driver.

(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

There is discontinuity at the peak of the mountain.
The circuit is integrating the input signal to get a steady state curve. The output will not instantly decrease as you go over the peak, which means the car will still be accelerating for a short time after traveling down hill. The graph below shows a triangular input and the system response; let the triangular input be the set speed by driver. The red curve (System Response) does not instantly react to change of input.