Name: Tan Ngiap Teen

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$$x(t) = A\cos(\omega_0 t) \rightarrow y(t) = x^2(t) \rightarrow y(t) = ???$$

Given that,

$$x(t) = A\cos(\omega_0 t),$$

and $y(t) = x^2(t)$

therefore,

$$y(t) = \left(A\cos(\omega_0 t)\right)^2$$

$$y(t) = A^2\cos^2(\omega_0 t) \qquad \text{EQ(1)}$$

From Trignometry Identity,

$$\cos^2 v = \frac{1}{2}(1+\cos 2v)$$

By substituing the above identity into EQ(1),

$$y(t) = A^2\left(\frac{1}{2} + \frac{\cos 2(\omega_0 t)}{2}\right)$$

$$y(t) = \frac{A^2}{2} + \frac{A^2\cos 2(\omega_0 t)}{2}$$

cosine series is an even function,

where,

$$a_0 = \frac{1}{T_0}\int_0^{T_0} y(t)dt \ ; \qquad ;\ b_n = 0$$

The fourier series of a sine or cosine is its own fourier series

Thus, $a_0 = \dfrac{A^2}{2}$

and $a_n = \dfrac{A^2}{2} \qquad a_n \neq 0$

Fourier Series for the equation can be expressed as

$$y(t) = a_0 + a_n\cos 2(\omega_0 t)$$

Therefore the answer is:

$$y(t) = \frac{A^2}{2} + \frac{A^2\cos 2(\omega_0 t)}{2}$$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

From (a), the output was found to be

$$y(t) = \frac{A^2}{2} + \frac{A^2 \cos 2(w_0 t)}{2}$$

Energy is infinte since output is a periodic function with infinite area under the curve, and thus

$$Energy, E = \infty$$

$$Power, P = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |y(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|\left(\frac{A^2}{2} + \frac{A^2 \cos 2w_0 t}{2}\right)^2\right| dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|\left(\frac{A^4}{4} + \frac{A^4}{2}\cos 2w_0 t + \frac{A^4 \cos^2 2w_0 t}{4}\right)\right| dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left|\left[\frac{A^4}{4} + \frac{A^4}{2}\cos 2w_0 t + \frac{A^4}{8} + \frac{A^4}{8}\cos 4w_0 t\right]\right| dt$$

The integral of the cosine terms will be equal to zero, the equation can be rewritten as follows:

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \left[\frac{A^4}{4} + \frac{A^4}{8}\right] dt$$

$$= \frac{A^4}{4} + \frac{A^4}{8} = \frac{3A^4}{8}$$

Therefore we found

$$Power, P = \frac{3A^4}{8}$$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

Given that $y(t) = x^2(t)$, we substitute into the equation below

$$x(t) = -t + 2, \qquad [0,2]$$

By rewriting the above equation in terms of y(t),

$$y(t) = x^2(t) = (-t+2)^2$$

$$Energy, E = \lim_{t\to\infty} \int_{-T}^{T} \left(y(t)\right)^2 dt$$

$$= \int_{0}^{2} \left((-t+2)^2\right)^2 dt$$

$$= \int_{0}^{2} (-t+2)^4 dt$$

Let u=-t+2, and du=-dt

$$= \int_{0}^{2} u^4 du$$

$$= \left.\frac{u^5}{5}\right|_{0}^{2}$$

$$= \frac{32}{5} = 6.4 Joules$$

$$Power, P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left|y(t)\right|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{2} \left|\left((-t+2)^2\right)\right|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} (6.4) = 0 Watt$$

Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, if x(t) is the unit step function. Explain.

This is a RLC band-pass filter.
Using circuit analysis:

$$V_s(t) = V_R(t) + V_c(t) + V_L(t)$$
$$= R_i + \frac{1}{C}\int_{t_0}^{t} i(x)dx + L\frac{di}{dt}$$
$$= L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = \frac{dV_s}{dt} \qquad \text{EQN(1)}$$

the characteristic equation of the network is

$$\Rightarrow s^2 + s + 1 = 0$$
$$\Rightarrow s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$\Rightarrow \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm 3j}{2} = -0.5 \pm 1.5j$$

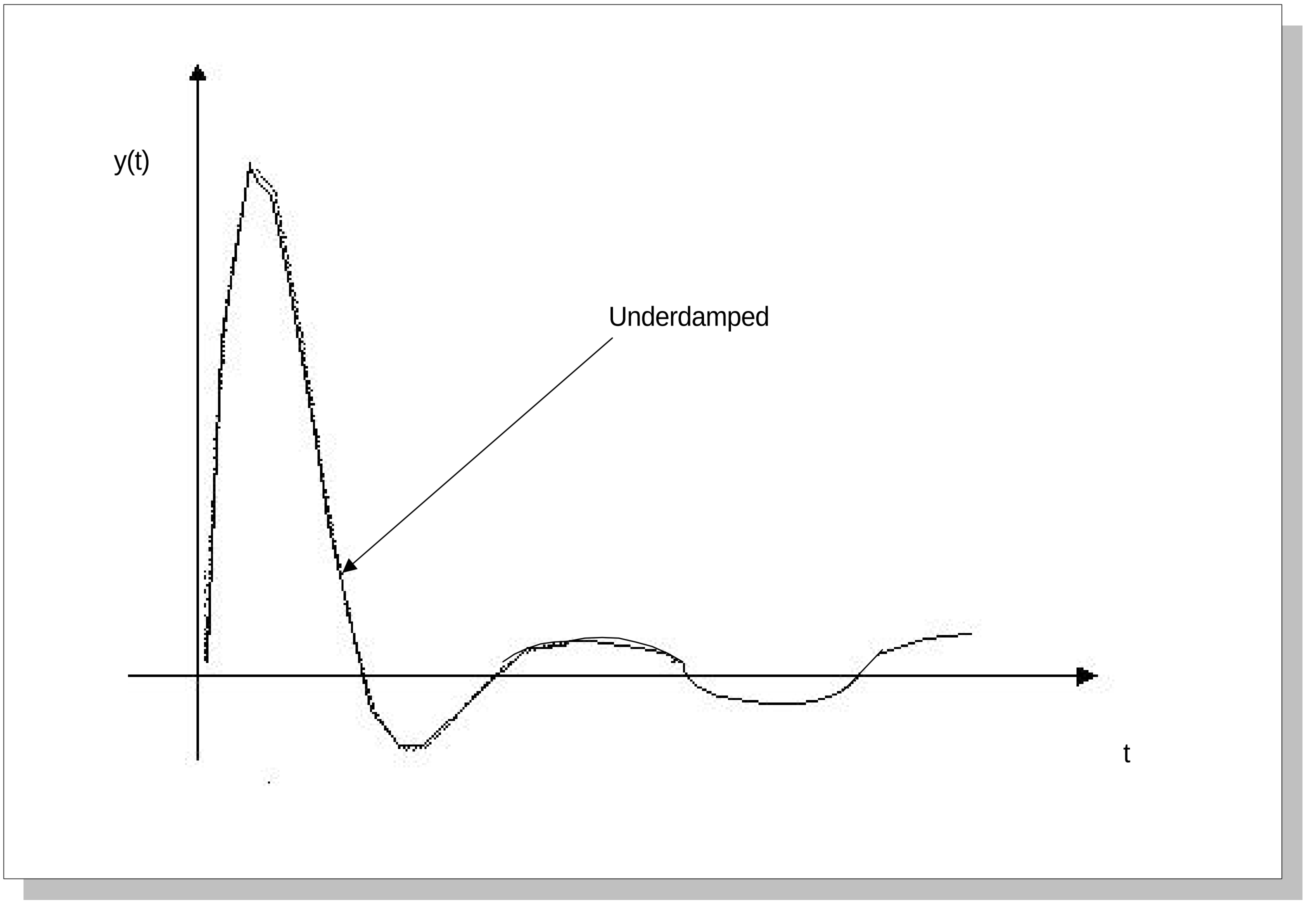
the circuit is underdamped

$$y(t) = K_1 e^{S_1 t} + K_2 e^{S_2 t}$$
$$= K_1 e^{(-0.5+1.5j)t} + K_2 e^{(-0.5-1.5j)t}$$

using Euler's identity:

$$y(t) = e^{-at}(A_1 \cos w_d t + A_2 \sin w_d t)$$

The output signal is as follows:



There is no voltage across an inductor if the current through it is not changing with time.

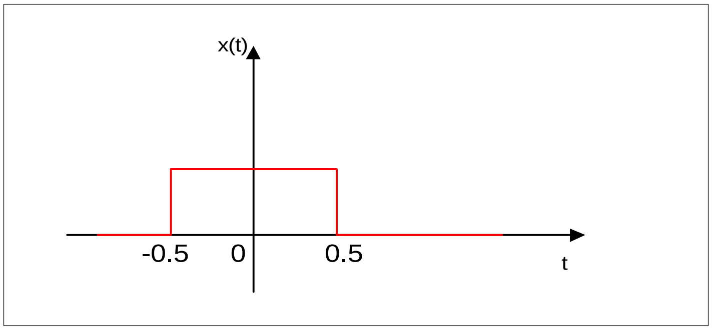
$$v_L(t) = L.\frac{di(t)}{dt}$$

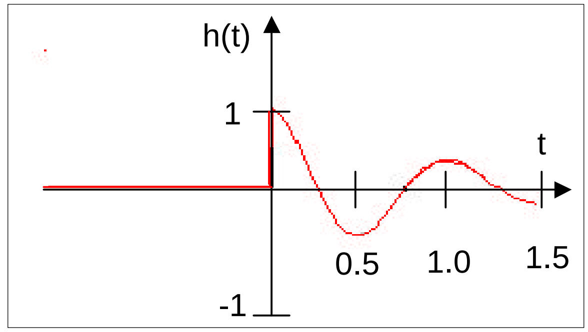
and

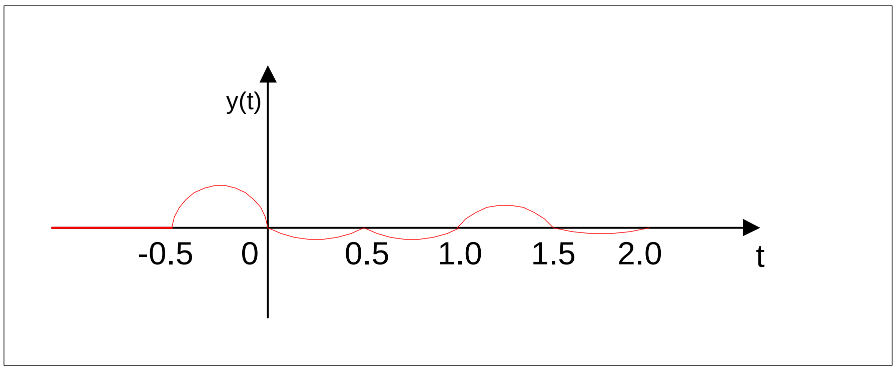
$$i(t) = C.\frac{dv(t)}{dt}$$

The current through the inductor cannot be changed instantaneously. The capacitor acts as an open circuit and the output voltage equals zero.There will be no current flow through the circuit in infinite time, so the voltage across the resistor will be zero for infinite time. The current will flow across the resistor during the transient response caused by the discontinuity at t=0.

(b) Sketch the output if x(t) is a unit pulse. Explain.

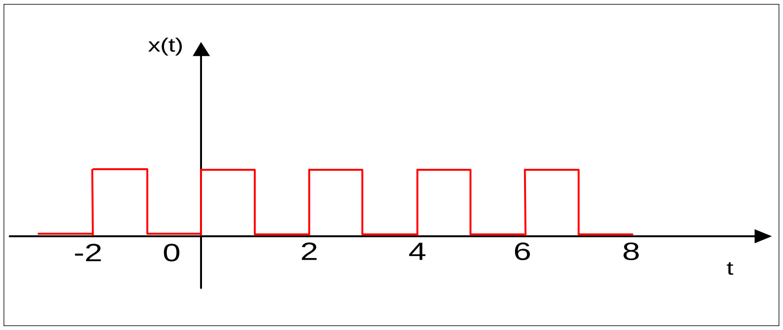


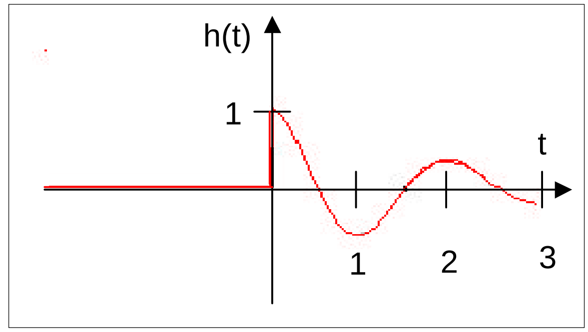


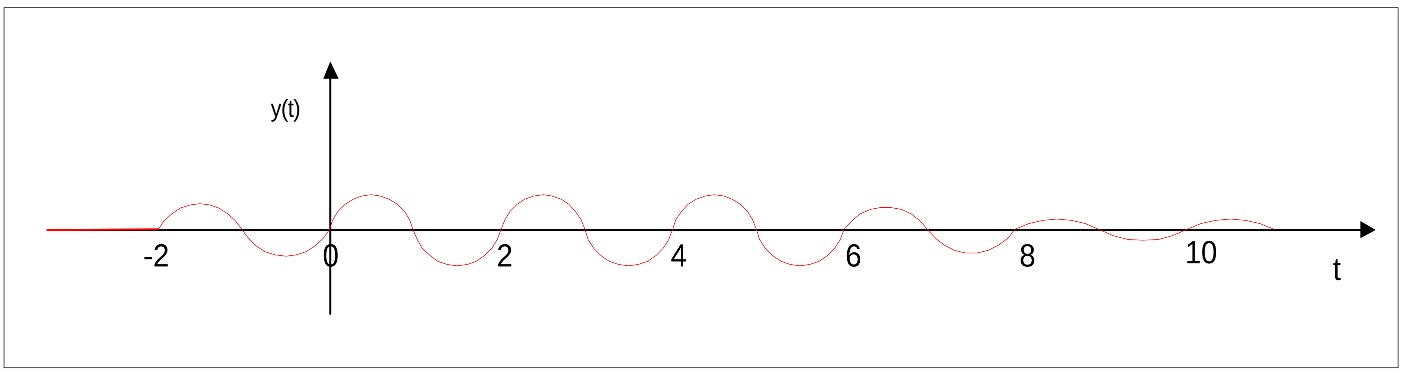


The output above is obtained by using the Java Convolution tool. The unit pulse is assumed to have a width of 1ms. x(t) have a value of 1V at t=-0.5ms and remains at 1V until t=0.5ms. As x(t) becomes 1V, the voltage across y(t) starts to increase as the resistance across the inductor decreases. This allows current to flow through the circuit. At t=0.5ms, x(t) becomes 0 voltage. The inductor and capacitor will be charged and then discharged when the voltage source x(t) becomes 0V. This will cause the y(t) voltage drop to be a slight curve as it is dropped back to zero.

(c) Sketch the output if x(t) is a periodic sequence of unit pulses with a period of 10 secs.







This result was obtained by using the Java Convolution tool. The output is also periodic.

(d) Suppose the period in (c) is decreased to 2 secs. How does this affect the shape of the output?

If the period is decreased, the output will have a narrower bandwidth and a sharper amplitude.

Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

The characteristic of the cruise control can be categorized as below:

i). Causal : Response to the output does not depend on future values of that input. The system cannot possibly anticipate the terrain that lies ahead.

ii). Linear : The system is linear. When an input(speed) is applied or the speed is increased, the response of the system is the speed of the car and the increase. Superposition holds.

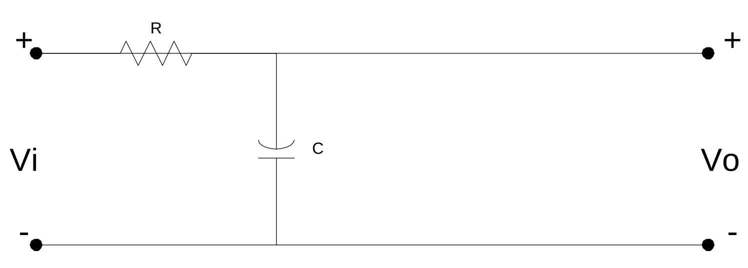
iii).Instantaneous : The output is a function of the input at the present time only.

iv). Fixed: The input-output relationship does not change with time.

v).Continuous-Time: The system is always inputting a signal to maintain the desired speed.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

The system can be implemented with a RC low - pass filter. The circuit shown below preserves lower frequencies signals, $w << \frac{1}{RC}$, while attenuating (reduce amplitude of signal) the frequencies above the cutoff frequency, $w_0 = \frac{1}{RC}$. The voltages, V_i, and V_0, are the filter input and output voltages, respectively.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

There is discontinuity at the peak of the mountain. The system can be represented as the response of the RC circuit to a triangular signal. The graph below shows the triangular input, which represents the signal for the mountain and the system response of the circuit. The curve will not decrease rapidly as you go over the peak. For RC <<1, the output closely approximates the input, if RC=1 the output does not resemble the input. When going downhill, cars tend to overrun the set speed for a short amount of time. We can extend the use of causality to address diagnosis of dynamic systems and use the evolution of the system over time to narrow down the set of possible malfunctions.