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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos^2 w_o t$

identity: $\cos^2 u = ½ (1 + \cos 2u)$
applying the identity you get

$y(t) = A^2/2 + A^2 /2 \cos (2w_o t)$

knowing that cosine is an even function you get $b_n = 0$ & $a_n \neq 0$
therefore you only have a_n terms

knowing that the fourier series of a sine or cosine is its own fourier series we obtain

$a_0 = A^2 /2$ $\quad\quad$ $a_2 = A^2 / 2$, where $n = 2$

so the fourier series is $y(t) = a_o + a_2 \cos 2w_o t$

Hence,

$y(t) = A^2 / 2 + A^2 / 2 \cos 2w_o t$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

output is $y(t) = A^2 / 2 + A^2 / 2 \cos 2wot$
The energy, E, is given as

$E = \int_{-\infty}^{\infty} |y(t)|^2 dt$

$E = \int_{-\infty}^{\infty} [A^4 / 4 + A^4 /2 \cos(2w_o t) + A^4/4 \cos^2(2w_o t)]\, dt$
$E = A^4 / 4 \int_{-\infty}^{\infty} dt + A^4 /2 \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/4\int_{-\infty}^{\infty} \cos^2(2w_o t)\, dt$
$E = A^4 / 4 \int_{-\infty}^{\infty} dt + A^4 /2 \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/4\int_{-\infty}^{\infty} (½ + ½ \cos(4w_o t))\, dt$
$E = A^4 / 4 \int_{-\infty}^{\infty} dt + A^4 /2 \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/8\int_{-\infty}^{\infty} dt + A^4/8\int_{-\infty}^{\infty} \cos(4w_o t)\, dt$

At this point, it is easily observed that the integrals with only constant terms will evaluate to infinity, and the integrals with cosine terms will oscillate forever between the plus and minus values of a constant.

Therefore, $E = \infty$
This could have also been seen intuitively by noting that y(t) is a periodic signal with infinite area under its curve.

The power, P is given as

$P = \lim_{T\to\infty} 1 / 2T \int_{-T}^{T} |y(t)|^2\, dt$

$P = 1 /2T \int_{-T}^{T} [A^4 / 4 + A^4 /2 \cos(2w_o t) + A^4/4 \cos^2(2w_o t)]\, dt$

$P = A^4 / (8T) \int_{-\infty}^{\infty} dt + A^4 /(4T) \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/(8T)\int_{-\infty}^{\infty}\cos^2(2w_o t)\, dt$

using trigonometric identity from problem 1a on the $\cos^2$ term we obtain

$P = A^4 / (8T) \int_{-\infty}^{\infty} dt + A^4 /(4T) \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/(8T)\int_{-\infty}^{\infty} (½ + ½ \cos(4w_o t))\, dt$

$P = A^4 / (8T) \int_{-\infty}^{\infty} dt + A^4 /(4T) \int_{-\infty}^{\infty} \cos(2w_o t)\, dt + A^4/(16T)\int_{-\infty}^{\infty} dt$
$+ A^4/(16T)\int_{-\infty}^{\infty} \cos(4w_o t)\, dt$

$= A^4/4 + 0 + A^4/8 + 0$

$= A^4/4 + A^4/8$

$= 3A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

$x(t) = -t + 2 \qquad 0 \le t \le 2$

$y(t) = x^2(t) = (-t + 2)^2$

Since the signal has no input for t less than zero or greater than two:

$E = \int_0^2 (-t + 2)^4\, dt$

using U substitution $\quad u = -t + 2$
$du = -dt$

We have $E = - \int u^4\, du$

$-1/5 (-t + 2)^5$ evaluated from [0,2]

0-(-32 / 5) = 32/5 Joules = 6.4 Joules

$P = \lim_{T\to\infty} 1 / 2T \int_0^2 (t^2 - 4t + 4)^2\, dt$

Taking the same integral that was taken for energy will yeild a constant over 2T, and at infinity:

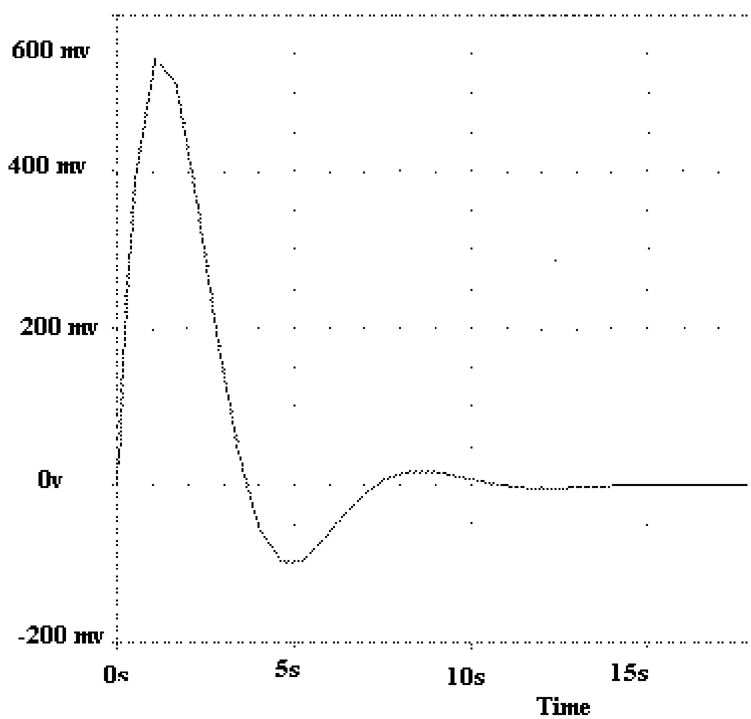
$P = 0$

Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, if x(t) is the unit step function. Explain.

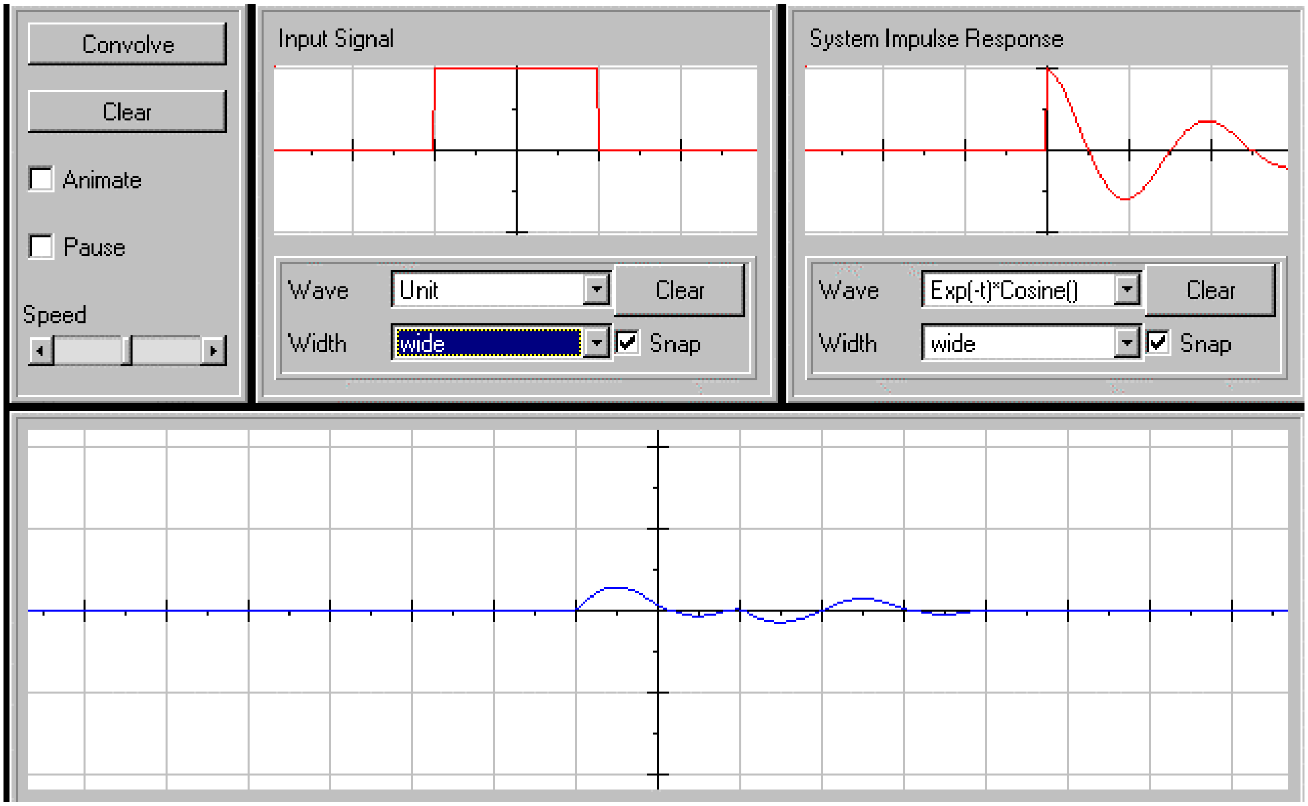
Because the output of the circuit can be written as a second order differential equation with complex roots, and since we are only concerned about the real portion of the output, a general output equation can be written as:

$$v(t) = A_1 e^{(-a_1 * t)} \cos(w_o t + \theta_1)^{+} A_2 e^{(-a_2 * t)} \cos(w_o t + \theta_2)$$



The natural response of an underdamped RLC circuit with a unit step input is an exponentially damped sinusoid. It is known that there is no voltage across an inductor if the current through it is not changing with time. The differential equations *(v = L * di /dt and I = C * dv /dt)* define the current and voltage relationships with inductors and capacitors. It is impossible to change the current through an inductor instaneously. In a capacitor there is no current passing through the capacitor because it is an open circuit at DC. Also it is impossible to change the voltage across a capacitor by a finite amount in zero time. Therefore, the steady state of this circuit for infinite time there will be no current flow across the resistor and therefore the voltage across the resistor will be zero for infinite time. The only time current will flow through the resitor is during the transient response caused by the discontinuity at 't' equal to zero. Also this circuit is an underdamped series RLC circuit. Therefore the output on the resistor is a damped sinusoidal response.

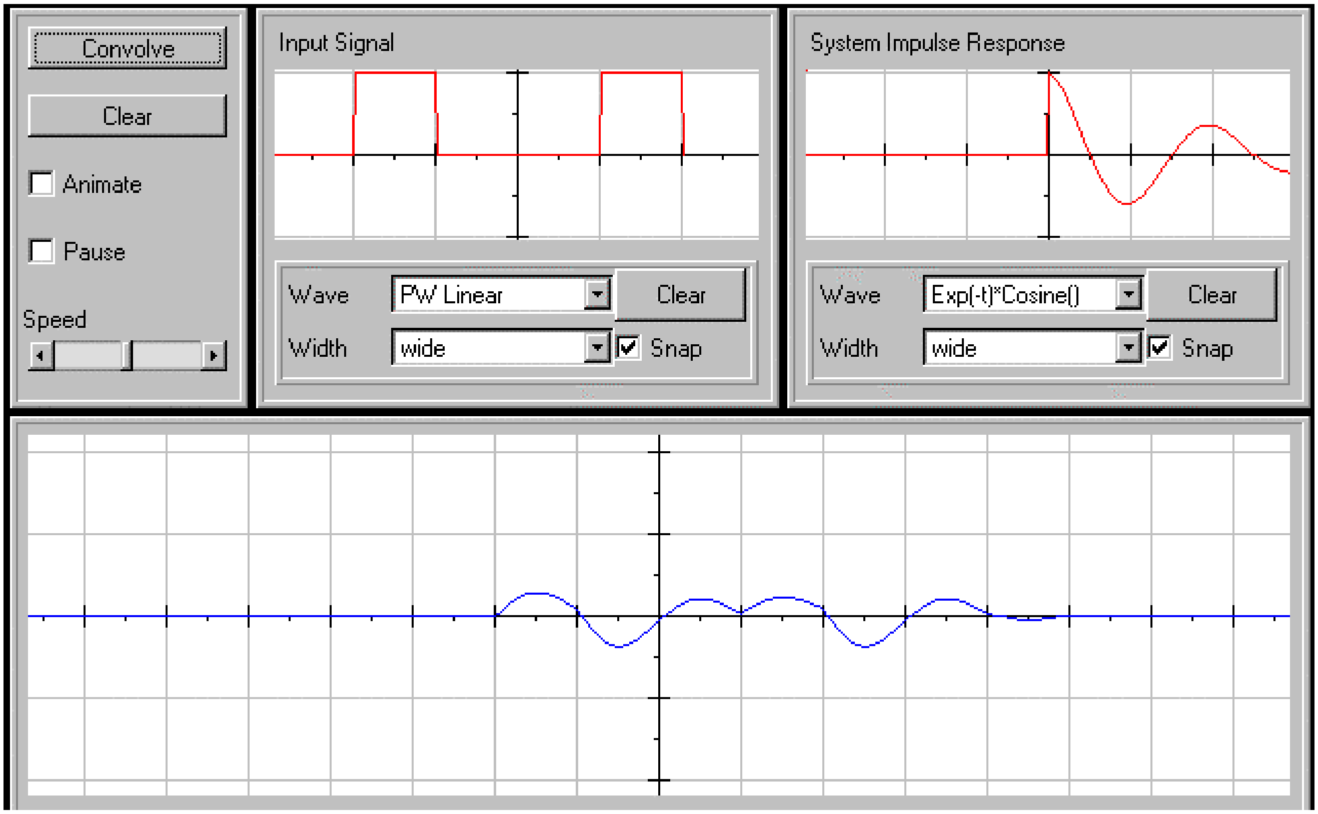
(b) Sketch the output if x(t) is a unit pulse with width of 1 millsecond. Explain.



The graph is a unit step function convolved with a decaying exponential sinusoid. Assuming the pulse to have a width of 1 ms: At t= - 0.5 ms x(t) becomes 1V and remains 1V until t= 0.5ms. When x(t) becomes 1V the voltage across y(t) starts to increase as the resistance across the inductor decreases and allows current to pass through the circuit. From here the signal will drop and take on some negative values before climbing positive again as dictated by the oscillating impulse response. This simply indicates a change in output current direction. At t=0.5 ms. x(t) becomes 0V. The y(t) voltage will have a slight curve as it goes back to zero since the capacitor and inductor will have charged up and will discharge when the voltage source x(t) becomes 0V.

Note: We used the Java Convolution tool to generate the graph, but due to the limitations in time constraints, we were not able to extend the periodic output for all time.

(c) Sketch the output if x(t) is a train of pulses of width 1 millsecond and a period of 10 seconds.



This graph would like the one in part b, but now the output would become periodic. An output identical to part b would occur every 10 seconds.

Note: We used the Java Convolution tool to generate the graph, but due to the limitations in time constraints, we were not able to extend the periodic output for all time.

(d) Suppose the period is dropped to 0.1 seconds. How does this affect the shape of the output?

By decreasing the period that just brings our output pulses together. Creates a narrower bandwidth and a sharper amplitude.

Problem No. 3: The dreaded thought problem...

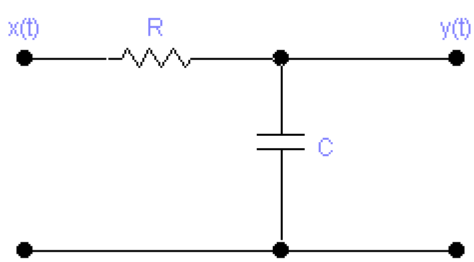
(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

The cruise control of a car is a low pass filter because it cuts off all high frequency inputs to create a smooth steady change of speed. The system is causal because it does not anticipate it's future input. In fact it is not possible for the system to anticipate the terrain it may encounter ahead because there is no completely described path. The system is also instantaneous because it does not use past values of the input to create an output. The system is also linear because when you increase the speed of the car or when it receives an input you get the output to be the speed of the car plus the increase. Therefore, this is a linear relationship in the fact that in the fact superposition holds. The system is also fixed because the input output relationship does not change with time. The system is also a continuous - time system because the input signal is defined at all points in time.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

For a good cruise control system the circuit should attenuate the output for high frequencies and give a smooth curve output.

To implement this in a circuit, run the signal through a low-pass filter as seen below. X(t) is the input Y(t) is the output.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

A cruise control system is not optimal for a mountainous terrian because at the peak of each mountain there is a discontinuity. The elevation changes so frequently that the car's speed will vary significantly, thus defeating the purpose of cruise control, which is to maintain a constant velocity.

The circuit is modifying the input signal to give you a steady curve so your curve will not decrease rapidly as you go over the peak and therefore your accelerator will still be accelerating the car for a short time after you are in fact going down hill. This can be seen below. The triangular signal would be the same as the signal for a mountain and the output signal is how much acceleration your system is producing. Sensors could be set up on the automobile that read the level of the car as well as possibly using a GPS satellite system that would identify the road ahead to indicate the elevation. The weight of the car will have to be maintained by sensors in the shocks so that the vehicle will know how much to increase or decrease the speed of the vehicle.