Name: **Mark Mckinney**

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
3c	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Fourier Series

(a) For the system below, compute the Fourier series of the output.

$y(t) = x^2(t) = (A \cos w_o t)^2$

$y(t) = A^2 \cos {}^2 w_o t$

identity: $\cos^2 u = ½ (1 + \cos 2u)$
applying: identity you get

$y(t) = A^2/2 + A^2 /2 *\cos (2w_o t)$

knowing that cosine is an even function you get $b_n = 0$ & $a_n \neq 0$
therefore you only have a_n terms

knowing that the fourier series of a sine or cosine is its own fourier series we obtain

$a_0 = A^2 /2$ $a_2 = A^2 / 2$

so the fourier series is $y(t) = a_o + a(2) \cos 2w_o t$

the answer is $y(t) = A^2 / 2 + A^2 / 2 *\cos 2w_o t$

(b) Compute the energy and power of the output. (Comment: Think about orthogonality before you write the answer that is most obvious.)

output is $y(t) = A^2 / 2 + A^2 / 2 \cos^2 w_o t$
because the output is a periodic function with infinite area under the curve its energy is infinite.
$E = \lim_{T\to\infty} \int_{-T}^{T} | y(t) |^2 dt$
$E = \lim_{T\to\infty} \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)] dt$
integrating it gives us $= \lim_{T\to\infty} (A^4 / 4 + A^4/8) 2T_o$
therefore $E = \infty$
$P = \lim_{T\to\infty} 1 / 2T \int_{-T}^{T} | y(t) |^2 dt$
$P = \lim_{T\to\infty} 1 /2T\int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)] dt$
substituting $wo = 2\pi f / T$ the first cosine term goes to zero we then have
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos^2(w_o T)] dt$
using trigonometric identity from 1A on the $\cos^2$ term we get
$1 / 2T_o \int_{-T}^{T} [A^4 / 4 + A^4/8 + A^4 /4 \cos^2(w_o T) + A^4/4 \cos(4w_o T)] dt$

after substituting $wo = 2\pi f / T$ the cosine term goes to zero we have
$1 / 2T_o \int_{-T}^{T} (A^4 / 4 + A^4/8)$
integrating $1 / 2T_o (A^4 / 4 + A^4/8) 2T_o$; the $2T_o$ cancelled leaving a power of $A^4 / 4 + A^4/8$

(c) Now assume x(t) is as shown to the right. Compute the energy and power of y(t).

$x(t) = -t + 2 \qquad 0 \leq t \leq 2$

$y(t) = x^2(t) = (-t + 2)^2$

$F = \lim_{T \to \infty} \int_{-T}^{T} (-t + 2)^4 \, dt$

using U substitution $\quad u = -t + 2$
$\qquad du = -dt$

We have $E = -\int u^4 \, du$

$-1/5 (-t + 2)^5$ evaluated from [0,2]

$0-(-32 / 5) = 32/5$ Joules $= 6.4$ Joules

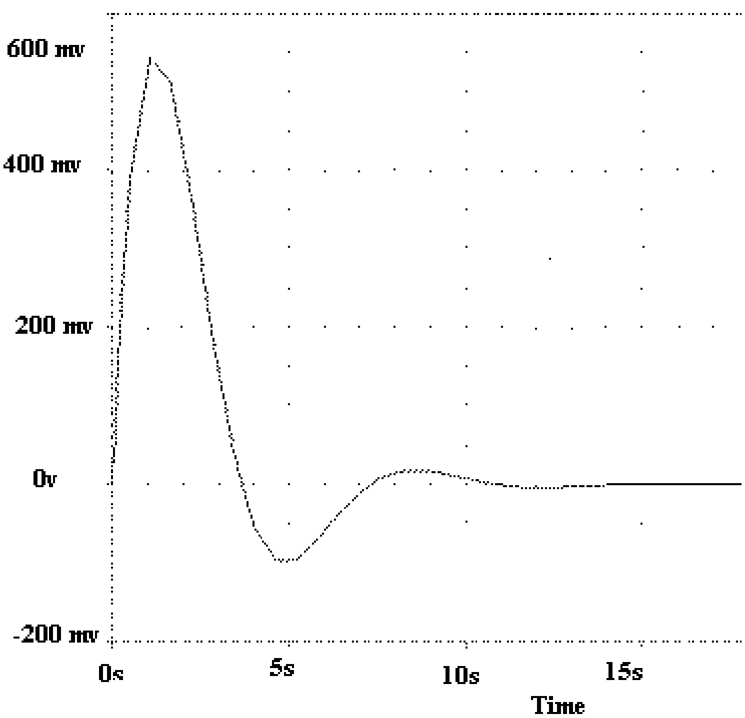
$P = \lim_{T \to \infty} 1 / 2T \int_{-T}^{T} | (t^2 - 4t + 4) |^2 \, dt$

P = 0 because you have infinity in the denominator which will give you zero.

Problem No. 2: Time-Domain Solutions

(a) For the circuit shown, using concepts developed in this class, sketch the output signal, , if x(t) is the unit step function. Explain.

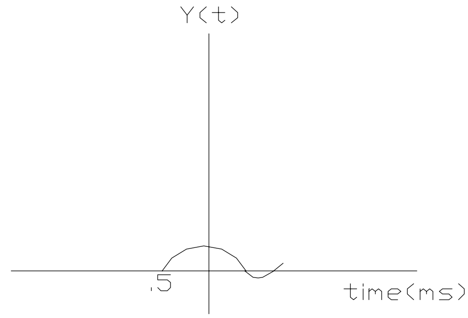
v(t) = e ^(-a*t) (b1cos(ωd*t) + b2 sin(ωd*t));This is the transient response of an underdamped RLC network.



The reason y(t) looks as it does when a unit step is the input is because this signal is the natural response of the circuit. It is known that three is no voltage across an inductor if the current through it is not changing with time. Also, it is impossible to change the current through an inducator by a finite amount in zero time. Also, in the capacitor there is not current passing through the capacitor if the voltage across it is not changing with time. A capacitor is an open circuit to DC. Also it is impossible to change the voltage across a capacitor by a finite amount in zero time. Therefore, the steady state of this circuit, for infinite time, will be no current flow across the resitor and therfore the votlage across the resistor will be zero for infinte time. The only time current will flow through the resitor is during the transient response cause by the discontinuity at 't' equal to zero. Also this circuit is an underdamped series RLC circuit. It is an underdamped network because omega is larger than alpha. Therefore the output on the resistor is a damped sinusoidal response.

$\omega = \frac{1}{\sqrt{LC}} = 1$ $\qquad$ $\alpha = \frac{1}{2RC} = .5$

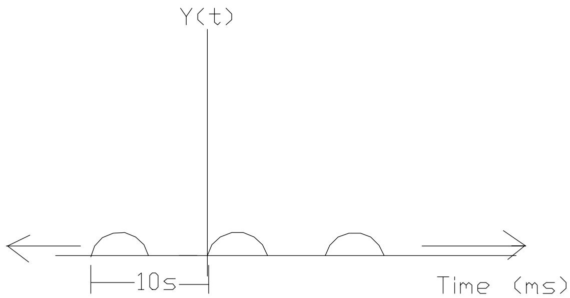
(b) Sketch the output if x(t) is a unit pulse with width of 1 millsecond. Explain.



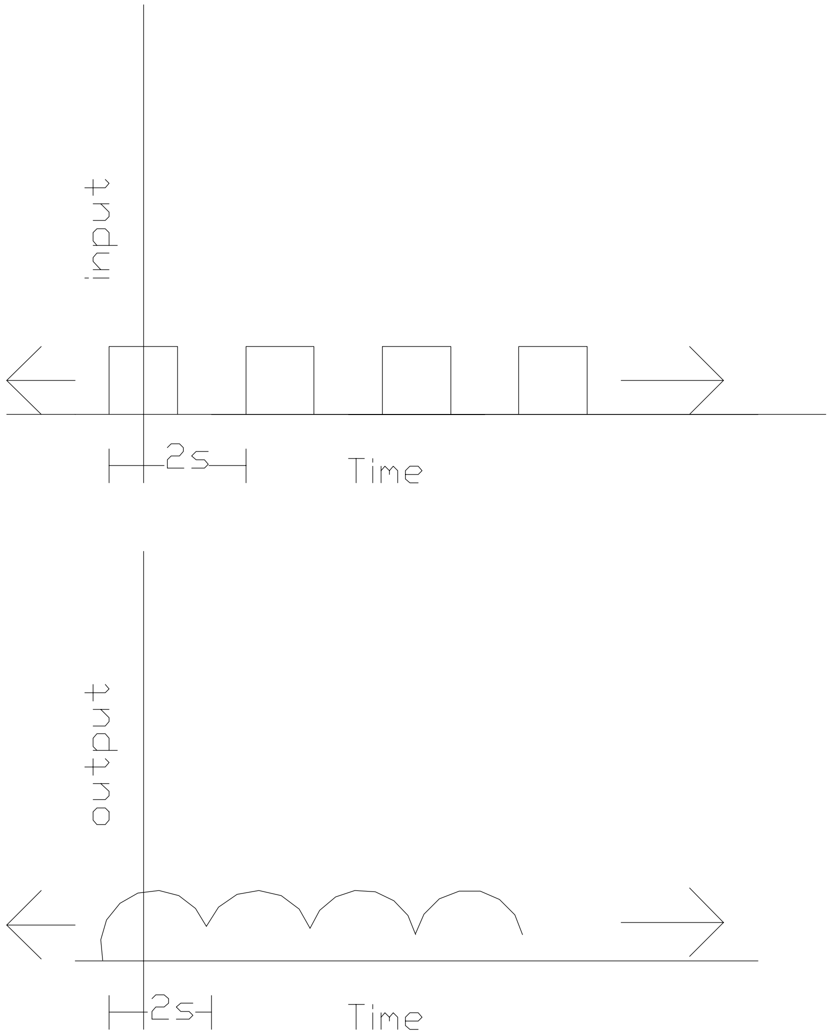
Assuming the pulse to have a width of 1 ms: At t= - 0.5 ms x(t) becomes 1V and remains 1V until t= 0.5ms. When x(t) becomes 1V the voltage across y(t) starts to increase as the resistance across the inductor decreases and allows current to pass through the circuit, but at t=0.5 ms. x(t) becomes 0V.

(c) Sketch the output if x(t) is a train of pulses of width 1 millsecond and a period of 10 seconds.

This graph would like the one in part b, but now the output would become periodic. An output identical to part b would occur every 10 seconds.



(d) Suppose the period is dropped to 0.1 seconds. How does this affect the shape of the output?



The period of the pulse is 2 seconds but the pulse width is still one second, this causes overlap. The pulses have overlap this means that they add one onto another. The output signal does not have time to approach zero before the next impulse begins.

*The impact this problem has on modem design is seen on the output of this pulse train. As the signal is being put into the system the response of the system rounds the edges of the signal. The rounded edgesof the output would cause a problem for designing a modem. These rounded edges could be seen as pieces of the signal being lost during the transfer across the line of the modem

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Problem No. 3: The dreaded thought problem...

(a) Using concepts developed in the first three chapters of the book, characterize the cruise control circuit in an automobile as a system.

The cruise control of a car is a low pass filter because it cuts off all high frequency inputs to create a smooth steady change of speed.

The system is causal because it does not anticipate it's future input. In fact it is not possible for the system to anticipate the terrain it may encounter ahead because there is no completely described path.

The system is also instantaneous because it does not use past values of the input to create an output.

The system is also linear because when you increase the speed of the car or when it receives an input you get the output to be the speed of the car plus the increase. Therefore, this is a linear relationship in the fact that in the fact superposition holds.

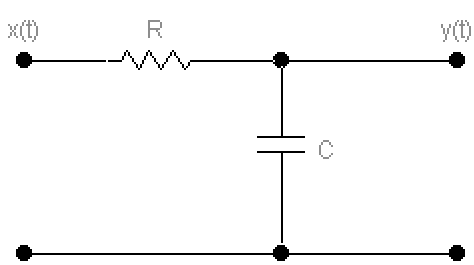
The system is also fixed because the input output relationship does not change with time.

The system is also a continuous - time system because is always inputting a signal.

(b) If this system were a linear time-invariant system, design the impulse response of a "good" system. Explain how you might implement this in a circuit.

For a good cruise control system the impulse responses for the sysem would be an integrator where it smoothes out the input discontinuities and gives a smooth curve output.

To implement this in a circuit run the signal through a series RC circuit as seen below. X(t) is the input Y(t) is the output.



(c) It is well-known that cruise controls are not optimal for terrain that is mountainous (such as ski country in Colorado). Why (explain this in terms of concepts discussed in this class)? What modifications would you need to make (again, explain this in terms of concepts such as causality)?

A cruise control system is not optimal for a mountaineous terrian because at the peak of a mountain you have a discountinutity.

Your circuit is integrating your input singnal to give you a steady curve so your curve will not decrease rapidly as you go over the peak and therefore your accelerator will still be accelerating the car for a short time after you are in fact going down hill. This can be seen below for the triangular signal would be the same as the signal for a mountain and the output signal is how much acceleration your system is producing.

*The triangle pulse is the input to the cruise control such as a mountain. The smooth curves are the output, and the slope of the curve depends on the value of R and C.