Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

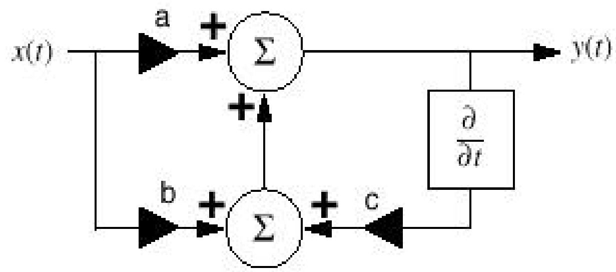
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.
The equation can be written by inspection

$a*x(t)+b*x(t)+c*dy/dt=y(t)$

$y(t)-c*dy/dt=a*x(t)+bx(t)$

(b) Find the transfer function.
Transfer to the s domain and isolate Y(s)/X(s)

$$Y(s) - s\cdot c\cdot Y(s) = (a+b)\cdot X(s)$$

$$Y(s)\cdot(1 - s\cdot c) = (a+b)\cdot X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{a+b}{1 - s\cdot c}$$

$$\frac{Y(s)}{X(s)} = \frac{-c(a+b)}{s - \frac{1}{c}}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

This system will be unstable when c is positive because there will be a pole in the right half s plane.

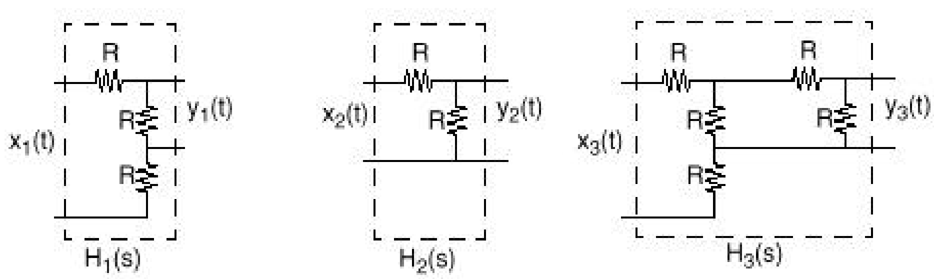
(d) Find the impulse response.

$$H(s) = -c \cdot (a + b) \cdot \frac{1}{\left(s - \frac{1}{c}\right)}$$

$$h(t) = \left[-c \cdot (a + b) \cdot \exp\left(\frac{-1}{c}\right)\right] \cdot u(t)$$

Stable when c is negative.

Problem No. 2: Transfer Functions
For the circuit shown below:



(a) Find H1(s):

$$H_1(s) = \frac{Y_1(s)}{X_1(s)}$$

$$Y_1(s) := \frac{R}{3 \cdot R} \cdot X_1(s)$$

$$\frac{Y_1(s)}{X_1(s)} = 0.333$$

$$H_1(s) := \frac{1}{3}$$

(b) Find H2(s):

$$H_1(s) = \frac{Y_2(s)}{X_2(s)}$$

$$Y_2(s) := \frac{R}{2 \cdot R} \cdot X_2(s)$$

$$\frac{Y_2(s)}{X_2(s)} = 0.5$$

$$H_2(s) := \frac{1}{2}$$

(c) Find H3(s):

Start by writing two loop equations and an equation for Y_3.

Loop 1 :

$$X_3(s) = I_1 \cdot R + (I_1 - I_2) \cdot R + I_1 \cdot R$$
$$X_3(s3) = R \cdot I_1 - R \cdot I_2$$

Loop 2 :

$$(I_2 - I_1) \cdot R + I_2 \cdot R + I_2 \cdot R = 0$$
$$3 \cdot I_2 - R \cdot I_1 = 0$$

Y_3 :

$$Y_3(s) = I_2 \cdot R$$

Solve for the equations for I_2 and I_1 simultaneously by using matrix algebra, $X*Y^{-1}=I$.

$$\begin{bmatrix} 3 \cdot R & -R \\ -R & 3 \cdot R \end{bmatrix} \cdot \begin{bmatrix} X_3(s) \\ 0 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$I_1 = \frac{.375 \cdot X_3(s)}{R}$$

$$I_2 = \frac{.125 \cdot X_3(s)}{R}$$

$$Y_3(s) = \frac{.125 \cdot X(s)}{R} \cdot R$$

$$H_3(s) = .125 \qquad H_3(s) = \frac{1}{8}$$

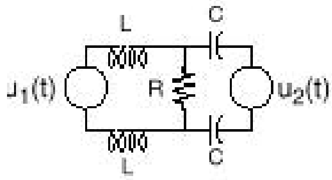
(d) Is ? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

No. Justification: You can't simply say that just by connecting these two circuits together that they the new transfer function is the superposition of both transfer functions because when you add the circuits together they become interdependant. That is you could say that the right half of the circuit is loading the left half of the circuit and therefore that changes the overall system response.

$$H_1(s) \cdot H_2(s) = \frac{1}{6}$$

$$\frac{1}{6} \neq \frac{1}{8}$$

Problem No. 3: The "Interesting" Problem



(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

The current through both inductors and the voltage on both capacitors wil be the same and neither will affect the output v_o. Therefore only two state variables are needed.

Equation 1 $L\cdot\frac{d}{dt}i_L = u_1(t) - v_c$

$\frac{d}{dt}i_L = \frac{u_1(t)}{L} - \frac{v_c}{L}$

Equation 2 $C\cdot\frac{d}{dt}v_c = I_L - I_R$ $I_R = \frac{u_1(t) - v_c}{R}$

$C\cdot\frac{d}{dt}v_c = I_L - \frac{(u_1(t) - v_c)}{R}$

$\frac{d}{dt}v_c = \frac{I_L}{C} - \frac{u_1(t)}{R\cdot C} + \frac{v_c}{R\cdot C}$

Output Eq, $v_o = \frac{u_1(t) - v_c}{R}\cdot R$

$v_o = u_1(t) - v_c$

And the state-variable matrices are as follows :

$$\begin{bmatrix} \frac{d}{dt}i_L \\ \frac{d}{dt}v_c \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} \\ \frac{1}{C} & \frac{1}{R} \end{bmatrix}\cdot\begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ \frac{-1}{R} \end{bmatrix}\cdot u_1(t)$$

$$v_o = u_1(t) + (0 \;\; -1)\cdot\begin{bmatrix} i_l \\ v_c \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

I used two state variables. One for each the top capacitor and the top inductor. The state variables represented the voltage across the capacitor and the current through the inductor.

$$\frac{d}{dt} v_c = -u_2(t)$$

$$\frac{d}{dt} i_L = \text{current in loop one}$$