Name: Dontraé Caldwell

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

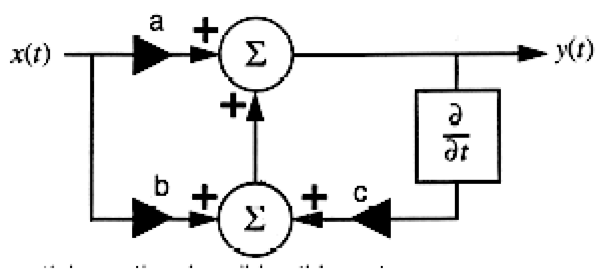
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams

(a) Write a differential equation describing this system.

$$Ax(t) + [bx(t) + c * (dy/dt) * y(t)] = y(t)$$



(b) Find the transfer function.

$$Ax(s) + bx(s) + csy(s) = y(s)$$
$$(a + b)\, x(s) = y(s)(1\text{-}sc)$$
$$H(s) = y(s) / x(s) = (a + b) / (1 - sc)$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

The system is stable if there is no zeros in the Right Hand Plane. For this to be true, all values of c need to be less than 0. For all values for a & b cannot be equal to zero.

(d) Find the impulse response.

$Y(s) = {}^{-(a+b)}\!/_{sc-1} = {}^{-a}\!/_{sc-1} - {}^{b}\!/_{sc-1}$

$Y(s) = L^{-1}\{y(s)\} = -aL^{-1}\{ {}^{1}\!/_{sc-1}\} - bL^{-1}\{ {}^{1}\!/_{sc-1}\}$

$= -aL^{-1}\{ {}^{1/c}\!/_{s-1/c}\} - bL^{-1}\{ {}^{1/c}\!/_{s-1/c}\}$

$= -(a/c)L^{-1}\{ {}^{1}\!/_{s-1/c}\} - bL^{-1}\{ {}^{1}\!/_{s-1/c}\}$

$y(t) = -(a/c)\, e^{(1/c)\, t} - (b/c)\, e^{(1/c)\, t}$

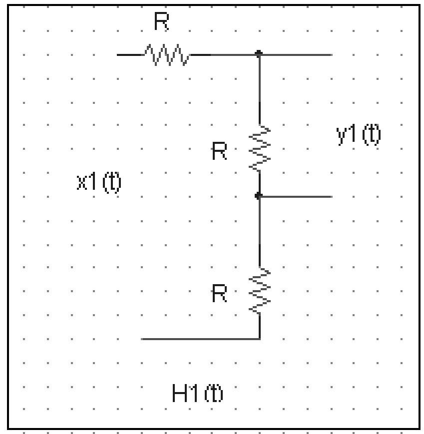
$y(t) = ({}^{-(a+b)}\!/_{c})e^{(1/c)\, t}$

Problem No. 2: Transfer Functions

For the circuit shown below:

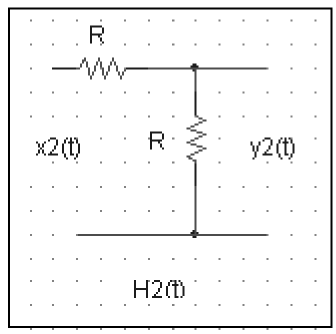
(a) Find $H_1(s)$:

$$H_1(s) = R / (R + 2R) = 1/3$$



(b) Find $H_2(s)$:

$$H_2(s) = R / 2R = 1/2$$

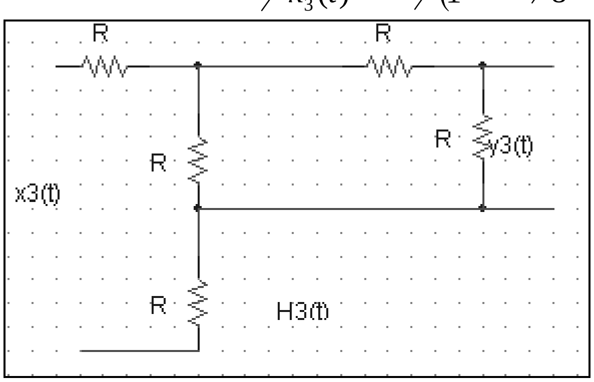


(c) Find $H_3(s)$:

By finding the equivalent impedance for the top three resistors:
2R // R = 2R / 3
(2R/3) / (2R + 2R /3) = ¼

y3(t) = 1/8

$$H_3(s) = \frac{y_3(t)}{x_3(t)} = \frac{1/8}{(1} = 1/8$$



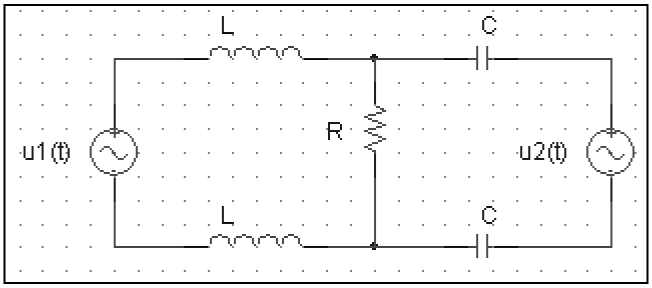
(d) Is $H_3(s) = H_1(s) * H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

No, because such an assumption is equivalent to neglecting all loading effects. We must take into account the effects of loading if we want accurate results. Also for the statement above to be true is is saying that when we connect $H_2(s)$ to $H_1(s)$ we will not be changing the characteristics of the circuit. In some such case there will be instances that this will hold true, but in this case it will not and has been verified mathematically.

$H_3(s) = H_1(s) * H_2(s) = (R / 2R) * (R / 3R) = 1/6$ which is not equal to 1 /8 found above in part c.

Problem No. 3: The "Interesting" Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.



Mesh 1 Mesh 2

KCL @ 1 $\frac{u1 - y}{sL} + \frac{y}{R}$ - C(dVc/dt)

Mesh 1: $-u_1 + Li_L + (i_L - i_2)R + Li_L = 0$

$2L = \frac{diL(t)}{dt} = u_1(t) - [i_L(t) - i_2(t)]R$

$\frac{diL(t)}{dt} = \frac{u1(t)}{2L} - \frac{iL(t)R}{2L} + \frac{i2(t)R}{2L}$

Mesh2: $R(i_2 - i_1) + Vc(t) + u_2 + Vc = 0$

$[i_2(t) - i_L(t)]R + u_2(t) + 2Vc(t) = 0$

$i_2(t)R = i_L(t)R - u_2(t) - 2Vc(t)$

$i2(t) = C\frac{dVc(t)}{dt}$

$\frac{dVc(t)}{dt}RC = i_L(t)R - u_2(t) - Vc(t) \Rightarrow \frac{dVc(t)}{dt} = \frac{iL(t)}{C} - \frac{2Vc(t)}{RC} - \frac{u2(t)}{RC}$

$\frac{diL(t)}{dt} = \frac{u1(t)}{2L} - \frac{diL(t)R}{2L} + \frac{1}{2L}[i_L(t)R - u_2(t)\ 2Vc(t)]$

$\frac{diL(t)}{dt} = \frac{1}{2L}[u_1(t) + u_2(t)] - \frac{Vc(t)}{L}$

$y(t) = 2Vc(t) + u_2(t)$

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

$$\frac{dx1}{dt} = -\frac{1}{L}x2 + \frac{1}{L}x2 + \frac{1}{2L}u2$$

$$\frac{dx2}{dt} = \frac{1}{C}x_1 - \frac{2}{RC}x_2 - \frac{1}{RC}u_2$$

$$y = 2x_2 + u_2$$

$$\begin{bmatrix} \frac{dx1}{dt} \\ \frac{dx2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{2}{RC} \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2L} & \frac{1}{2L} \\ 0 & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} u1 \\ u2 \end{bmatrix}$$

$$[y] = [0 \quad 2] \begin{bmatrix} x1 \\ x2 \end{bmatrix} + [0 \quad 1] \begin{bmatrix} u1 \\ u2 \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

I should have used two state variables and did two loop equations. One to find i1(L) and the other to find the voltage through the capacitor. Using KVL the four equations are coupled together and then they are reduced down by a factor of two because of that. Duplicates circuit elements in a similar fashion would also just make solving for the 4 state variables redundant.