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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

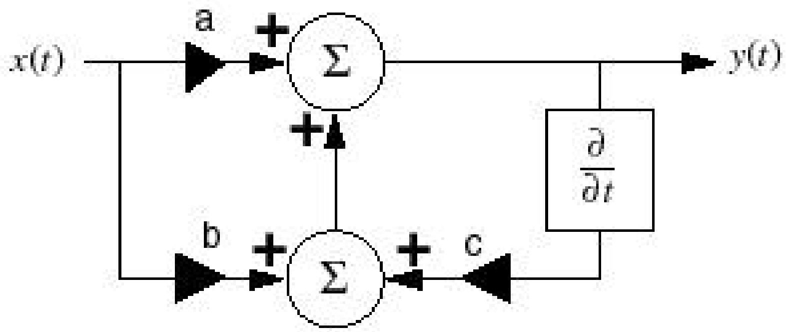
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

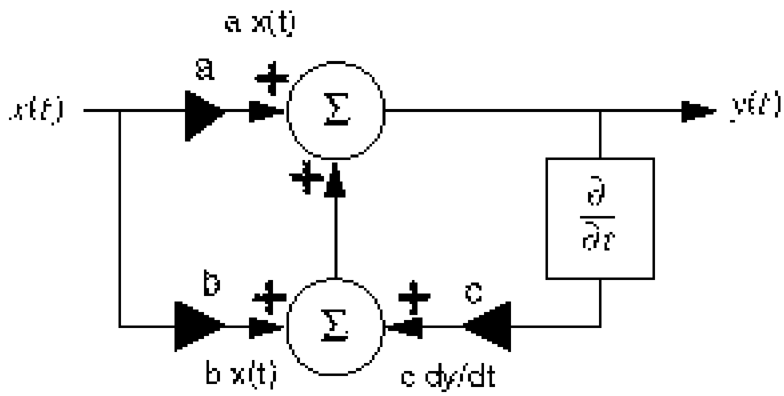
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.



The labels that were assigned to the block diagrams critical points are read and combined when necesssary. The values of the combined signals going into the top summer are added to get the output y(t). Below is the differential equation gained from adding the signals at the tope summer.

$$a\,x(t) + b\,x(t) + c\,\frac{dy(t)}{dt} = y(t)$$

(b) Find the transfer function.

The transfer function is found by taking the Laplace transform of the differential equation from section 1-a.

L {y(t)} = L { a x(t) + b x(t) + c dy(t)/dt }

y(s) = a x(s) + b x(s) + c [s y(s) - y(0^-)]

the initial value, y(0^-), is assumed to be zero in our case therefore we now have:

y(s) = a x(s) + b x(s) + cs y(s)

dividing by x(s) throughout the equation to get H(s) where H(s)=y(s) / x(s)

a + b + cs H(s) = H(s)

after doing a little simple algebra to get H(s) on one side we get the equation of the transfer function in its final form.

$$H(s) = \frac{(a+b)}{(1-cs)}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

for the system described above to be stable:

'a' and 'b' can be any postive real number
'c' must be a negative number

(d) Find the impulse response.

given our transfer function:

$$H(s) = \frac{(a+b)}{(1-cs)}$$

must must take the inverse Laplace transform to get the impulse response of the system.

$$L^{-1}\{H(s)\} = L^{-1}\{\frac{(a+b)}{(1-cs)}\}$$

pulling the constant, (a+b) outside of the inverse Laplace you get

$$h(t) = (a+b)\, L^{-1}\{\frac{1}{(1-cs)}\}$$

divide throughout the right hand side by '- c' and pull this constant value outside of the Laplace transform you get

$$h(t) = (a+b)\left(\frac{-1}{c}\right) L^{-1}\left\{\frac{1}{\left(s - \frac{1}{c}\right)}\right\}$$

now that we have the argument of the Laplace transform in a form found on a table we can transform it, we get the impulse response to be

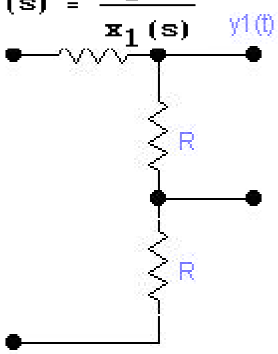
$$h(t) = (a+b)\left(\frac{-1}{c}\right) e^{\frac{t}{c}}\, u(t)$$

Problem No. 2: Transfer Functions

For the circuits shown below:

(a) Find $H_1(s)$:

$$H_1(s) = \frac{y_1(s)}{x_1(s)}$$



$$H_1(s) = \frac{y_1(s)}{x_1(s)}$$

writing out a voltage divider equation for the voltage across R

$$y_1(s) = x_1(s)\left(\frac{R}{3R}\right)$$

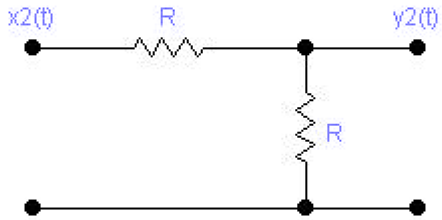
simplifying the equation:

$$y_1(s) = x_1(s)\left(\frac{1}{3}\right)$$

dividing both sides by $x_1(s)$ to get our transfer function $H_1(s) = y_1(s)/x_1(s)$ we get

$H_1(s) = 1/3$

(b) Find $H_2(s)$:



$$H_2(s) = \frac{y_2(s)}{x_2(s)}$$

writing out a voltage divider equation for the voltage across R

$$y_2(s) = x_2(s)\left(\frac{R}{2R}\right)$$

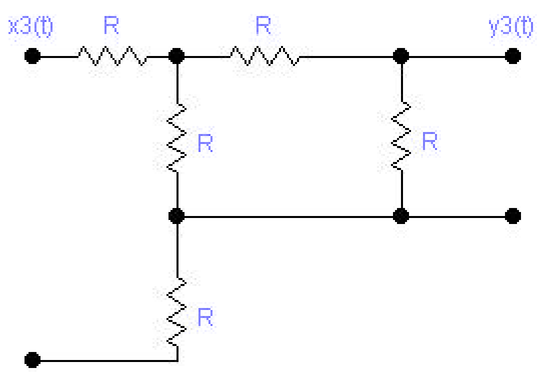
simplifying the equation by cancelling R's

$$y_2(s) = x_2(s)\left(\frac{1}{2}\right)$$

dividing both sides by $x_2(s)$ to get the transfer function $H_2(s)$ we get

$$H_2(s) = \left(\frac{1}{2}\right)$$

(c) Find $H_3(s)$:



$$H_3(s) = \frac{y_3(s)}{x_3(s)}$$

We are going to do a voltage division calculation to get our transfer function. To better calculate our problem we will assign the voltage across the middle resistor (the $x_1(s)$ resistor) to be w(s).

solving this voltage division for w(s) we get

$$w(s) = \left(x_3(s)\,\frac{(R\ ||\ 2R)}{(2R + R\,||\,2R)}\right)$$

further simplifying

$$w(s) = x_3(s)\,\frac{\frac{2R}{3}}{\left(2R + \frac{2R}{3}\right)}$$

from combining and cancelling resistors we get

$$w(s) = \frac{x_3(s)}{4}$$

solving $y_3(s)$ in terms of w(s) we get

$$y_3(s) = \frac{w(s)}{2}$$

substituting in w(s) in the previous equation we get

$$y_3(s) = x_3(s)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = x_3(s)\left(\frac{1}{8}\right)$$

now we get the transfer function $H_3(s)$ by dividing both sides by $x_3(s)$ to get

$$H_3(s) = \frac{1}{8}$$

(d) Is $H_3(s) = H_2(s) \cdot H_1(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

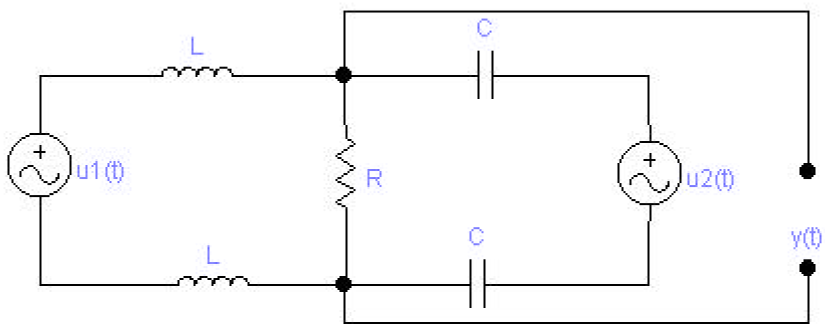
$H_3(s) = 1/8$; $H_2(s) = 1/2$; $H_1(s) = 1/3$

$1/8 \neq (1/2) \cdot (1/3) = (1/6)$

1/8 does not not equal 1/6

Therefore the answer is NO. Even though circuit three is a combination of circuits one and two, the tranfer function of the two combined circuits is not equal to their individual transfer functions multiplied together. This discrepency is due to loading of the circuit. Loading is the dependence of the individual circuit element's voltages or currents on the other circuit element's characteristics in the system. In our case the loading effect is seen in the voltage division that occurs when circuits one and two are combined to create circuit three.

Problem No. 3: The "Interesting" Problem



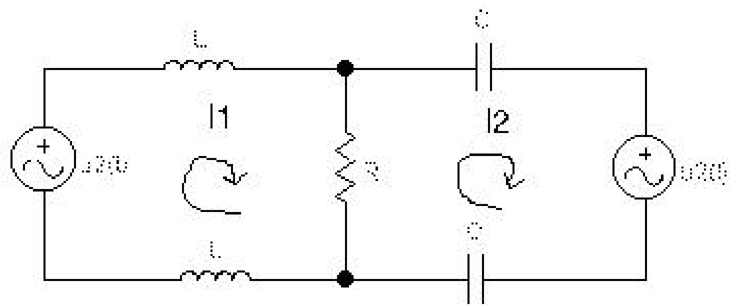
(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

First we write our general state equations.

x' = Ax + Bu	also:	i_L' is the derivative of i_L
y = Cx + Du		V_c' is the derivative of V_c

where i_L and V_c are our state variables (x) for this system and our inputs (u) are $u_1(t)$ and $u_2(t)$. Our output (y) is y(t), and our variables A,B,C,D desribe the system interconnection and behavior.

To obtain some equations to work with we need to create two mesh currents and write the KVL equations for those loops. The first is on the left and the second is on the right.



KVL 1) $-u_1(t) + L\, i_1' + R(i_1 - i_2) + L\, i_1' = 0$

$-u_1(t) + 2\, L\, i_1' + R\, i_1 - R\, i_2 = 0$

KVL 2) $u_2(t) + v_c + R\,(i_2 - i_1) + v_c = 0$

$u_2(t) + 2\, v_c - R\, i_1 + R\, i_2 = 0$

assuming that: $i_1 = i_L$ and $i_2 = C\, V_c'$

we now use these values and plug them into the obtained KVL equations and simplify

from KVL 1) $-u_1(t) + 2\, L\, i_L + R\, i_L - R\, C\, V_c' = 0$

$2\, L\, i_L' = R\, C\, V_c' + u_1(t) - R\, i_L$

from KVL 2) $u_2(t) + 2\, v_c + RC\, V_c' - R\, i_L = 0$

$RC\, V_c' = -2\, v_c + R\, i_L - u_2(t)$ we call this equation (3)

dividing through by RC in the above equation will give us one of the necessary state equations. We get:

$$(4) \qquad V_C' = \frac{-2\, v_C}{RC} + \frac{i_L}{C} + \frac{-u_2(t)}{RC}$$

now we get a useful equation by substituting equation (3) into KVL 1

$$2\, L\, i_L' = -2\, v_c + R\, i_L - u_2(t) + u_1(t) - R\, i_L$$

canceling out the R i_L terms and dividing through by 2L to get i_L' on a side by itself we get:

$$(5) \qquad i_L' = \frac{-v_C}{L} - \frac{-u_2(t)}{2L} + \frac{u_1(t)}{2L}$$

this is our other input state variable equation.

Now we map equations four and five into matrix form and we get the following:

$$\begin{bmatrix} V_c' \\ I_L' \end{bmatrix} = \begin{bmatrix} -2V_c/RC & 1/C \\ -V_c/L & 0 \end{bmatrix} \begin{bmatrix} V_c \\ I_L \end{bmatrix} + \begin{bmatrix} 0 & -1/RC \\ 1/2L & 1/2L \end{bmatrix} \begin{bmatrix} u1(t) \\ u2(t) \end{bmatrix}$$

Now we must solve for the output matrix.

From the problem definition we know that:

$y(t) = R\ (i_1 - i_2)$

substituting our earlier assumption of the mesh currents into this equation we get

$y(t) = R\ i_L - RC\ V_c'$

now, substituting equation four into the previous equation gives us the following:

$y(t) = R\ i_L - RC\ (-\ 2\ v_c /RC + i_L /C - u_2(t)/RC)$

distributing RC through, cancelling out and combining terms we get:

$y(t) = 2\ R\ i_L + 2\ v_c + u_2(t)$

now mapping this equation into a state variable matrix we obtain the output matrix

$$\begin{bmatrix} y(t) \end{bmatrix} = \begin{bmatrix} 2R & 2 \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} u1(t) \\ u2(t) \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

You should have only used two state variables to solve this system. If you had used four, your matrix would have had some values of zero in it since this is a symmetric circuit and redundance would have been encountered. By symmetric I mean L1 = L2 and C1 = C2. This fact along with the elements placement in the circuit (ie symmetrically about each source), leads to the conclusion that to solve this you only need to examine one of the capacitors and one of the inductors. It doesn't matter which ones you examine because the answer will work out to be the same.

The first state variable corresponds to the inductor current. The second state variable corresponds to the capacitor voltage. Again is does not matter which inductor or capacitor you choose to analyze as a state variable.

If part A had been worked with four state variables, these being: inductor one and two currents, and capacitor one and two voltages, it would have become apparent that this was unnecessary. We would have labeled our inductor currents as one and two in the original KVL equation. Since this is a mesh loop and the inductors are on the outside edge of the loop, and they have the same inductance, it is totally unecessary to label them seperately because the magnitude of the current on the outside of a loop is always the same. Knowing this, the derivative of the currents would also equal. Therefore you only need one of the inductor currents as a state variable. As for the two capacitor's voltages that would have been labeled seperately, we also see that this was unecessary also. We assumed that current in loop two was equal to the capacitance times the derivative of the voltage on the capacitor. Both of the capacitors had the same value for capacitance and also were on the outside of the mesh loop (same current), therefore the magnitude of the voltage across each of the capacitors is equal, and consequently the derivative of the voltage of the capacitors is the same. Simple nodal and mesh analysis proves you only need two state variables for this circuit.