Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams

(a) Write a differential equation describing this system.

$ax(t) + [bx(t) + c\,dy/dt] = y(t)$

(b) Find the transfer function.

$aX(s) + bX(s) + csY(s) = Y(s)$

$(a + b)X(s) = Y(s)(1 - sc)$

$H(s) = Y(s)/X(s) = (a + b)/(-cs + 1)$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

A stable system has no poles in the right half plane. For the previous system to be stable the a and b constants may take on any value other than the designated zero values and the c constant can be any value less than zero.

(d) Find the impulse response.

$H(s) = (a + b)/(-cs +1) = (-a – b)/(cs – 1) = -a/(sc – 1) – b/(sc – 1)$

$= -a\{(1/c)/(s – 1/c)\} – b\{(1/c)/(s – 1/c)\}$

$= -a/c\{1/(s – 1/c)\} – b/c\{1/(s – 1/c)\};$

$h(t) = L^{-1}\{H(s)\} = -a/c\ L^{-1}\{1/(s – 1/c)\} – b/c\ L^{-1}\{1/(s – 1/c)\}$

$= (-a/c)*e^{t/c} – (b/c)*e^{t/c}$

$= -[(a + b)/c]*e^{t/c}$

Problem No. 2: Transfer Functions

For the circuit shown below:

(a) Find $H_1(s)$:

$H_1(s) = R/3R = 1/3$

(b) Find $H_2(s)$:

$H_2(s) = R/2R = 1/2$

(c) Find $H_3(s)$:

First, finding an equivalent impedence for the top three resistors:

$2R//R = 2R^2/3R = 2R/3$

Now taking X3(t) = 1V we get a voltage divider of:

(2R/3)/(2R +2R/3) = ¼

Using this computed value as the voltage across the last two resistors, Y3(t) = 1/8

Making the transfer function Y3/X3 = (1/8)/1 = 1/8

(d) Is $H_3(s) = H_1(s) * H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

$H_3(s) =? H_1(s) * H_2(s)$

1/8 =? 1/2 * 1/3

No, $1/8 \neq 1/6$

As the values show, the total circuit transfer function will not be eqivalent to the product of the 2 independent transfer functions because the total independent product fails to take connectivity phenomena into account. Voltages and currents are altered by connecting the two circuits in that physically connecting the circuits alters circuit properties by means of voltage division among the resistors. This does not occur if the individual circuits are viewed independently.

Problem No. 3: The “Interesting” Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

1: KVL (mesh 1): $-u_1 + Li_L^\bullet + (i_L - i_2)R + Li_L^\bullet = 0$

2: KVL (mesh 2): $(i_2 - i_L)R + v_c + u_2 + v_c = 0$

$2\,Li_L^\bullet = u_1 - (i_L - i_2)R$

$2\,Li_L^\bullet = u_1 - i_LR + i_2R$

3: $i_L^\bullet = u_1/2L - (i_LR)/2L + (i_2R)/2L$

$(i_2 - i_L)R + 2v_c + u_2 = 0$

$i_2R = i_LR - u_2 - 2v_c$

4: $i_2 = Cv_c^\bullet$

5: $Cv_c^\bullet R = i_LR - u_2 - 2v_c$

$v_c^\bullet = i_L/C - u_2/RC - 2v_c/RC$

substituting 4 into 3:

6: $i_L^\bullet = u_1/2L - (i_LR)/2L + (Cv_c^\bullet R)/2L$

substituting 5 into 6:

$i_L^\bullet = u_1/2L - (i_LR)/2L + (i_LR - u_2 - 2v_c)/2L$

$i_L^\bullet = (u_1 - u_2)/2L - v_c/L$

by inspection:

$y(t) = 2v_c + u_2$

now writing equations in standard form:

$x^\bullet = Ax + Bu$
$y = Cx + Du$

$i_L^\bullet = \quad - v_c/L \quad + u_1/2L - u_2/2L$

$v_c^\bullet = i_L/C - 2v_c/RC \qquad + u_2/RC$

$y =$ $2v_c$ $+ u_2$

$$\begin{bmatrix} \mathrm{iL}\bullet \\ \mathrm{vc}\bullet \end{bmatrix} = \begin{bmatrix} 0 & -1/L \\ 1/C & -2/RC \end{bmatrix} \begin{bmatrix} \mathrm{iL} \\ \mathrm{vc} \end{bmatrix} + \begin{bmatrix} 1/2L & -1/2L \\ 0 & -1/RC \end{bmatrix} \begin{bmatrix} \mathrm{u1} \\ u2 \end{bmatrix}$$

$$[y] = [0 \quad 2] \begin{bmatrix} iL \\ vc \end{bmatrix} + [0 \quad 1] \begin{bmatrix} \mathrm{u1} \\ u2 \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

2 state variables should have been used and they should correspond to the total change in inductor current and the total change in capacitor voltage.