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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

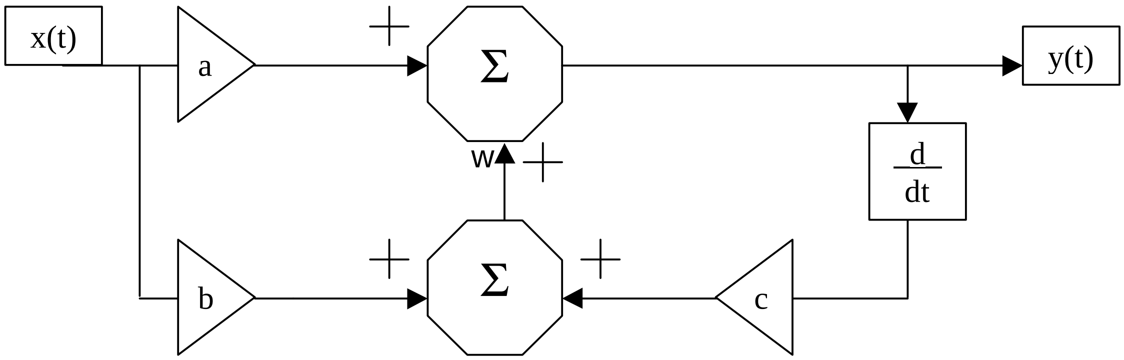
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.

Labelling the input to the top adder from the bottom adder as w:

$$y(t) = ax(t) + w$$
$$w = bx(t) + c\frac{dy(t)}{dt}$$
$$y(t) = ax(t) + bx(t) + c\frac{dy(t)}{dt}$$

$$y(t) = (a+b)x(t) + c\frac{dy(t)}{dt}$$

(b) Find the transfer function.

Using Laplace to convert to the frequency domain:

$$Y(s) = (a+b)X(s) + c[sY(s) - y(0^-)]$$

If all initial conditions are zero:

$$Y(s) = (a+b)X(s) + csY(s)$$
$$\frac{Y(s) - CsY(s)}{X(s)} = a+b$$
$$\frac{Y(s)}{X(s)} = H(s) = \frac{a+b}{1-cs}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

The values of a and b have no effect on stability because they are constant. If c has a positive value, there will be a pole in the right half s-plane. This means that in thetime-domain, there will be a non-decaying exponential. Because of this, the system will be unstable if c has a value greater than zero.

(d) Find the impulse response.

Taking the the inverse Laplace of the transfer function:

$$L^{-1}(H(s)) = L^{-1}\left(\frac{a+b}{1-cs}\right) = (a+b)L^{-1}\left(\frac{1}{1-cs}\right) = -\frac{a+b}{c}L^{-1}\left(\frac{1}{s-\frac{1}{c}}\right) = -\frac{a+b}{c}e^{\left(\frac{1}{c}\right)t}$$

The impulse response is:

$$H(t) = L^{-1}(H(s)) = -\frac{a+b}{c}e^{\left(\frac{1}{c}\right)t}$$

Problem No. 2: Transfer Functions

For the circuit shown below:

(a) Find $H_1(s)$:

$$y_1(t) = \frac{x_1(t) * R}{R + R + R} = \frac{x_1(t)}{3}$$

$$Y_1(s) = \frac{X_1(s)}{3}$$

$$\frac{Y_1(s)}{X_1(s)} = H_1(s) = \frac{1}{3}$$

(b) Find $H_2(s)$:

$$y_2(t) = \frac{x_2(t) * R}{2R} = \frac{x_2(t)}{2}$$

$$Y_2(s) = \frac{X_2(s)}{2}$$

$$\frac{Y_2(s)}{X_2(s)} = H_2(s) = \frac{1}{2}$$

(c) Find $H_3(s)$:

Labelling the voltage across the resistor formerly at the output of $H_1(s)$ as w(t), we can say:

$$w(t) = \frac{x_3(t) * R \parallel 2R}{2R + R \parallel 2R} = \frac{{}^2\!/_3\, x_3(t)R}{2R + {}^2\!/_3\, R} = \frac{{}^2\!/_3\, x_3(t)}{{}^8\!/_3} = \frac{x_3(t)}{4}$$

$$y_3(t) = \frac{w(t)}{2} = \frac{x_3(t)}{8}$$

$$Y_3(s) = \frac{X_3(s)}{8}$$

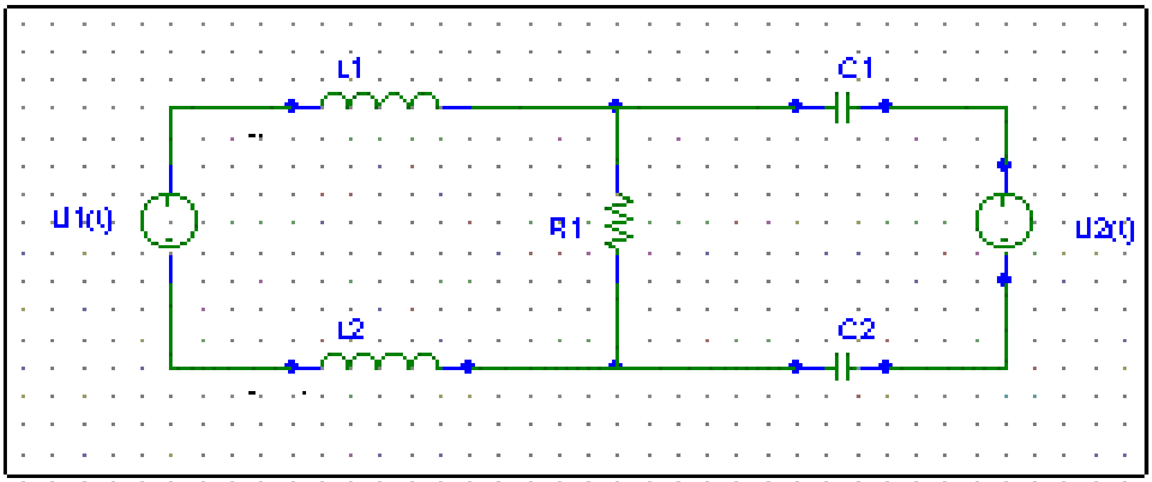
$$\frac{Y_3(s)}{X_3(s)} = H_3(s) = \frac{1}{8}$$

(d) Is $H_3(s) = H_1(s)*H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

No, because of loading. Even though the third circuit is a combination of the first two, the load of the second circuit of the first will cause there to be a different transfer function.

Problem No. 3: The “Interesting” Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.



Substituting il for i_1 and CV_c for i_2

$$\overline{X} = \underline{A}\overline{x} + \underline{B}\overline{u}$$

$$\overline{Y} = \underline{C}\overline{x} + \underline{D}\overline{u}$$

$$\overline{X}_1 = \frac{dil_1}{dt}$$

$$\overline{X}_2 = \frac{dil_2}{dt}$$

$$\overline{X}_3 = \frac{v_1}{dt}$$

$$\overline{X}_4 = \frac{v_2}{dt}$$

$$KVL_1 : -u_1(t) + L\dot{i}_1 + R(i_1 - i_2) + L\dot{i}_1 = 0$$

$$-u_1(t) + 2L\dot{i}_1 + Ri_1 - Ri_2 = 0$$

$$KVL_2 : u_2(t) + V_c + R(\dot{i}_2 - i_1) + V_c = 0$$

$$u_2(t) + 2V_c - Ri_1 + R\dot{i}_2 = 0$$

Substituting

$$i_1 = \dot{i}l \;\&\; i_2 = C\dot{V}c \therefore$$

$$-u_1(t) + 2Li_l + Ri_l - RcV_c = 0$$

$$i_l(2L + R) = Rc\dot{V}_c + u_1(t)$$

$$u_2(t) + Vc_1 + Vc_2 + Rc\dot{V}c - Ri_l = 0$$

$$\dot{V}_c = -Vc_1 - Vc_2 + Ri_1 - u_2(t)$$

$$i_1 2L = Rc\dot{V}c + u_1(t) - Ri_1$$

$$2Li_1 = -2V_c + Ri_1 - u_2(t) + u_1(t) - Ri_1$$

$$\begin{bmatrix}\dot{Vc}\\ \dot{il}\end{bmatrix}=\begin{bmatrix}\frac{-2VC}{Rc} & \frac{1}{C}\\ \frac{-Vc}{L} & 0\end{bmatrix}\bullet\begin{bmatrix}Vc\\ il\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{Rc}\\ \frac{1}{2L} & \frac{1}{2L}\end{bmatrix}\bullet\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$$

$$y(t)=R(i_1-i_2)$$

$$y(t)=Ri_1-RcVc$$

$$y(t)=Ril-R(\frac{-2}{Rc}Vc+\frac{1}{c}il-\frac{1}{Rc}u_2(t)$$

$$\text{simplified}:Ril+\frac{2}{c}Vc-\frac{R}{c}il+\frac{1}{c}u_2(t)$$

$$[y(t)]=\begin{bmatrix}\frac{2}{c} & R-\frac{R}{c}\end{bmatrix}\bullet\begin{bmatrix}Vc\\ il\end{bmatrix}+\begin{bmatrix}0 & \frac{1}{c}\end{bmatrix}\bullet\begin{bmatrix}u_1(t)\\ u_2(t)\end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

There should only be two state variables because the current flowing through the two inductors is the same, as is the voltage across the capacitors. The state variables should correspond to this current and voltage.