Name: Eddie McDaniel

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

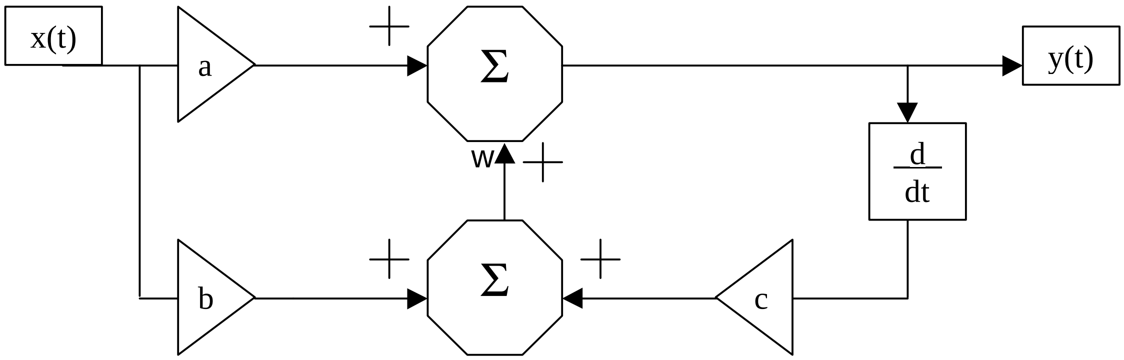
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.

Labelling the input to the top summer from the bottom one as w, we have:

$$y(t) = ax(t) + w$$
$$w = bx(t) + c\frac{dy(t)}{dt}$$
$$y(t) = ax(t) + bx(t) + c\frac{dy(t)}{dt}$$

Thus we have:

$$y(t) = (a+b)x(t) + c\frac{dy(t)}{dt}$$

(b) Find the transfer function.

Using Laplace to convert to a frequency domain representation we get:

$$Y(s) = (a+b)X(s) + c[sY(s) - y(0^-)]$$

Assuming initial conditions to be zero, we have:

$$Y(s) = (a+b)X(s) + csY(s)$$
$$\frac{Y(s) - CsY(s)}{X(s)} = a+b$$
$$\frac{Y(s)}{X(s)} = H(s) = \frac{a+b}{1-cs}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

a and b are constants in the numerator, and thus they have no effect on stability. Any positive c will result in a pole in the right half s-plane. Since any pole in the right half s-plane results in a non-decaying exponential in the time domain, we know that the system will be unstable for any c greater than zero.

(d) Find the impulse response.

For the impulse response, we need to take the the inverse Laplace transform of the transfer function.

$$L^{-1}(H(s)) = L^{-1}\left(\frac{a+b}{1-cs}\right) = (a+b)L^{-1}\left(\frac{1}{1-cs}\right) = -\frac{a+b}{c}L^{-1}\left(\frac{1}{s-\frac{1}{c}}\right) = -\frac{a+b}{c}e^{\left(\frac{1}{c}\right)t}$$

Thus, the impulse response becomes:

$$H(t) = L^{-1}(H(s)) = -\frac{a+b}{c}e^{\left(\frac{1}{c}\right)t}$$

Problem No. 2: Transfer Functions

For the circuit shown below:

(a) Find $H_1(s)$:

$$y_1(t) = \frac{x_1(t) * R}{3R} = \frac{x_1(t)}{3}$$

$$y_1(s) = \frac{x_1(s)}{3}$$

$$\frac{y_1(s)}{x_1(s)} = H_1(s) = \frac{1}{3}$$

(b) Find $H_2(s)$:

$$y_2(t) = \frac{x_2(t) * R}{2R} = \frac{x_2(t)}{2}$$

$$y_2(s) = \frac{x_2(s)}{2}$$

$$\frac{y_2(s)}{x_2(s)} = H_2(s) = \frac{1}{2}$$

(c) Find $H_3(s)$:

Labelling the voltage across the resistor formerly at the output of $H_1(s)$ as w(t), we can say:

$$w(t) = \frac{x_3(t) * R // 2R}{2R + R // 2R} = \frac{\frac{2}{3} x_3(t)R}{2R + \frac{2}{3} R} = \frac{\frac{2}{3} x_3(t)}{\frac{8}{3}} = \frac{x_3(t)}{4}$$

$$y_3(t) = \frac{w(t)}{2} = \frac{x_3(t)}{8}$$

$$y_3(s) = \frac{x_3(s)}{8}$$

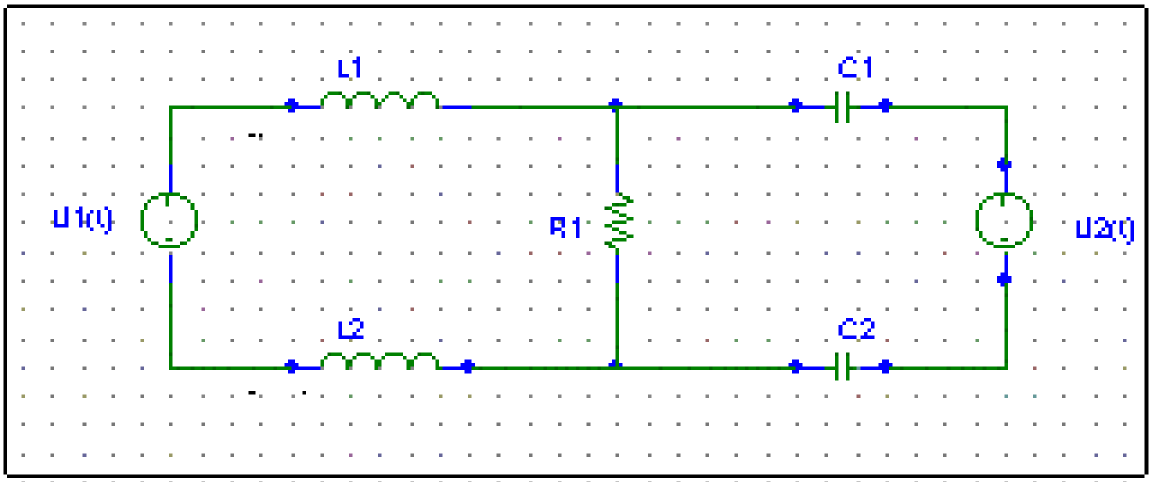
$$\frac{y_3(s)}{x_3(s)} = H_3(s) = \frac{1}{8}$$

(d) Is $H_3(s) = H_1(s)*H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

No, because although $H_3(s)$ is just the combination of $H_1(s)$ and $H_2(s)$, we have loading effects from the combination. Thus, when we solve for the resulting network, we get a different transfer function than would be expected.

Problem No. 3: The "Interesting" Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.



Substituting i_L for i_1 and CV_c for i_2

$$\overline{X} = \underline{A}\overline{x} + \underline{B}\overline{u}$$

$$\overline{Y} = \underline{C}\overline{x} + \underline{D}\overline{u}$$

$$\overline{X}_1 = \frac{di_{L1}}{dt}$$

$$\overline{X}_2 = \frac{di_{L2}}{dt}$$

$$\overline{X}_3 = \frac{v_1}{dt}$$

$$\overline{X}_4 = \frac{v_2}{dt}$$

$$KVL_1 : -u_1(t) + L\dot{i}_1 + R(i_1 - i_2) + L\dot{i}_1 = 0$$

$$-u_1(t) + 2L\dot{i}_1 + Ri_1 - Ri_2 = 0$$

$$KVL_2 : u_2(t) + V_c + R(\dot{i}_2 - i_1) + V_c = 0$$

$$u_2(t) + 2V_c - Ri_1 + R\dot{i}_2 = 0$$

$$i_1 = \dot{i}_L$$

$$i_2 = C\dot{V}_c$$

Thus,

$$-u_1(t) + 2Li_1 + Ri_1 - RCV_c = 0$$

$$i_1(2L + R) = RC\dot{V}_c + u_1(t)$$

$$u_2(t) + V_{c1} + V_{c2} + RC\dot{V}_c - Ri_1 = 0$$

Substituting

$$\dot{V}_c = -V_{c1} - V_{c2} + Ri_l - u_2(t)$$

$$i_l 2L = RC\dot{V}_c + u_1(t) - Ri_l$$

$$2Li_l = -2V_c + Ri_l - u_2(t) + u_1(t) - Ri_l$$

$$\begin{bmatrix} \dot{V}_c \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} \frac{-2V_c}{RC} & \frac{1}{C} \\ \frac{-V_c}{L} & 0 \end{bmatrix} \bullet \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{RC} \\ \frac{1}{2L} & \frac{1}{2L} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t) = R(i_1 - i_2)$$

$$y(t) = Ri_l - RCV_c$$

$$y(t) = Ri_L - R(\frac{-2}{RC}V_c + \frac{1}{C}i_L - \frac{1}{RC}u_2(t)$$

$$Ri_L + \frac{2}{C}V_c - \frac{R}{C}i_L + \frac{1}{C}u_2(t)$$

$$[y(t)] = \begin{bmatrix} \frac{2}{C} & R - \frac{R}{C} \end{bmatrix} \bullet \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{C} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

We only need two state variables for this problem because the current through the two inductors is the same. This is also true for the capacitors. Also, we know that we want circuit elements that are independent of each other. Since the inductors and capacitors in this circuit depend on the same currents, they are not independent of each other. We should have used I_L and V_c as our variables. They represent the current through an inductor and the voltage across a capacitor respectively. It won't matter which inductor or capacitor we use for these variables.