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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

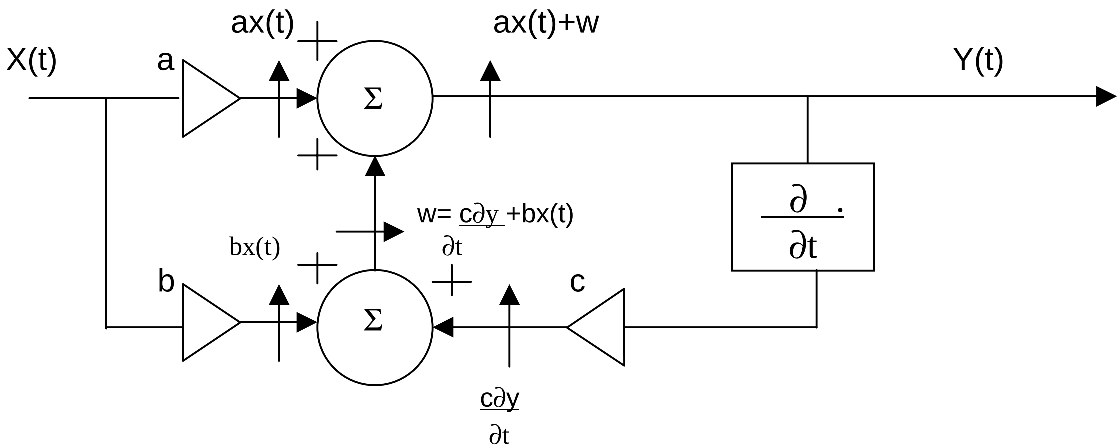
1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams

(a) Write a differential equation describing this system.



By using the simplified graph above the following expression was determined.

$y(t) = ax(t) + w$

$w = bx(t) + \frac{c\partial y(t)}{\partial t}$

By substituting w into the equation for y(t) you get the following:

$y(t) = \frac{c\partial y(t)}{\partial t} + ax(t) + bx(t)$

(b) Find the transfer function.

The transfer function is the laplace transform of the output of the system divided by the laplace transform of the input of the system.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{1 - sc} * (X(s)a + X(s)b)}{X(s)} = \frac{(a + b)}{1 - sc}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

A condition for the stability of a linear, time-invariant system is that the system poles all lie in the left-half s-plane. The system poles are simply the roots of the characteristic polynomial, which is the denominator of the overall system transfer function. The validity of the stability condition is readily established. The system response will contain terms of the form e^at, where a are the system poles. If any of the poles have positive real parts, the response will clearly increase without bound.

The denominator of the transfer function tells the location of the poles.
Therefore:
$1 - sc$ is the characteristic polynomial.
by solving for s you get the roots.
$s = \frac{1}{c}$

So it can be seen that to satisfy the conditions above for stability. The system is only stable when c is less than 0.

(d) Find the impulse response.

The impulse response of the system is the inverse laplace of the transfer function of the system. Therefore :

$$h(t) = \mathcal{L}^{-1}\left[\frac{(a+b)}{(1-sc)}\right] = -\frac{(a+b)}{c}\mathcal{L}^{-1}\left[\frac{1}{s-\frac{1}{c}}\right]$$

therefore by using the following laplace transform taken off a laplace transform table the inverse laplace transform can be eaisly solved.

$$\frac{t^n e^{(-\alpha t)} u(t)}{n!} \Rightarrow \frac{1}{(s+\alpha)^{n+1}}$$

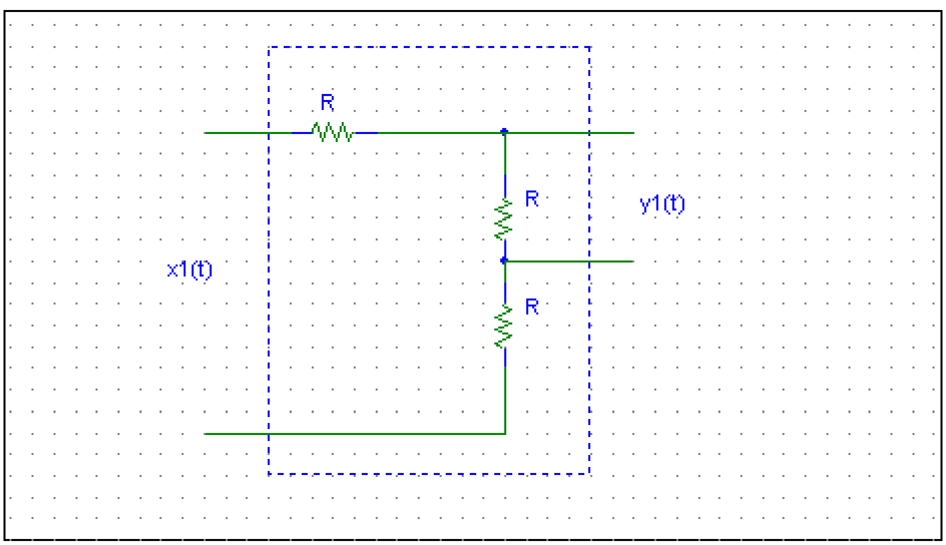
This is therefore the impulse response of the system

$$h(t) = \frac{-(a+b)}{c} e^{\frac{t}{c}} u(t)$$

Problem No. 2: Transfer Functions

For the circuit shown below:

(a) Find $H_1(s)$:



By noticing that the two other resistors are in series their equivalent can be found by simply adding them together. To find the output of this circuit is to simply use a voltage divder as seen below.

$$y_1(t) = \frac{R}{R+2R} x_1(t)$$

By pulling the R out of the equation and and just having a R divided by an R you can cancel them out and then by taking the laplace transform of the output the output becomes:

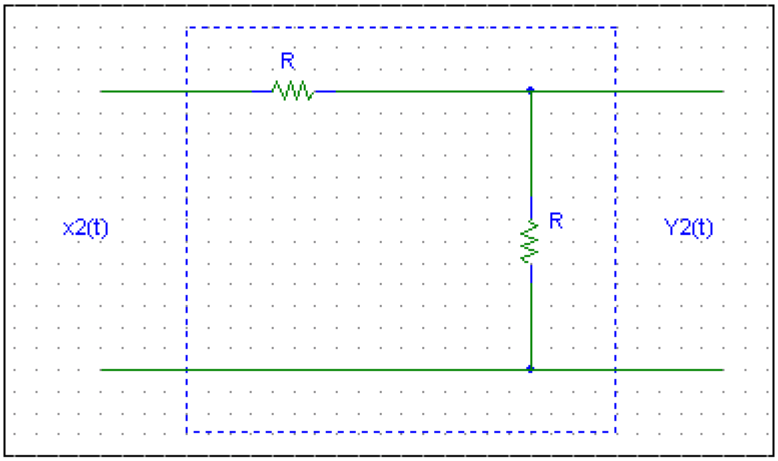
$$Y_1(S) = \frac{1}{3}X_1(S)$$

The transfer function of a system is the laplace transform of the output divided by the laplace transform of the output. So therefore the transfer function of the system can be seen below

$$H_1(S) = \frac{Y_1(S)}{X_1(S)} = \frac{{}^{1}\!/_{3}X_1(S)}{X_1(S)}$$ By eliminating X(S) the transfer function becomes

$$H_1(S) = \frac{1}{3}$$

(b) Find H_2(s):



It is possible to just use a voltage divider to get the output of the system above. So therefore the output and it's laplace transform can be seen below.

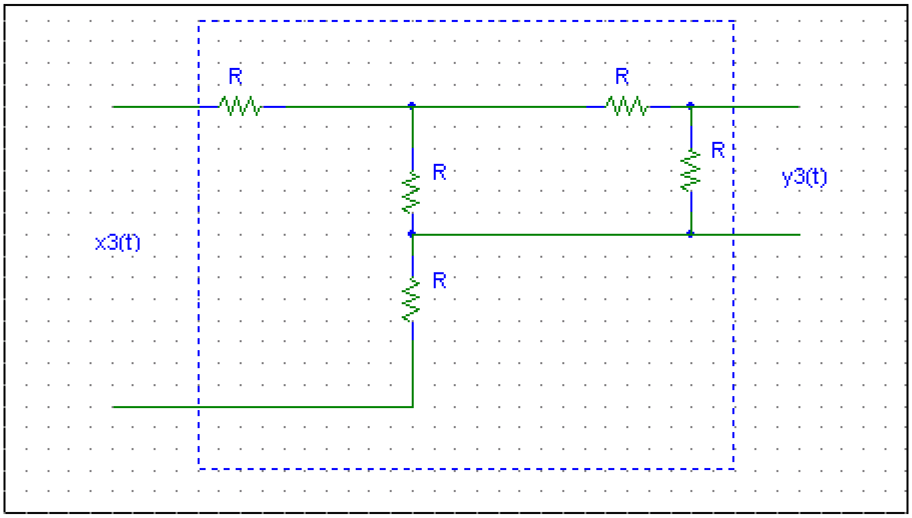
$$y_2(t) = \frac{R}{R+R}x_2(t) = \frac{R}{2R}x_2(t) = \frac{1}{2}x_2(t)$$

$$Y_2(S) = \frac{1}{2}X_2(S)$$

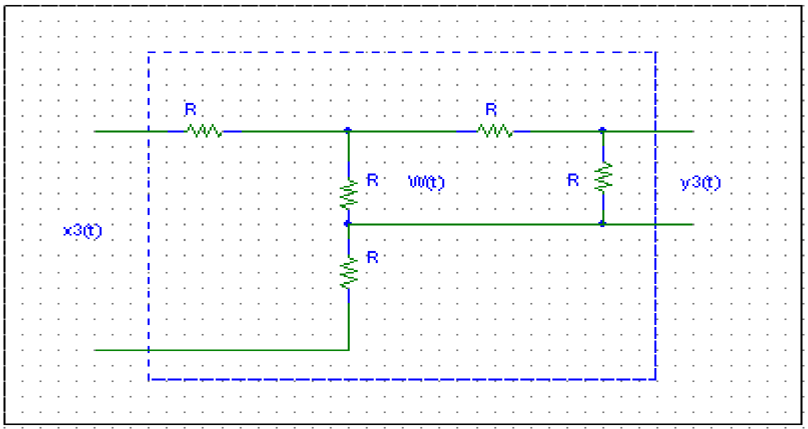
The transfer function of a system is the laplace transform of the output divided by the laplace transform of the output. So therefore the transfer function of the system can be seen below

$$H_2(S) = \frac{Y_2(S)}{X_2(S)} = \frac{{}^{1}\!/_{2}X_2(S)}{X_2(S)} = \frac{1}{2}$$

(c) Find H_3(s):



By leting W be the output of the first stage the output of the circuit can be eaisly found and thus also the transfer function.



$$H_3(s) = \frac{Y_3(s)}{X_3(s)}$$

$$W(s) = \frac{X_3(s) \bullet R \parallel 2R}{2R + R \parallel 2R}$$

$$W(s) = \frac{\frac{2}{3} \bullet X_3(s) \bullet R}{2R + \frac{2}{3}R}$$

$$W(s) = \frac{\frac{2}{3} \bullet X_3(s)}{\frac{8}{3}}$$

$$W(s) = \frac{X_3(s)}{4}$$

The output of the circuit can now eaisly be obtained which allows the deriavation of the transfer function. Since, the transfer function of a system is the laplace transform of the output divided by the laplace transform of the output. So therefore the transfer function of the system can be seen below.

$$Y_3(s) = \frac{W(s)R}{R+R} = \frac{W(s)R}{2R} = \frac{W(s)}{2} = \frac{\frac{X_3(s)}{4}}{2} = \frac{X_3(s)}{8} \therefore$$

$$H_3(s) = \frac{Y_3(s)}{X_3(s)} = \frac{\frac{X_3(s)}{8}}{X_3} = \frac{1}{8}$$

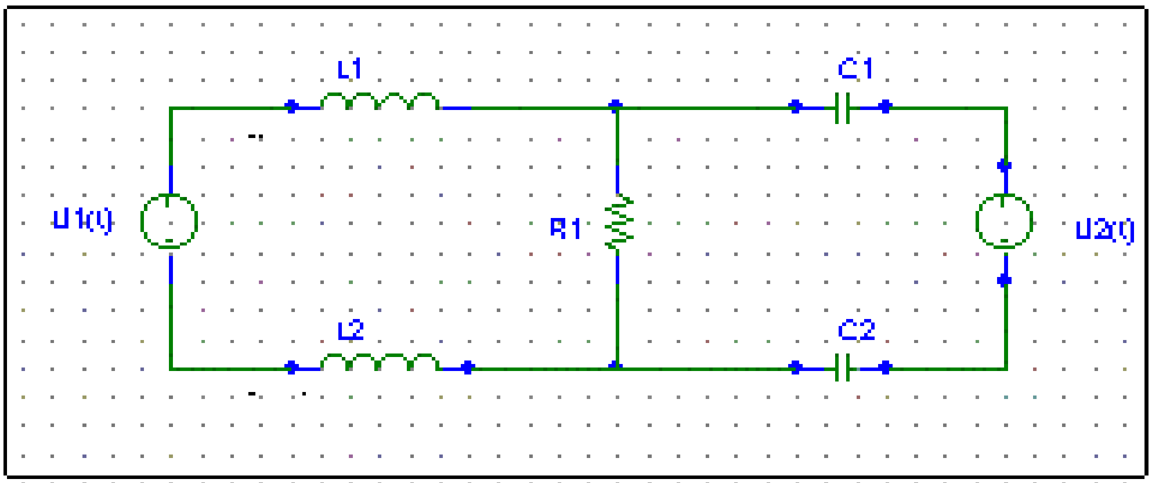
(d) Is ? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

$H_3(S) \neq H_2(S) \bullet H_1(S)$ The third transfer function is not equal to the first times the second because of loading. By attaching the second system to the first and visulaizing this by using a block diagram such as the one below it would be assumed that H3 is H2 times H1 but when H2 is hooked together to H1 the value for the transfer function H1 changes and thus H3 cannot equal H1 times H2.

Problem No. 3: The "Interesting" Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

The circuit for which we are to get the state variables can be seen below along with the derivation for the state variables.



The state variable technique is a way of handling systems with many inputs and outputs within precisely the same notational framework that we will use for single-input, single output solutions. We can find solutions to multiple input, output systems using the same technique as for systems with one output and one input.

$$\overline{X} = \underline{A}\overline{x} + \underline{B}\overline{u}, \overline{Y} = \underline{C}\overline{x} + \underline{D}\overline{u}$$

$$\overline{X}_1 = \frac{dil_1}{dt}, \overline{X}_2 = \frac{dil_2}{dt}, \overline{X}_3 = \frac{v_1}{dt}, \overline{X}_4 = \frac{v_2}{dt}$$

$$KVL_1 : -u_1(t) + L\dot{i}_1 + R(i_1 - i_2) + L\dot{i}_1 = 0$$

$$simplified : -u_1(t) + 2L\dot{i}_1 + Ri_1 - Ri_2 = 0$$

$$KVL_2 : u_2(t) + Vc + R(\dot{i}_2 - i_1) + Vc = 0$$

$$simplified : u_2(t) + 2vc - Ri_1 + R\dot{i}_2 = 0$$

Substituting,

$$i_1 = \dot{i}l, \& i_2 = C\dot{V}c \therefore$$
$$-u_1(t) + 2Li_l + Ri_l - RcV_c = 0$$
$$i_l(2L+R) = Rc\dot{V}_c + u_1(t)$$
$$u_2(t) + Vc_1 + Vc_2 + Rc\dot{V}c - Ri_l = 0$$

$$\dot{V}_c = -Vc_1 - Vc_2 + Ri_l - u_2(t)$$
$$i_l 2L = Rc\dot{V}c + u_1(t) - Ri_l$$
$$2Li_l = -2V_c + Ri_l - u_2(t) + u_1(t) - Ri_l$$

The state variable equations can now be seen below.

$$\begin{bmatrix} \dot{V}c \\ \dot{i}l \end{bmatrix} = \begin{bmatrix} \frac{-2VC}{Rc} & \frac{1}{C} \\ \frac{-Vc}{L} & 0 \end{bmatrix} \bullet \begin{bmatrix} Vc \\ il \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{Rc} \\ \frac{1}{2L} & \frac{1}{2L} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$
$$y(t) = R(i_1 - i_2)$$
$$y(t) = Ri_l - RcVc$$
$$y(t) = Ril - R(\frac{-2}{Rc}Vc + \frac{1}{c}il - \frac{1}{Rc}u_2(t)$$
$$simplified: Ril + \frac{2}{c}Vc - \frac{R}{c}il + \frac{1}{c}u_2(t)$$
$$\begin{bmatrix} y(t) \end{bmatrix} = \begin{bmatrix} \frac{2}{c} & R - \frac{R}{c} \end{bmatrix} \bullet \begin{bmatrix} Vc \\ il \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{c} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

You should have two state variables becase, the current (I2) through the two capacitors is the same. Also, the the current (I1) through the two inductors is the same.

State variable one is

i_l

State variable two is

V_c