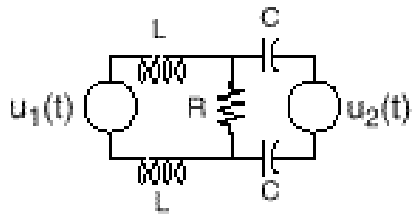


(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

$$-\frac{1}{s} + sLI_1(s) - Li_L + R[I_1(s) - I_2(s)] + sLI_1(s) - Li_L = 0$$

$$I_1(s)[2sL + R] - I_2(s)R = \frac{1 + sLi_L}{s}$$

$$-R[I_2(s) - I_1(s)] + I_2(s)\frac{1}{sc} - \frac{V_c}{s} + \frac{1}{s} + I_2(s)\frac{1}{sc} - \frac{V_c}{s} = 0$$

$$I_1(s)R + I_2(s)\left[\frac{2}{sc} - R\right] = \frac{2V_c - 1}{s}$$

$$\begin{bmatrix} 2sL + R & R \\ R & \frac{2}{sc} - R \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1 + 2sLi_L}{s} \\ \frac{2V_c - 1}{s} \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

There should be two state variables, one that corresponds to the voltage across the capacitors, and one that corresponds to the current through the inductors.

(c) Find $H_3(s)$:

$$x_3(t) \Rightarrow x_3(s); \; y_3(t) \Rightarrow y_3(s)$$

$$z(s) = \frac{x_3(s)\,{}^2\!/_3\,R}{2\,{}^2\!/_3\,R}$$

$$z(s) = \frac{1}{4}R$$

$$y_3(s) = \frac{1}{2}z(s)$$

$$= \frac{1}{2}\left[\frac{1}{4}x_3(s)\right]$$

$$\frac{y_3(s)}{x_3(s)} = \frac{1}{8}$$

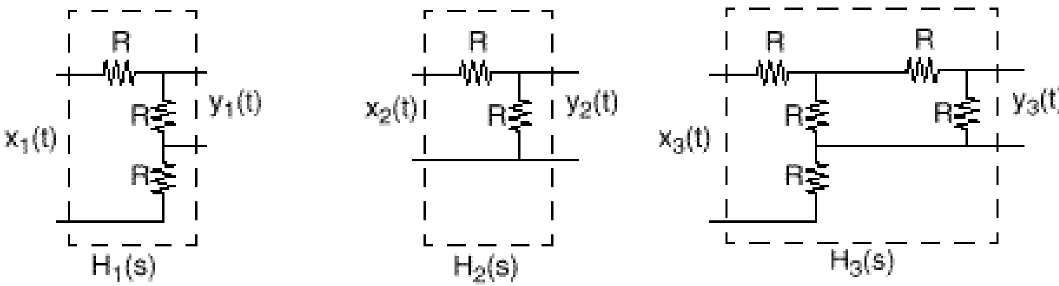
(d) Is $H_3(s) = H_1(s) \bullet H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

No. When the two circuits are combined, the voltage across the middle resistor changes because it is in parallel with two other resistors.

Problem No. 3: The "Interesting" Problem

Problem No. 2: Transfer Functions

For the circuit shown below:



(a) Find $H_1(s)$:

$$x_1(t) \Rightarrow x_1(s);\, y_1(t) \Rightarrow y_1(s)$$
$$y_1(s) = \frac{x_1(s)R}{3R}$$
$$\frac{y_1(s)}{x_1(s)} = \frac{1}{3}$$

(b) Find $H_2(s)$:

$$x_2(t) \Rightarrow x_2(s);\, y_2(t) \Rightarrow y_2(s)$$
$$y_2(s) = \frac{x_2(s)R}{2R}$$
$$\frac{y_2(s)}{x_2(s)} = \frac{1}{2}$$

(d) Find the impulse response.

$$c\frac{\partial y(t)}{\partial (t)} - y(t) = -x(t)[a+b]$$

$$\frac{-c}{a+b}\frac{\partial y(t)}{\partial t} + \frac{1}{a+b}y(t) = x(t)$$

$$x(t) = \delta(t) = 0, t < 0$$

$$y(t) = h(t)$$

$$\frac{-c}{a+b}\frac{\partial h(t)}{\partial t} + \frac{1}{a+b}h(t) = 0$$

$$-c\frac{\partial h(t)}{\partial t} + h(t) = 0$$

$$h(t) = Ae^{\alpha t}$$

$$-c\alpha Ae^{\alpha t} + Ae^{\alpha t} = 0$$

$$Ae^{\alpha t}(1 - c\alpha) = 0$$

$$\alpha = \frac{1}{c}$$

$$h(t) = Ae^{t/c}, t > 0$$

$$h(t) = 0, t < 0$$

$$\text{for t} > 0$$

$$-c\frac{\partial h(t)}{\partial t} + h(t) = \delta(t)$$

$$-c\int_{0^-}^{0^+}\frac{\partial h(t)}{\partial t}dt + \int_{0^-}^{0^+}h(t)dt = \int_{0^-}^{0^+}\delta(t)dt$$

$$-c[h(0^+) - h(0^-)] + 0 = 1$$

$$h(0^+) = -\frac{1}{c}$$

$$A = -\frac{1}{c}$$

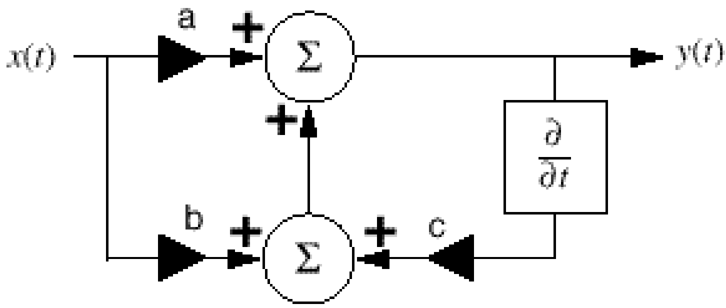
$$h(t) = -\frac{1}{c}e^{t/c}, \text{ t} \geq 0$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

The system is stable for the following values of a, b, and c,

$$-\infty<a<\infty,\ a\neq 0$$
$$-\infty<b<\infty,\ b\neq 0$$
$$-\infty<c<\infty,\ c\neq 0, s^{-1}$$

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.

$$
\begin{aligned}
y(t) &= ax(t) + bx(t) + c\frac{\partial y(t)}{\partial t} \\
&= x(t)[a+b] + c\frac{\partial y(t)}{\partial t} \\
\frac{\partial y(t)}{\partial (t)} &= \frac{y(t) - x(t)[a+b]}{c}
\end{aligned}
$$

(b) Find the transfer function.

$$
\begin{aligned}
y(t) &= x(t)[a+b] + c\frac{\partial y(t)}{\partial t} \\
y(s) &= x(s)[a+b] + csy(s) \\
y(s) - csy(s) &= x(s)[a+b] \\
y(s)[1-cs] &= x(s)[a+b] \\
\frac{y(s)}{x(s)} &= \frac{a+b}{1-cs}
\end{aligned}
$$

Name: **John E. Moore III**

Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)