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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

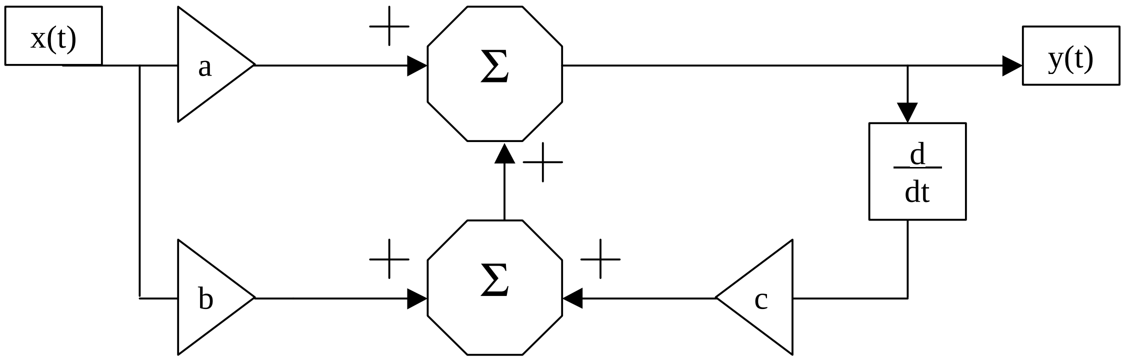
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

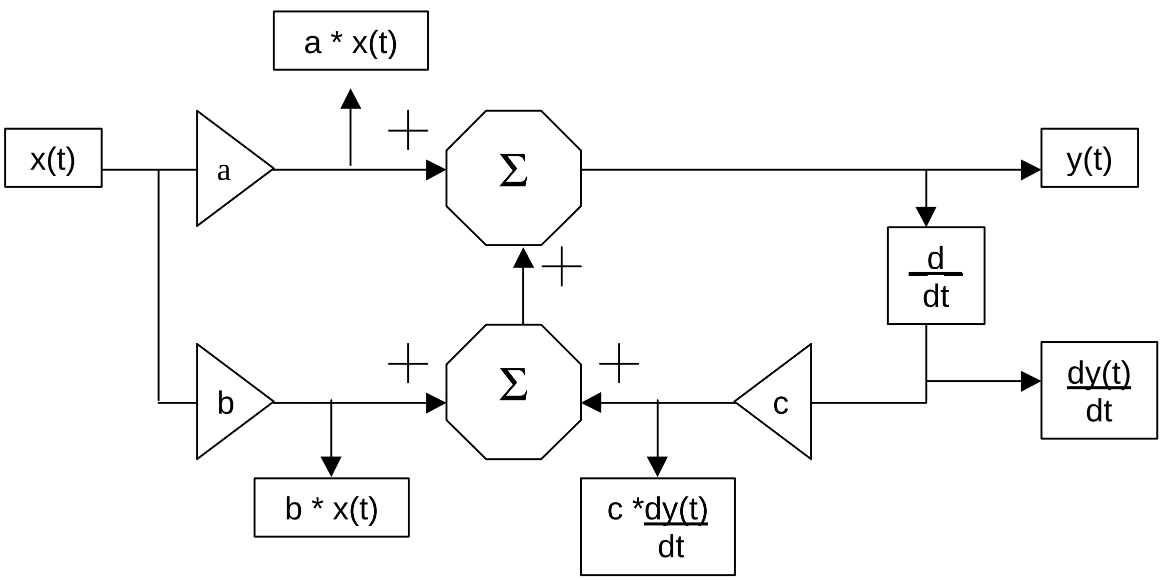
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.

First, label the output signal of every box that wasn't a summation.



Next, find a way to define y(t) with the given signals and summations.

$$y(t) = a \cdot x(t) + b \cdot x(t) + c \cdot \frac{dy}{dt}$$

(b) Find the transfer function.

$$H(s) = \frac{Y(s)}{X(s)}$$

and $X(s) = L\{ x(t) \}$

$a \cdot X(s) + b \cdot X(s) + c \cdot [s \cdot Y(s) + y(0^{-})] = Y(s)$ Assume initial conditions to be zero.

$X(s) \cdot (a + b) + c \cdot s \cdot Y(s) = Y(s)$ Divide everything by X(s)

$a + b + c \cdot s \cdot H(s) = H(s)$

$a + b = H(s) - c \cdot s \cdot H(s)$

$$H(s) = \frac{a + b}{1 - c \cdot s}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

a & b can be any real number because they have no effect on stability. c must be less than zero, otherwise the system will blowup. A good way to see this is to look at part (d) of this problem. If c is positive, then the output of

$$\lim t\text{-}> \infty \ [x(t) * d(t)] = \infty$$

Note: |c| determines the speed at which $e^{t/c}$ approaches infinity

Assuming that c is negative, there is a pole at $-\frac{1}{C}$

(d) Find the impulse response.

The impulse response returns the transfer function. So far this circuit, that would mean:

$$H(t) = L^{-1}\{H(s)\} = L^{-1}\{\frac{a+b}{1-c\cdot s}\}$$

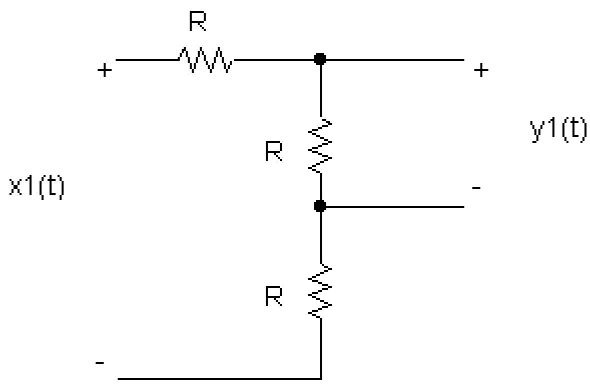
$$= (a+b)\cdot L^{-1}\{\frac{1}{1-c\cdot s}\} = -\frac{(a+b)}{c}\cdot L^{-1}\{\frac{1}{s-\frac{1}{c}}\}$$

$$H(t) = -\frac{(a+b)}{c}\cdot e^{(\frac{1}{c})t}$$

Problem No. 2: Transfer Functions

For the circuit shown below:

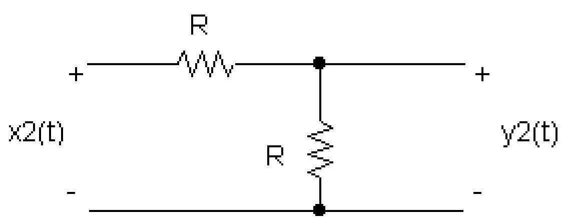
(a) Find $H_1(s)$:



$$Y_1(s) = X_1(s) \cdot \frac{R}{R+R+R}$$

$$H_1(S) = \frac{1}{3}$$

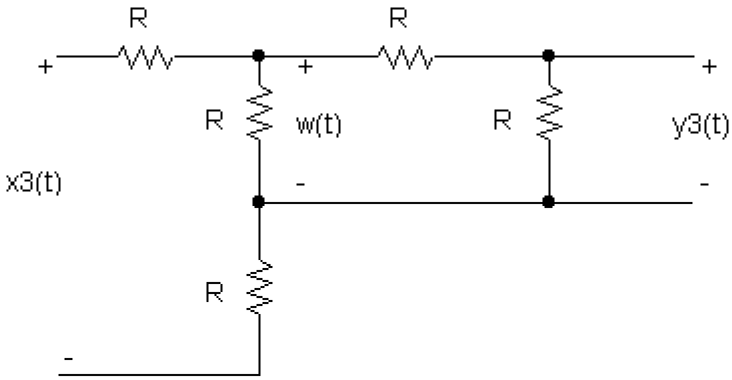
(b) Find $H_2(s)$:



$$Y_2(s) = X_2(s) \cdot \frac{R}{R+R}$$

$$H_2(s) = \frac{1}{2}$$

(c) Find $H_3(s)$:



The two resisters in on the right are in series :

$$\frac{R \cdot 2R}{3R} = \frac{2}{3}R$$

$$w(s) = x(t)\frac{\frac{2}{3} \cdot R}{2R + \frac{2}{3}R} = \frac{1}{4}x_3(s)$$

$$y_3(s) = w(s) \cdot \frac{1}{2} = [x_3(s) \cdot \frac{1}{4}] \cdot \frac{1}{2} = \frac{1}{4}x_3(s)$$

$$H_3(s) = \frac{1}{8}$$

(d) Is $H_3(s) = H_1(s) \cdot H_2(s)$? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

$$H_3(s) \neq H_1(s) \cdot H_2(s)$$

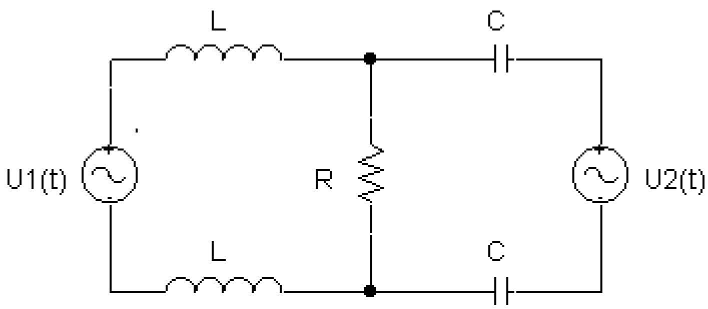
$$H_3(s) = \frac{1}{8}$$

$$H1(s) \cdot H2(s) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$$

$$\frac{1}{6} \neq \frac{1}{8}$$

The reason this does not work on this circuit is because of loading. In this case, each part of the circuit effects the other part. In essence, adding $H_2(s)$ to $H_1(s)$ caused different values of resistance in what was once $H_1(s)$. This in turn effected the transfer function.

Problem No. 3: The “Interesting” Problem



(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

(The L and C in the top of the circuit will be given the subscript 1 for reference and the L and C in the bottom will be given a subscript of 2--Note: both L's are the same value, and the subscript is used for current and voltages across the corresponding L. The same goes for C).

$$\overline{X} = \underline{A}\overline{x} + \underline{B}\overline{u}$$
$$\overline{Y} = \underline{C}\overline{x} + \underline{D}\overline{u}$$

Assume that:

x1(t) = i_{L1}
x2(t) = i_{L2}
x3(t) =V_{C1}
x4(t) = V_{C2}

Use loop analysis to determine equations:
i_1 is the current through the first loop (left hand of the circuit) and
i_2 is the current through the second loop (right hand of the circuit).

(1) $-u_1(t) + L\cdot\dot{i}_1 + R(i_1 - i_2) + L\cdot\dot{i}_1 = 0$ $-u_1(t) + 2L\cdot\dot{i}_1 + R\cdot i_1 - R\cdot i_2 = 0$	(2) $u_2(t) + V_{C1} + R(i_2 - i_1) + V_{C2} = 0$ $u_2(t) + V_{C1} + R\cdot i_2 - R\cdot i_1 + V_{C2} = 0$

Substite: $i_1 = \dot{i}_{L1}$
$i_2 = C \cdot \dot{V}_{C1} = C \cdot \dot{V}_{C2}$ into equations 1 and 2.
If we assme 0 for initial conditions, then $V_{C1} = V_{C2}$

We get:

$$-u_1(t) + 2 \cdot L \cdot \dot{i}_l + R \cdot i_l - R \cdot C \cdot V_c = 0$$

$$\dot{i}_l \cdot (2 \cdot L + R) = R \cdot C \cdot \dot{V}_c + u_1(t)$$

$$u_2(t) + 2 \cdot V_C + R \cdot C \cdot \dot{V}c - R \cdot i_l = 0$$

$$(3)\ R \cdot C \cdot \dot{V}c = R \cdot i_l - 2 \cdot V_C - u_2(t)$$

$$\dot{V}_c = -2 \cdot V_C + R \cdot i_l - u_2(t)$$

$$\dot{i}_L \cdot 2 \cdot L = R \cdot C \cdot \dot{V}c + u_1(t) - Ri_l$$

Substitute equation 3 in to get:

$$(4)\ 2 \cdot L \cdot \dot{i}_L = -2V_c + R \cdot i_L - u_2(t) + u_1(t) - R \cdot i_L$$

Equation 3 makes up the top rows of this matrix and Equation 4 makes up the bottom

$$\begin{bmatrix} \dot{V}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} \frac{-2V_C}{R \cdot C} & \frac{1}{C} \\ \frac{-V_C}{L} & 0 \end{bmatrix} \bullet \begin{bmatrix} Vc \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{R \cdot C} \\ \frac{1}{2 \cdot L} & \frac{1}{2 \cdot L} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

row of this matrix.

Now for the $\overline{Y} = \underline{C}\overline{x} + \underline{D}\overline{u}$

$$y(t) = R(i_1 - i_2)$$

$$y(t) = Ri_l - R \cdot C \cdot \dot{V}c$$

$$y(t) = R \cdot i_L - R(\frac{-2}{R \cdot C} V_C + \frac{1}{c} i_L - \frac{1}{Rc} u_2(t)$$

$$\text{simplified}: R \cdot i_L + \frac{2}{c} Vc - \frac{R}{c} i_L + \frac{1}{c} u_2(t)$$

$$[y(t)] = \begin{bmatrix} \frac{2}{c} & R - \frac{R}{c} \end{bmatrix} \bullet \begin{bmatrix} V_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{c} \end{bmatrix} \bullet \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

Only two state variables are need. One for the current through both the inductors and one for the voltage across the capacitors.

It is obvious that the current through both inductors are the same, because they are in the same loop (you can see this if you try to do KVL equations or loop analysis.

The current through the capacitors is also the same, for the same reason the inductors have the same current. Therefore: $i_2 = C\frac{dV_{C1}}{dt} = C\frac{dV_{C2}}{dt}$ if we assume initial conditions to be zero, then $V_{C1}(t)$ must be equal to $V_{C2}(t)$.

This is why only two state variables are need:
1 for the current through the inductors and
1 for the voltage across one of the capacitors.