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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

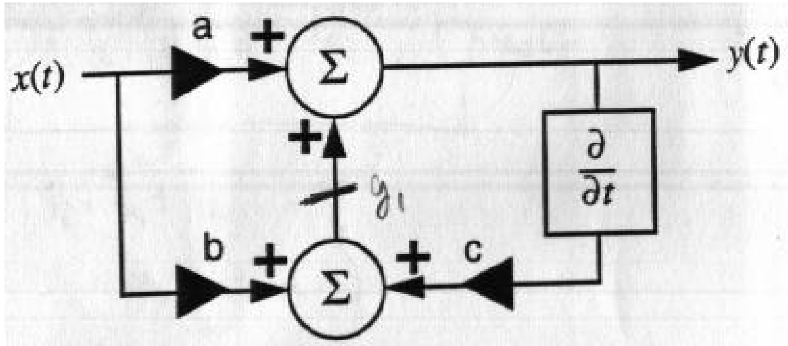
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams



(a) Write a differential equation describing this system.

Let $x = x(t)$, $\mathrm{y} = \mathrm{y(t)}$, and $\frac{\partial \mathrm{y}}{\partial \mathrm{t}} = \frac{\partial}{\partial t} y(t)$

Then y_1 can be written as, $y_1 = bx + c\frac{\partial \mathrm{y}}{\partial \mathrm{t}}$

$y = ax + y_1$

Substituting y_1 into the above equation, you will get

$y = ax + bx + c\frac{\partial \mathrm{y}}{\partial \mathrm{t}}$

Move y to one side and x to the other, you get this differential equation

$-c\frac{\partial \mathrm{y}}{\partial \mathrm{t}} + y = (a+b)x$

(b) Find the transfer function.

From the D.E above, $-c\frac{\partial \mathrm{y}}{\partial \mathrm{t}} + y = (a+b)x$, do a Laplace Transform on both side and you will get

$(-cs+1)Y(s) = (a+b)X(s)$

Then the transfer function of this system is just output, $Y(s)$, over input, $X(s)$

$$\frac{Y(s)}{X(s)} = \frac{(a+b)}{(-cs+1)}$$

Or you can write it as

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a+b}{1-cs}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a+b}{1-cs}$$

From the Transfer function above, the system is stable when the pole is on the left (negative) half plane.

Therefore, c have to be less than zero, and not equal to $\frac{1}{s}$

As for a and b, they can be any number as long as their sum is not equal to zero

$$\therefore \{a \in \Re, b \in \Re, but\ a+b \neq 0; c < 0, but\ c \neq \frac{1}{s}\}$$

(d) Find the impulse response.

$$H(s) = \frac{Y(s)}{X(s)} = \frac{a+b}{1-cs}$$

With the Transfer function above, to find the impulse response, you need to find h(t) by taking the inverse Laplace Transform,

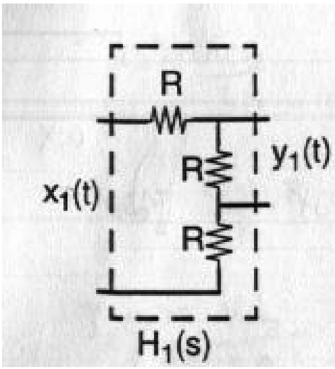
$$H(s) = \frac{a+b}{1-cs} = \frac{a+b}{1-cs} * \frac{1/c}{1/c} = \frac{a+b}{c} * \frac{1}{1/c - s} = -\frac{a+b}{c} * \frac{1}{s-1/c}$$

Then the inverse laplace of that is

$$h(t) = -\frac{a+b}{c} e^{\frac{1}{c}t} u(t)$$

Problem No. 2: Transfer Functions

(a) Find $H_1(s)$:



Find $H_1(s)$, to do so, first you must find the $I_1(s)$, the current around the loop, then you can find $Y_1(s)$ by multiply current to the resister, R.

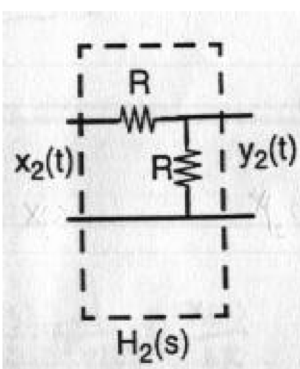
$$I_1(s) = \frac{X_1(s)}{3R}$$

$$Y_1(s) = I_1(s) * R = \frac{X_1(s)}{3R} * \mathrm{R} = \frac{X_1(s)}{3}$$

$$H_1(s) = \frac{Y_1(s)}{X_1(s)} = \frac{X_1(s)}{3} * \frac{1}{X_1(s)} = \frac{1}{3}$$

$$\therefore H_1(s) = \frac{1}{3}$$

(b) Find $H_2(s)$:



Find $H_2(s)$, to do so, first you must find the $I_2(s)$, the current around the loop, then you can find $Y_2(s)$ by multiply current to the resister, R.

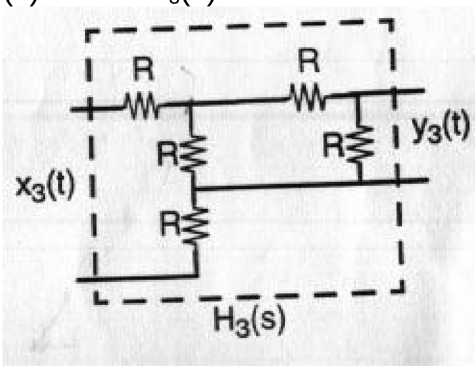
$$I_2(s) = \frac{X_2(s)}{2R}$$

$$Y_2(s) = I_2(s) * R = \frac{X_1(s)}{2R} * \mathrm{R} = \frac{X_1(s)}{2}$$

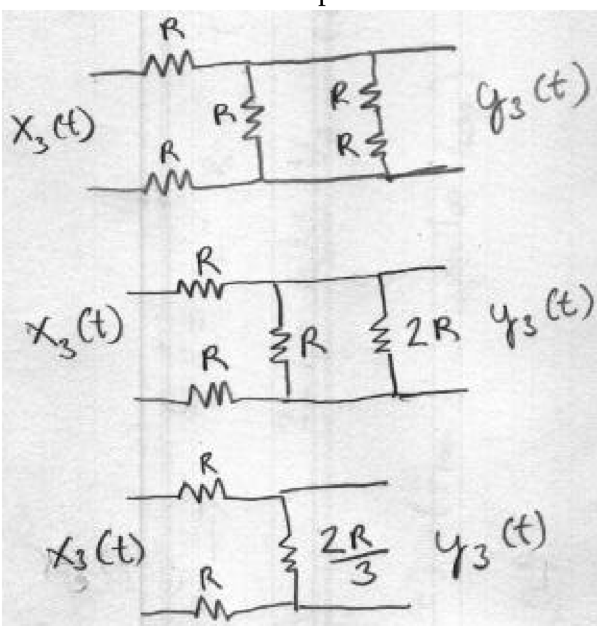
$$H_2(s) = \frac{Y_2(s)}{X_2(s)} = \frac{X_2(s)}{3} * \frac{1}{X_2(s)} = \frac{1}{2}$$

$$\therefore H_2(s) = \frac{1}{2}$$

(c) Find $H_3(s)$:



The circuit can be simplified like this:

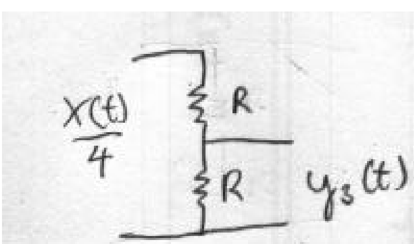


Find voltage across $\frac{2R}{3}$, first you must find $I_{\frac{2R}{3}}$ by voltage divis

$$I_{\frac{2R}{3}}(s) = X_3(s) * \frac{1}{R + R + \frac{2R}{3}} = X_3(s) * \frac{3}{8R}$$

$$V_{\frac{2R}{3}}(s) = I_{\frac{2R}{3}}(s) * R = X_3(s) * \frac{3}{8R} * \frac{2R}{3} = \frac{X_3(s)}{4}$$

The voltage across $\frac{2R}{3}$ is the same as the voltage across 2R (see simpifed picture 2 above)



The picture on left is the step before simplifing the far right half of the circuit

Knowing that the voltage across 2R is equal to $\frac{X_3(s)}{4}$, do the voltage division

$$Y_3(s) = \frac{X_3(s)}{4 * 2R} * R = \frac{X_3(s)}{8}$$

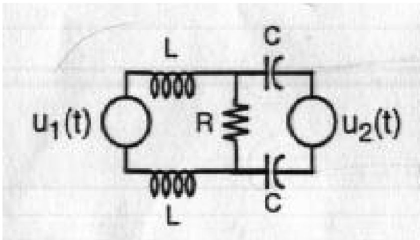
From here, we can find the transfer function

$$H(s) = \frac{Y(s)}{X(s)} = \frac{X_3(s)}{8} * \frac{1}{X(s)} = \frac{1}{8}$$

(d) Is ? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

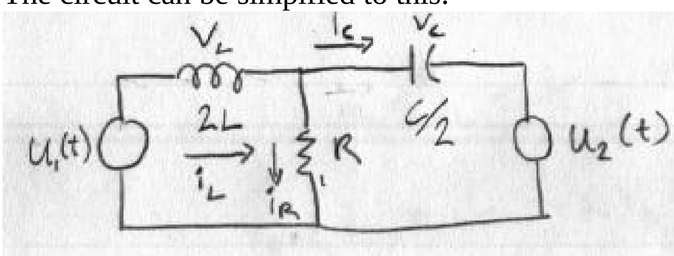
The answer is no, because the loading effect the entire circuit, and also for this example H1*H2 is not equal H3.

Problem No. 3: The "Interesting" Problem



(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

The circuit can be simpified to this:



(note Series simplifica tion, L = 2L, C = C/2)

From this circuit, we can write the KVL, and KCL to be

$i_C = i_L - i_R$

$v_L = u_1(t) - v_R$

Then, from knowing $i_c = C\frac{dv}{dt}$ and $v_L = L\frac{di}{dt}$, and the notation of $\frac{d}{dt}x$ is $\dot{x}$, the two equation above can be expressed as

$$(1)\ \frac{C}{2}\dot{v}_C = i_L - i_R \qquad \text{and} \qquad (2)\ 2L\dot{i}_C = u_1(t) - v_R$$

From the KVL on the capacitor side, $v_R = v_C + u_2(t)$

which means, $i_R = \frac{v_C}{R} + \frac{u_2(t)}{R}$

Substitute the v_R and i_R to the equation (1) and (2), and solve for v_C and i_C , you get

$$(3)\quad \dot{v}_C = \frac{2i_L}{C} - \frac{2i_R}{CR} - \frac{2u_2(t)}{CR} \qquad \text{and} \qquad (4)\quad \dot{i}_C = \frac{u_1(t) - v_C - u_2(t)}{2L}$$

Find $y(t)$, or v_R, fist must find i_R, from equation (1) $\frac{C}{2}\dot{v}_C = i_L - i_R$

$i_R = i_L - \frac{C}{2}\dot{v}_C$ then substitute $\dot{v}_C$ from equation (3) into it, you get

$$i_R = i_L - \frac{C}{2}\left(\frac{2i_L}{C} - \frac{2i_R}{CR} - \frac{2u_2(t)}{CR}\right) = \frac{-v_c}{R} - \frac{u_2(t)}{R},\ \text{then } v_R = R*\left(\frac{-v_c}{R} - \frac{u_2(t)}{R}\right) = -v_c - u_2(t)\ (5)$$

Then, from equation (3), (4), and (5), the matrix can be formed as followed.

$$\begin{bmatrix} \dot{v}_C \end{bmatrix} = \begin{bmatrix} \frac{-2}{CR} & \frac{2}{C} \end{bmatrix} [v_C] + \begin{bmatrix} 0 & \frac{-2}{CR} \end{bmatrix} [u_1]$$

$$\begin{bmatrix} \dot{i}_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{2L} & 0 \end{bmatrix} [i_L] + \begin{bmatrix} \frac{1}{2L} & \frac{-1}{2L} \end{bmatrix} [u_{21}]$$

$$y(t) = [-1 \quad 0] \begin{matrix} [v_C] \\ [i_L] \end{matrix} + [0 \quad -1] \begin{matrix} [u_1] \\ [u_2] \end{matrix}$$

(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

I used 2 state variables for this problem

The current corresponds to the inductor (where it's 2L)
and the voltage corresponds to the capacitor (where it's C/2)