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Problem	Points	Score
1a	10	
1b	10	
1c	10	
1d	10	
2a	10	
2b	10	
2c	10	
2d	10	
3a	10	
3b	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Block Diagrams

(a) Write a differential equation describing this system.

Answer:

$$\left(c\frac{d}{dt}y(t)\right)+\left(b.x(t)\right)+\left(a.x(t)\right)= y(t)$$

Therefore the equation for the block diagram can be written as,

$$\left(c\frac{d}{dt}y(t)\right)+\left((a+b).x(t)\right)= y(t)$$

(b) Find the transfer function.

Answer:

$$c.s.Y(s)+\left((a+b)X(s)\right)= Y(s)$$

$$Y(s)(cs-1)=\left[\left(-(a+b)\right).X(s)\right]$$

$$\frac{Y(s)}{X(s)}=\frac{-(a+b)}{(cs-1)}$$

The transfer function is therefore,

$$H(s)=\frac{a+b}{(1-cs)}$$

(c) For what values of a, b, and c is the system stable (consider only non-zero values of a, b, and c).

Answer:

The system will be stable for any value of a& b (non-zero values).
And for c, any value c except for 0 & (1/s).

(d) Find the impulse response.

Answer:

$$H(s) = \frac{-(a+b)}{c.s-1}$$

$$= \frac{-\frac{1}{c}(a+b)}{s-\frac{1}{c}}$$

$$h(t) = L^{-1}[H(s)]$$

$$= \left(\frac{-(a+b)}{c}.e^{\frac{t}{c}}\right).u(t)$$

Problem No. 2: Transfer Functions

For the circuit shown below:

(a) Find $H_1(s)$:

Answer:

Voltage Division,

$$y_1(t) = \frac{R}{3R}.x_1(t)$$

$$H_1(s) = \frac{1}{3}$$

(b) Find $H_2(s)$:

Answer:

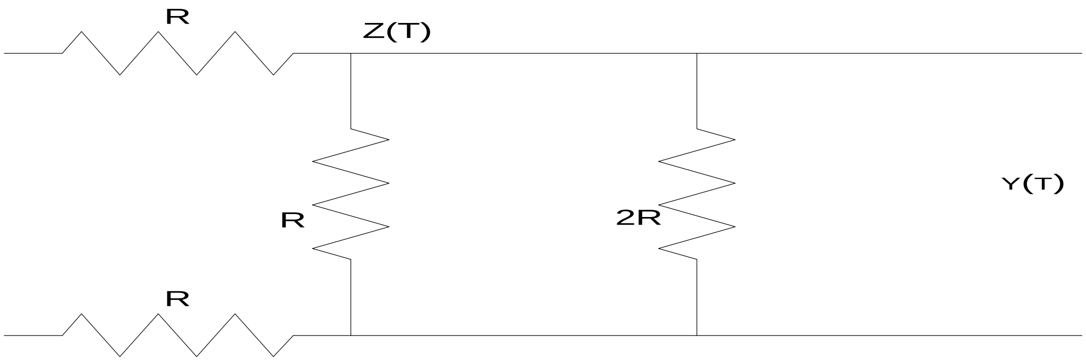
Using Voltage Division,

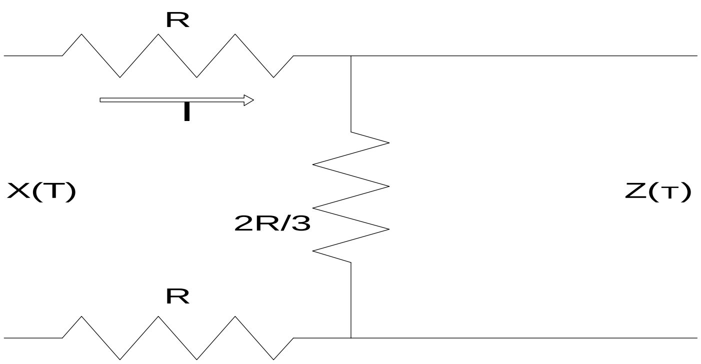
$$y_2(t) = \frac{R}{2R}.x_2(t)$$

$$H_2(s) = \frac{1}{2}$$

(c) Find $H_3(s)$:

Answer:





$x(t) = i.\frac{8R}{3}$

Voltage, z(t) over resistor, $\frac{2R}{3}$,

$$z(t) = i.\frac{2R}{3}$$
$$= \frac{x(t)}{8R}.3\left(\frac{2R}{3}\right)$$
$$= \frac{x(t)}{4}$$

The voltage, z(t), is equal to y(t), thus,

$$\Rightarrow \frac{x(t)}{4} = i.2R$$

$$\Rightarrow y(t) = \frac{1}{8}x(t)$$

$$\Rightarrow H_3(s) = \frac{1}{8}$$

(d) Is ? Justify your answer. Use as many concepts developed in this course as possible. A yes/no answer with no explanation gets no credit.

Answer:

$H_3(s) \neq H_1(s).H_2(s)$

This is because of the loading effect from the balance of the network.

Problem No. 3: The "Interesting" Problem

(a) Assume the voltage across the resistor in the circuit above is the output voltage, y(t). Derive the state variables representation of this circuit.

Answer:

Choose as state variables the capacitor voltage V_c and the inductor current, i_L
Writing a KCL for the capacitor yields:

$$\frac{-c}{2}.\frac{dV_c}{dt} - i_R + i_L = 0$$

$$\frac{c}{2}.\frac{dV_c}{dt} = i_L - i_R \qquad \Rightarrow Eqn(1)$$

A KVL for the inductor gives:

$$2L.\frac{di_L}{dt} = u_1(t) - V_R \qquad \Rightarrow Eqn(2)$$

The capacitor voltage equation:

$$V_c = V_R - u_2(t)$$

$$V_R = V_c + u_2(t)$$

$$i_R = \frac{V_c}{R} + \frac{u_2(t)}{R} \qquad \Rightarrow Eqn(3)$$

Substituting Eqn(3) into Eqn(1) yields:

$$\frac{dV_c}{dt} = \frac{2i_L}{C} - \frac{2V_c}{CR} - \frac{2u_2(t)}{CR}$$

From Eqn(2) and Eqn(3),

$$\frac{di_L}{dt} = \frac{u_1(t)}{2L} - \frac{V_c}{2L} - \frac{u_2(t)}{2L}$$

Since $i_R = i_L - i_c$ *and*

$$i_c = \frac{c.dV_c}{dt}$$

we have $i_c = 2i_L - 2\frac{V_c}{R} - 2\frac{u_2(t)}{R}$

and $i_R = (-i_L) - \frac{2V_c}{R} + 2\frac{u_2(t)}{R}$

Con't 3) part a.

$$V_R = i_R . R$$

$$V_R = \left((-i_L) + \frac{2V_c}{R} + 2\frac{u_2(t)}{R} \right) . R$$

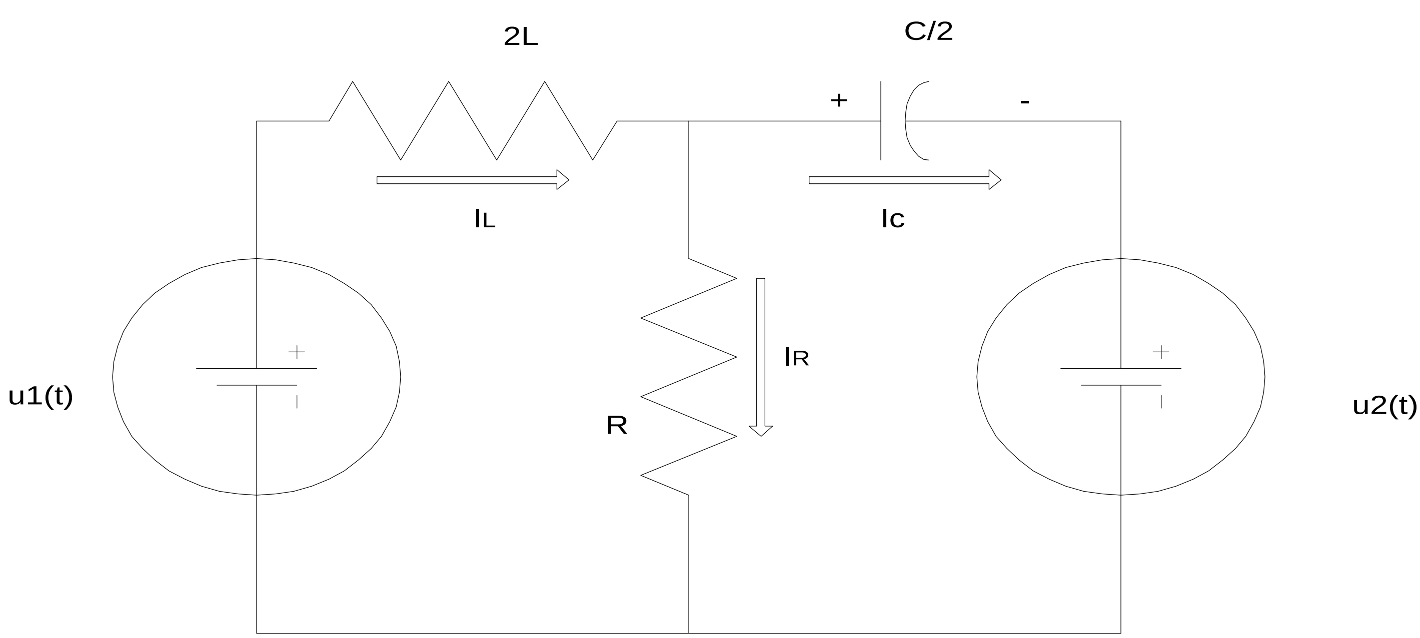
$$V_R = (-Ri_L) + 2V_c + 2u_2(t) \qquad \Rightarrow \textit{ output eqn}$$

The corresponding state equation in matrix form is:

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-2}{CR} & \frac{2}{C} \\ \frac{-1}{2L} & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{-2}{CR} \\ \frac{1}{2L} & \frac{-1}{2L} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

The output equation is therefore,

$$y(t) = \begin{bmatrix} 2 & -R \end{bmatrix} . \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 2 \end{bmatrix} . \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



(b) Now that you have finished (a), tell me how many state variables you should have used, and identify which voltages and/or currents they correspond to :)

Answer:

2 state variables were used, the capacitor voltage, V_C ,and the inductor current, i_L .