Name:

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

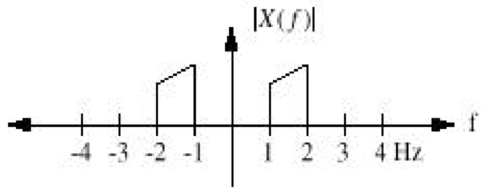
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

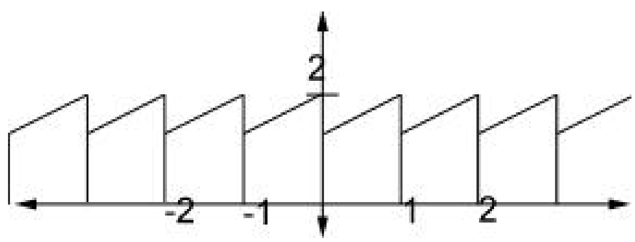
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.
The signal is complex because it is not symmetric about the origin.
Symmetric signals are real.

(b) Draw the spectrum of the sampled signal if f_s=1Hz.



The signal will be repeated every 1Hz and be double its original value.

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

1. Build a band pas filter that encompasses the freq range of x(t) and a magnitude response greater than the max value of X(f).

2. Pass the sampled signal through the filter to get X(f) back.

3. Take inverse Z transform to get x(n).

4. Use linear interpolation to connect the dots and reconstruct the original signal.

– The sampling theorem was not violated because the space between 0-1 Hz is dead space and can be thrown out. Therefore you can sample at 1 Hz.

Problem No. 2: Given the signal and impulse response shown below:

$$x(n) = \delta(n) + \delta(n+2) - \delta(n+4) - \delta(n+6)$$
$$h(n) = 3^{-1/2}\delta(n+1) + 3^{-1/2}\delta(n) + 3^{-1/2}\delta(n-1)$$

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

Facts	**Conclusion**
Convolution of 2 pulse trains is a pulse train.	Signal is an energy signal
Signal is time limited.	
Frequency is not band limited.	

(b) Compute described in (a) as the convolution of these two functions.

There are two easy ways to compute this answer. Discrete convolution and using the z transform. I will solve it with discrete convolution since I know my other classmates all solved it with the Z - transform method.

0	0	0	-1	0	-1	0	1	0	1	0	0	0
n=-6	$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$									
n=-5		$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$								
n=-4			$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$							
n=-3				$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$						
n=-2					$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$					
n=-1						$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$				
n=0							$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$			
n=1								$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$		
n=2									$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$	
n=3										$3^{-1/2}$	$3^{-1/2}$	$3^{-1/2}$

At n<-6 and n>2 the answer will be zero. So restraining n to between these two numbers will ease the calculations. This should give future students something to compare with.

$y(-6) = 3^{-1/2} * -1 = -3^{-1/2}$
$y(-5) = 3^{-1/2} * 0 + 3^{-1/2} * -1 = -3^{-1/2}$
$y(-4) = 3^{-1/2} * -1 + 0 + 3^{-1/2} * -1 = -6^{-1/2}$
$y(-3) = 0 + -1 * 3^{-1/2} + 0 = -3^{-1/2}$
$y(-2) = 3^{-1/2} * 1 + 0 + 3^{-1/2} * -1 = 0$
$y(-1) = 0 + 3^{-1/2} * 1 + 0 = 3^{-1/2}$
$y(0) = 3^{-1/2} * 1 + 0 + 3^{-1/2} * 1 = 6^{-1/2}$
$y(1) = 0 + 3^{-1/2} * 1 + 0 = 3^{-1/2}$
$y(2) = 3^{-1/2} * 1 = 3^{-1/2}$

If you are curious, Matlab gives the same results as well.

M - file :

```
x = [-1,  0,  -1,   0,  1,  0,  1];
h = [ sqrt(3),  sqrt(3),  sqrt(3) ];
y = conv(x, h);
```

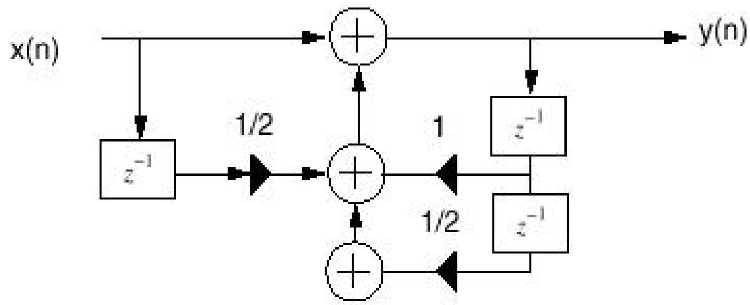
Ans :

```
    y = [ -1.7321   -1.7321   -3.4641   -1.7321   0    1.7321    3.4641  1.7321    1.7321 ]
```

(c) Assume in (a) was a periodic signal. Will the power in the output, be different than the power in the input? Explain.

The power output will be different because the weights of h(n) are not one. Therefore the weights of the output will change thereby changing the power output. Since power is defined as the square of the signal divided by half its period, this will result in a factor of 3 change in power from the input signal.

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

$$y(z) := x + \frac{1}{2}\cdot x\cdot z^{-1} + y\cdot z^{-1} + \frac{1}{2}\cdot y\cdot z^{-2}$$

$$y\cdot\left(1 - z^{-1} - \frac{1}{2}\cdot z^{-2}\right) = x\cdot\left(1 + \frac{1}{2}\cdot z^{-1}\right)$$

$$\frac{y(z)}{x(z)} = \frac{1 + \frac{1}{2}\cdot z^{-1}}{1 - z^{-1} - \frac{1}{2}\cdot z^{-2}}$$

(b) Find the impulse response.

Take inverse Z transform to find impulse response :

$$\frac{y(z)}{x(z)} = \frac{1+\frac{1}{2}\cdot z^{-1}}{1-z^{-1}-\frac{1}{2}\cdot z^{-2}}$$

Matlab can be used to find the partial fraction expansion by use of the residue function.

M - file :
Numerator = [0 .5 1]
Denominator = [-.5 -1 1]
[R, P, k] = residue (Numerator, Denominator)

R =

-0.2113
-0.7887

P =

-2.7321
0.7321

The two elements in the r matrix correspond to the partial fraction expansion coefficients for the denominator factors (z^{-1} + 2.7321) and (z^{-1} - 0.7321) respectively.

$$h(z) = \frac{-0.2113}{2.7321+z^{-1}} + \frac{0.7887}{.7321-z^{-1}} \qquad h(z) = \frac{-.577}{1+2.7321\cdot z^{-1}} + \frac{.577}{1-.7321\cdot z^{-1}}$$

$$y(nT) = -.577\cdot(2.7321)^{n} + .577\cdot(0.7321)^{n}$$

(c) Sketch the magnitude of the frequency response.

H(f) = H(z) when $z = e^{-j\cdot 2\cdot\pi\cdot\frac{f}{f_s}}$

$$\frac{y(f)}{x(f)} = \frac{1+\frac{1}{2}\cdot\exp\left(-j\cdot 2\cdot\pi\cdot n\cdot\frac{f}{f_s}\right)}{1-\exp\left(-j\cdot 2\cdot\pi\cdot n\cdot\frac{f}{f_s}\right)-\frac{1}{2}\cdot\exp\left(-j\cdot 4\cdot\pi\cdot n\cdot\frac{f}{f_s}\right)}$$

There are going to be two poles in this system. One at .7321 and the other at -2.7321.

(d) Convert *H(z) to H(s)* by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

Conversion can be accomplished by realizing that the z transform is just the Laplace transform applied to a discrete signal. z is just a substitution into the result of the transform. To solve for the poles in the s plane just solve the substitution equation for s, and then lug your poles into the resulting equation. Each term in the summation below contains the value of the sample and the occurrence in time of the sample. The coefficient *x(nT)*, denotes the sample value and z^{-n} denotes that the sample value occurs at n sample periods after *t=0*. The f in the result when we solved for s is simply the sample frequency. You can assume it to be 1 for plotting.

$$X(z) := \sum_{n=0}^{\infty} x(n\cdot T)\cdot z^{-n}$$

$z = e^{s\cdot t}$ s1 := ln(2.7321) s2 := ln(.7321)

$\ln(z) = s\cdot t$

$s = \frac{\ln(z)}{t} = \ln(z)\cdot f$ s1 = 1.005 s2 = −0.312