Name: Rachel Casey

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

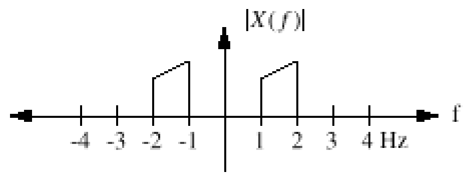
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

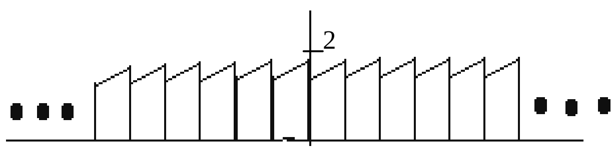
Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.

The signal is complex because it is not symmetric around the y-axis. If the signal were real, it would have to display symmetry.

(b) Draw the spectrum of the sampled signal if f_s = 1 Hz.



The sampled signal would be a saw tooth signal of magnitude 2 occuring at each multiple of $f_s/2$. The theorem $X_s = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$ is used to get the sampled signal.

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

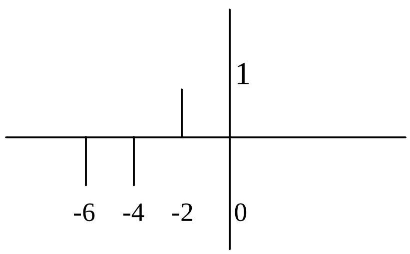
To recover a signal, the sample frequency needs to be greater than the highest frequency in the signal or $f_s \geq 2 f_h$. This condition is called the Nyquist rate. In this case, the sample frequency is 1 Hz and the highest frequency in the signal is 1 Hz. Therefore, the sample frequency just barely meets the Nyquist rate condition. The signal can be recovered using an ideal low pass filter.

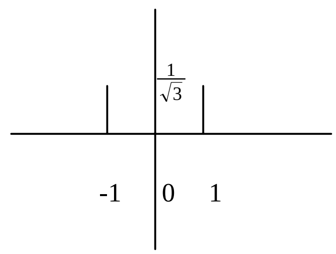
Problem No. 2: Given the signal and impulse response shown below:

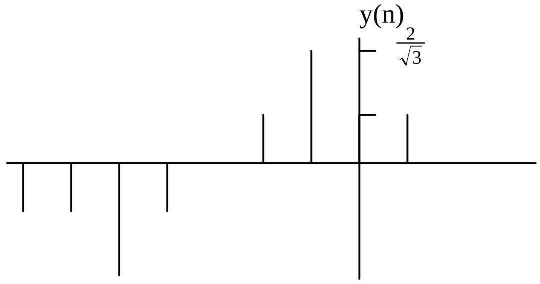
$$x(n) = \delta(n) + \delta(n+2) - \delta(n+4) - \delta(n+6)$$
$$h(n) = 3^{-1/2}\delta(n+1) + 3^{-1/2}\delta(n) + 3^{-1/2}\delta(n-1)$$

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

The signal convoution of the two signals produces a non-periodic function, therefore the signal is an energy signal. This can be seen by graphically convolving the two signals.







(b) Compute described in (a) as the convolution of these two functions.

Since convolution in the Z-domain is multiplication, it is easier to transform the signals into the Z-domian.

$$X(z) = 1 + z^2 - z^4 - z^6$$
$$H(z) = \frac{1}{\sqrt{3}}\left(z^1 + 1 + z^{-1}\right)$$

Next, multiply the signals together.

$$X(z) \bullet H(z) = \frac{1}{\sqrt{3}}\left[\left(1 + z^2 - z^4 - z^6\right)\left(z^1 + 1 + z^{-1}\right)\right]$$

$$= \frac{1}{\sqrt{3}}\left(z^1 + 1 + z^{-1} + z^3 + z^2 + z^1 - z^5 - z^4 - z^3 - z^7 - z^6 - z^5\right)$$
$$= \frac{1}{\sqrt{3}}\left(z^{-1} + 1 + z^1 + z^2 - z^4 - 2z^5 - z^6 - z^7\right)$$

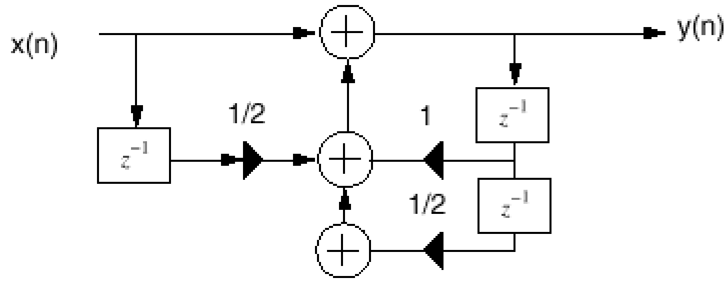
Once the solution is obtained, take the inverse z-transform to return the values to the discrete time domain.

$$y(n) = \frac{1}{\sqrt{3}}\left(\delta(n-1) + \delta(n) + \delta(n+1) + \delta(n+2) - \delta(n+4) - 2\delta(n+5) - \delta(n+6) - \delta(n+7)\right)$$

(c) Assume in (a) was a periodic signal. Will the power in the output, y(n), be different than the power in the input? Explain.

Assuming that x(n) was a periodic signal, the power of the input signal, x(n), and the output signal, y(n), will be approximately the same because of the filter h(n) that was used. This happens because the function h(n) is a type of averaging function. h(n)'s $\frac{1}{\sqrt{3}}$ coefficient causes the power to remain nearly the same for both signals since the squared value of the signal equals the average power of the input signal.

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

Convert the input and output signals to the z-domain and write the equation for the system.

$$X(z)+\frac{1}{2}z^{-1}X(z)+z^{-1}Y(z)+\frac{1}{2}z^{-2}Y(z)=Y(z)$$

Next, simplify the equation having only Y(z) on the left and X(z) on the right.

$$Y(z)\left[1-z^{-1}-\frac{1}{2}z^{-2}\right]=X(s)\left[1+\frac{1}{2}z^{-1}\right]$$

Divide to get the transfer function.

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+\frac{1}{2}z^{-1}}{1-z^{-1}-\frac{1}{2}z^{-2}}$$

(b) Find the impulse response.

To get the inpulse response, the inverse z transform must be taken of the transfer function. The quadratic formula will be needed to factor the denominator of the transfer function.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 - z^{-1} - \frac{1}{2}z^{-2}} = \frac{A}{1 - Mz^{-1}} + \frac{B}{1 - Nz^{-1}}$$

Solve for M and N using the quadratic formula. The general form of the quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Choosing a = 1, b = -1, c = -1/2, and letting x = z^{-1}, we get the following.

$$z^{-1} = \frac{-(-1) \pm \sqrt{1^2 - 4(1)\left(-\frac{1}{2}\right)}}{2(1)} = \frac{1 \pm \sqrt{1+2}}{2} = \frac{1 \pm \sqrt{3}}{2}$$

Using these results, we can define M and N.

$$M = \frac{1 + \sqrt{3}}{2} = 1.37 \qquad N = \frac{1 - \sqrt{3}}{2} = -.37$$

Substituting back into the transfer function and finding the values for A and B, we get

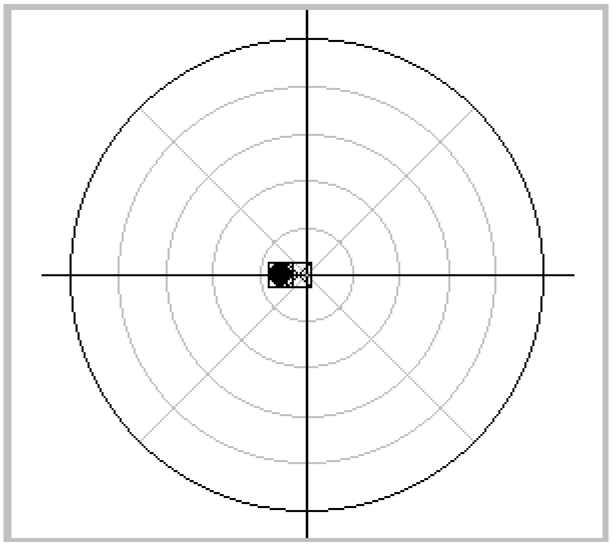
$$H(z) = \frac{A}{1 - Mz^{-1}} + \frac{B}{1 - Nz^{-1}} = \frac{-.211}{1 - Mz^{-1}} + \frac{-.788}{1 - Nz^{-1}}$$

$$h(n) = -.211(M)^n - .788(N)^n = -.211(1.37)^n - .788(-.37)^n$$

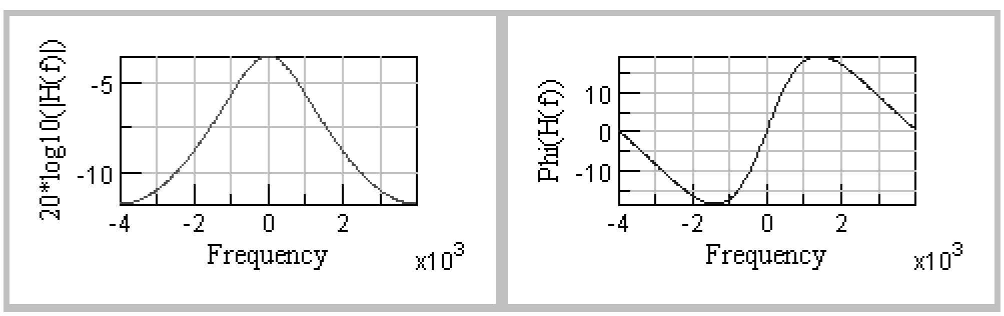
$$h(n) = -.211(1.37)^n - .788(-.37)^n$$

(c) Sketch the magnitude of the frequency response.

Using a sample frequency of 8000 Hz, the transfer function can be plotted on the z axes. This shows that there are two poles, one located at 0 Hz (bandwidth = -2459) and one at 4000 Hz (bandwidth = 8173). The values of the poles and zero were obtained from the transfer function. There is also one zero located at 4000 Hz (bandwidth = 5545). The pole with zero frequency lies outside of the plot area.



Using the Java pole and zero tool, the frequency response of the discrete signal is shown below as

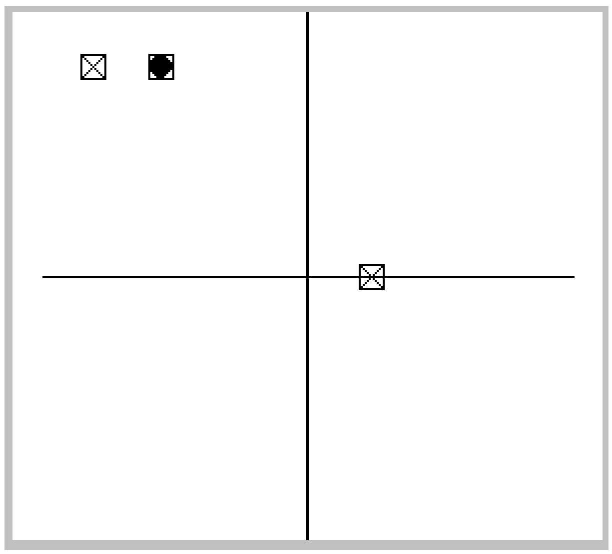


(d) Convert H(z) to H(s) by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

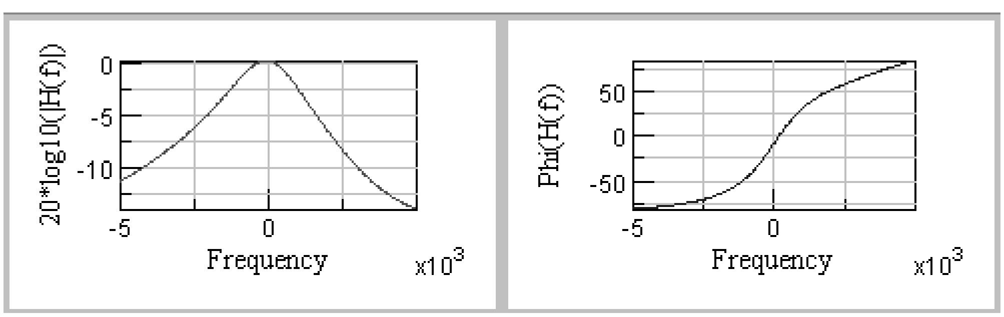
To convert H(z) to H(s), H(z) needs to be evaluated for $z = e^{sT}$

$$H(z)\Big|_{z=e^{sT}} = \frac{1+\frac{1}{2}e^{-sT}}{1-e^{-sT}-\frac{1}{2}e^{-2sT}}$$

Using 8000 Hz as the sample frequency as in part c, the pole (4000 Hz, bandwidth = 8173) and two zeros (4000 Hz, bandwidth = 5545 and 0 Hz, bandwidth = -2945 can be plotted.



The frequency response of H(s) is



In comparing the magnitude of the frequency response of H(z) and H(s), both signals have similar shape. H(s)'s magnitude has a different rise and fall than that of H(z) but the shapes and maximum magnitude are the same. The phase of the two different responses is different due to the location of the pole and zeros. For H(z), one of the zeros is not within the z transform circle, therefore it has an effect on changing the phase response.