Name: Matthew Gunter

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

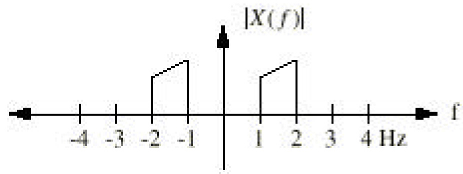
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

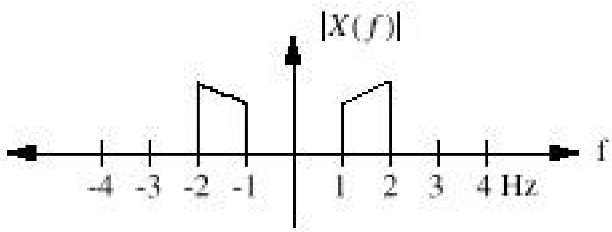
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Sampling

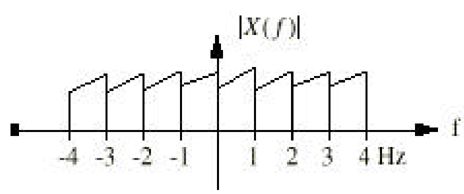


(a) Is this signal real or complex? Justify your answer.

The signal shown above is a complex signal. If the signal were a real signal the plot of its frequency spectrum would be symmetrical about the zero frequency axis. A real signal similar to this one could look like the one shown below.



(b) Draw the spectrum of the sampled signal if f_s = 1 Hz .



Because our f_h=2Hz and we are sampling at 1Hz we will have aliasing. This is shown in the above graph of our periodic spectrum obtained by sampling the signal. The amplitude of the sampled signal is twice that of the original signal because of overlap.

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

To recover the signal from the sampled signal you need an ideal low pass filter with a bandwidth of 0.5 fs and run the sampled signal through the filter to obtain the original signal. Using the modulation theorem, fs can be lower than 8 Hz because you can recover the positive frequency side just fine and it is a periodic signal. Therefore the sampling theorem was not violated. If this were a real signal this would not be so because symmetry would prevent this trick.

Problem No. 2: Given the signal and impulse response shown below:

$$x(n) = \delta(n) + \delta(n+2) - \delta(n+4) - \delta(n+6)$$
$$h(n) = 3^{-1/2}\delta(n+1) + 3^{-1/2}\delta(n) + 3^{-1/2}\delta(n-1)$$

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

$$y(n) = x(n) * H(n)$$

This is an energy signal. It is a segment over a limited space of time consitsting of impulse at discrete points over that time. It is not a periodic signal so therefore it can not be a power signal.

(b) Compute described in (a) as the convolution of these two functions.

$$y(n) = x(n) * H(n)$$

Below is shown the impulse functions as a group of discrete points, and padded with some of the zeroes where necessary.

$$x(n) = \{-1,\ 0,\ -1,\ 0,\ 1,\ 0,\ 1,\ 0,\ 0\}$$
$$H(n) = \left\{0,\ 0,\ 0,\ 0,\ 0,\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ 0\right\}$$

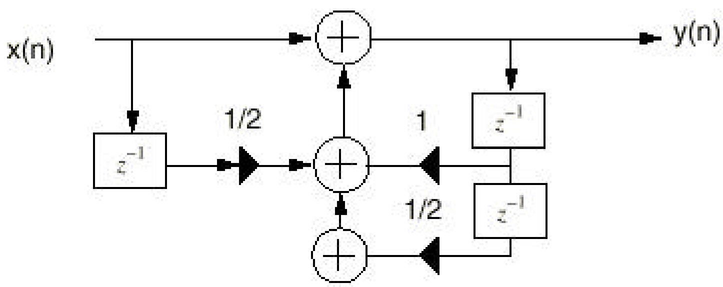
To obtain the convolution of these two signal we simply add the impulses at the respective points in time. Adding the two together we obtain y(n) to be:

$$y(n) = \left\{-1,\ 0,\ -1,\ 0,\ 1,\ \frac{1}{\sqrt{3}},\ 1 + \frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}}\right\}$$

(c) Assume in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

Because of the coefficients of the transfer function, the power of the output would be different than the power of the input. The transfer function is an averager and also an amplifier. The amplification of the input gives way to different power at the output.

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

The transfer function of the above system can be taken from inspection and simply written down since it is only the sum of three time delays that are amplifed and adding to the input signal to get the output. The transfer function was obtained and simplified as follows.

$$H(z) = \frac{y(z)}{x(z)}$$

from inspection

$$y(z) = z(z) + y(z)\,z^{-1} + \frac{1}{2}\,y(z)\,z^{-2} + \frac{1}{2}\,x(z)\,z^{-1}$$

grouping our y(z) and x(z) on both sides we get

$$y(z) - y(z)\,z^{-1} - \frac{1}{2}\,y(z)\,z^{-2} = z(z) + \frac{1}{2}\,x(z)\,z^{-1}$$

pulling out the y(z) and x(z) terms on both sides

$$y(z)[1 - z^{-1} - \frac{1}{2} z^{-2}] = x(z)[1 + \frac{1}{2} z^{-1}]$$

dividing by x(z) on one side and the term after y(z) into the other side we get our transfer function to be

$$H(z) = \frac{y(z)}{x(z)} = \frac{1 + \frac{1}{2} z^{-1}}{1 - z^{-1} - \frac{1}{2} z^{-2}}$$

(b) Find the impulse response.

To find the impulse response we must take the inverse Z-transform of the transfer function

First we multiply the transfer function by two to simplify things a bit.
Doing so we obtain.

$$H(z) = \frac{2 + z^{-1}}{2 - 2z^{-1} - z^{-2}}$$

Once this is done, multiply by (-1/-1) and re-write the transfer function in quadratic form and take the take the roots of the denominator.

$$H(z) = \frac{-2 - z^{-1}}{z^{-2} + 2z^{-1} - 2}$$

Taking the roots of the denominator we obtain

$$z = -1 + \sqrt{3}\,;\ -1 - \sqrt{3}$$

Therefore:

$$z^{-1} = 1.366,\ -0.366$$

Rewriting our transfer function in the with roots taken we have:

$$H(z) = \frac{-2 - z^{-1}}{(z^{-1} - 1.366)(z^{-1} + 0.366)}$$

We must now do partial fractions expansion on the above equation to be able to take the inverse Z-transform.

$$H(z) = \frac{A}{z^{-1} - 1.366)} + \frac{B}{(z^{-1} + 0.366)}$$

Doing partial fraction routine we obtain the following two equations that allow us to find the numerators of the new equation:

$$-1 = A + B$$

$$-2 = -A(1.366) - B(-0.366)$$

Solving simultaneously for A and B we get

A = 0.943 and B = -1.943

$$H(z) = \frac{0.943}{(z^{-1} - 1.366)} + \frac{-1.943}{(z^{-1} + 0.366)}$$

We are now ready to take the inverse Z transform of the transfer function since it is in a form found on a transform table. Below is or formatted transfer function from above.

$$H(z) = \frac{-0.69}{(1 + 0.732z^{-1})} + \frac{-5.308}{(1 + 2.732z^{-1})}$$

Looking on the transform table and finding the form that fits out above equation we get.

$$H(z) = -0.69\left(\frac{1}{(1 - Kz^{-1})}\right) - 5.308\left(\frac{1}{(1 - Kz^{-1})}\right)$$

K = -0.732 and -2.732

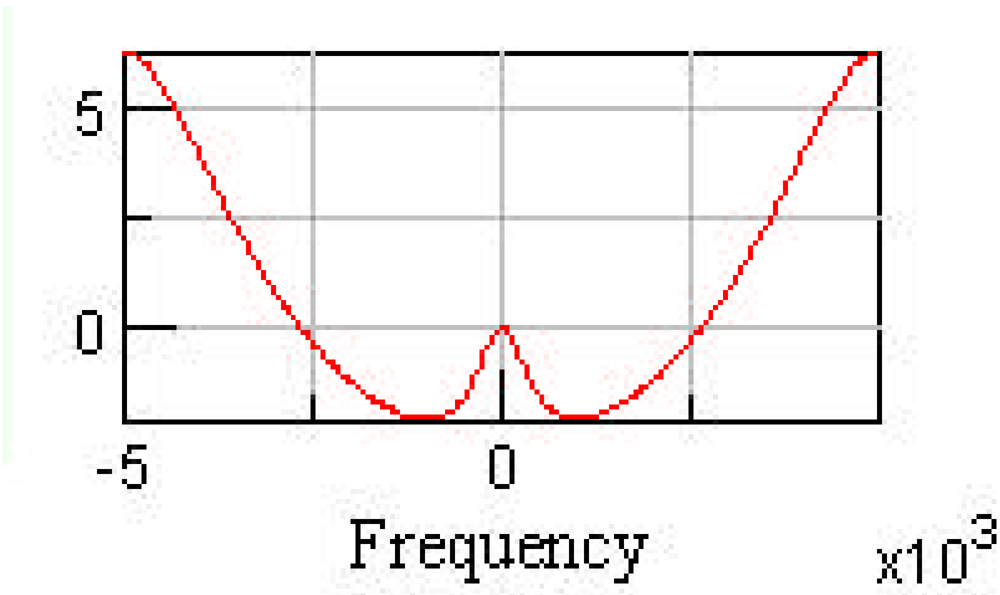
according to the table we have

$$H(nT) = -0.69(-0.732^{n}) - 5.308(-2.732^{n})$$

this is our impulse response

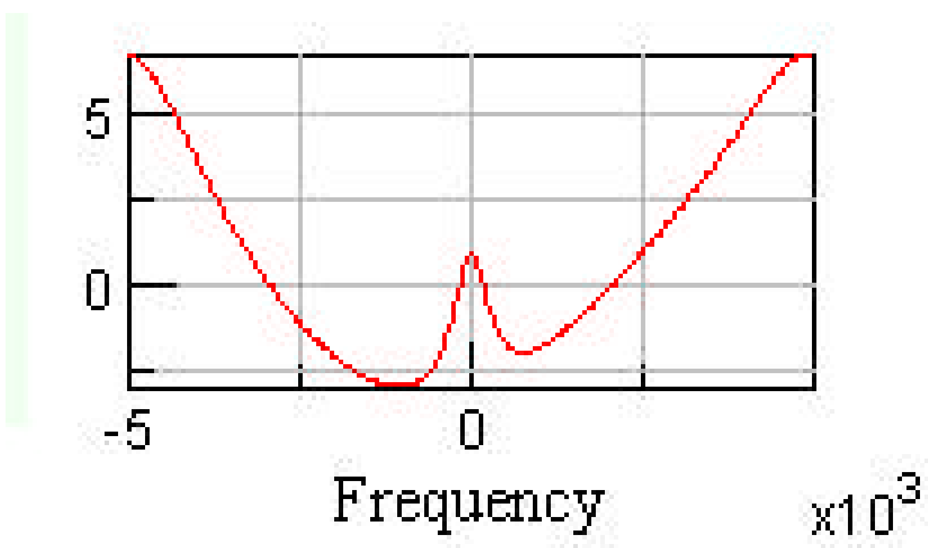
(c) Sketch the magnitude of the frequency response.

Using the pole zero java applet, I found the frequency response. Below is the graph of the magnitude of the frequency response.



note: this is a rough estimate of the graph

(d) Convert to by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.



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The differences found between the two frequency responses is due to the transformation technique used to go from the Z to the S planes. Our transformation used which was $z = e^{st}$ is a non-linear transformation and this is evident in the discrepency between the two graphs. This effect is called frequency warping.