Name: Tuni Kern

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

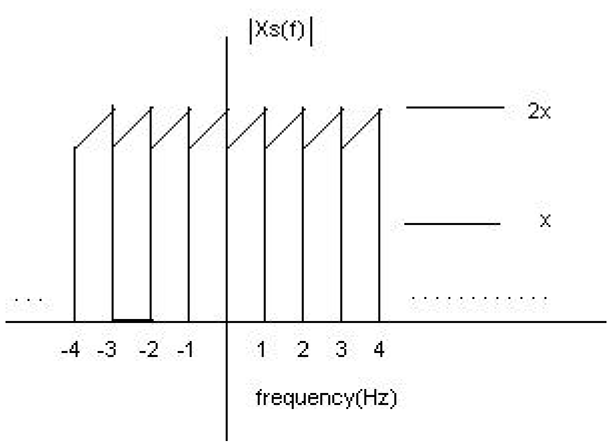
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Sampling

(a) Is this signal real or complex? Justify your answer.

The signal is complex because it is not symmetric around the y-axis

(b) Draw the spectrum of the sampled signal if f_s = 1 Hz.



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

In this case I would use an ideal reconstruction filter twice with an f_c of ½ Hz since f_c must be greather than or equal to f_h or ½ while simultaneously being less than or equal to $f_s - f_h$ yeilding the only possible value of ½ Hz. This yeilds a filter bandwidth of f_s or 1Hz ($f_c = f_s/2$). For an ideal filter, the filter amplitude would also need to be set to T. The sampling theorem states that for a signal to be completely specified and properly reconstructed, its samples must be taken at a frequency of twice the highest signal frequency. Obviously our signal in (b) was sampled at a frequency less than twice the highest frequecny or 4Hz and was sampled at 1Hz violating the sampling theory. However, because of the shape of our particular signal and the sampling frequency being precisely the bandwidth of a single “sawtooth” of our original signal, we get perfect overlapping in our sampled signal and are able to reconstruct the original signal to within a constant of its original amplitude.

Problem No. 2: Given the signal and impulse response shown below:

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

As can be seen in part b, the convolution of X and H are not periodic. This means the signal has finite energy and zero power and is thus an energy signal.

(b) Compute Y(n) described in (a) as the convolution of these two functions.

$$X(z) = 1 + z^2 - z^4 - z^6$$

$$H(z) = 3^{\frac{-1}{2}}\left[z^1 + 1 + z^1\right]$$

$$Y(z) = X(z) \bullet Y(z)$$

$$Y(z) = 3^{\frac{-1}{2}}\left[z^1 + 1 + z^1\right] \bullet \left[1 + z^2 - z^4 - z^6\right]$$

$$Y(z) = 3^{\frac{-1}{2}}\left[z^{-1} + 1 + 2z^1 + z^2 - z^4 - 2z^5 - z^6 - z^7\right]$$

$$Y(n) = 3^{\frac{-1}{2}}\left[\mathrm{d}(n-1) + \mathrm{d}(n) + 2\mathrm{d}(n+1) + \mathrm{d}(n+2) - \mathrm{d}(n+4) - 2\mathrm{d}(n+5) - \mathrm{d}(n+6) - \mathrm{d}(n+7)\right]$$

(c) Assume in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

In a real world application, with our given H(n) there would be little or no power lost because of our $3^{-1/2}$ term on each of the pulses. This serves as a scale factor to make up for energy lost due to the non-ideal filter. If H(n) was a true ideal filter the coefficients on the pulses would be 1/3 for perfect reconstruction of X(n), but for a real application the $3^{-1/2}$ term gives us a negligible power loss and the power of X(n) and Y(n) will be approximately the same.

Problem No. 3: Z-Transforms

(a) Find the transfer function of the system shown above.

$$Y(z) = X(z) + \frac{1}{2}X(z)z^{-1} + Y(z)z^{-1} + \frac{1}{2}Y(z)z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{X(z) + \frac{1}{2}X(z)z^{-1} + Y(z)z^{-1} + \frac{1}{2}Y(z)z^{-2}}{X(z)}$$

$$H(z) = 1 + \frac{1}{2}z^{-1} + H(z)z^{-1} + \frac{1}{2}H(z)z^{-2}$$

$$H(z) - H(z)z^{-1} - \frac{1}{2}H(z)z^{-2} = 1 + \frac{1}{2}z^{-1}$$

$$H(z)\left[1 - z^{-1} - \frac{1}{2}z^{-2}\right] = 1 + \frac{1}{2}z^{-1}$$

$$H(z) = \frac{1 + \frac{1}{2}z^{-1}}{\left[1 - z^{-1} - \frac{1}{2}z^{-2}\right]}$$

(b) Find the impulse response.

$$H(z) = \frac{2 + z^{-1}}{2 - 2z^{-1} - 1 \cdot z^{-2}}$$

$$\frac{2 + \mathrm{z}^{-1}}{(1 - ((1 + \sqrt{3})/2)\mathrm{z}^{-1})(1 - ((1 - \sqrt{3})/2)\mathrm{z}^{-1})} = \frac{\mathrm{A}}{(1 - ((1 + \sqrt{3})/2)\mathrm{z}^{-1})} + \frac{\mathrm{B}}{(1 - ((1 - \sqrt{3})/2)\mathrm{z}^{-1})}$$

$$2 + \mathrm{z}^{-1} = \mathrm{A} \cdot (1 - (1 - \sqrt{3})/2)\mathrm{z}^{-1} + \mathrm{B} \cdot (1 - (1 + \sqrt{3})/2)\mathrm{z}^{-1}$$

$$2 + \mathrm{z}^{-1} = \mathrm{A} - (\mathrm{A} \cdot (1 - \sqrt{3})/2)\mathrm{z}^{-1} + \mathrm{B} - (\mathrm{B} \cdot (1 + \sqrt{3})/2)\mathrm{z}^{-1}$$

$$1 = -\mathrm{A} \cdot (1 - \sqrt{3})/2 - \mathrm{B} \cdot (1 + \sqrt{3})/2$$

$$2 = A + B$$

$$\mathrm{A} = 2.1547$$

$$\mathrm{B} = -0.1547$$

$$\mathrm{H(z)} = \frac{2.1547}{1 - ((-1 + \sqrt{3})/2)\mathrm{z}^{-1}} - \frac{0.1547}{1 - ((-1 - \sqrt{3})/2)\mathrm{z}^{-1}}$$

$$\mathrm{H(n)} = \mathrm{Z}^{-1}\{\mathrm{H(z)}\}$$

$$\mathrm{H(n)} = 2.1547 \cdot ((-1 + \sqrt{3})/2)^{\mathrm{N}} - 0.1547 \cdot ((-1 - \sqrt{3})/2)^{\mathrm{N}}$$

$$\mathrm{H(n)} = 2.1547 \cdot (0.366025)^{N} - 0.1547 \cdot (-1.366025)^{N}$$

(c) Sketch the magnitude of the frequency response.

(d) Convert H(z) to H(s) by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.