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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

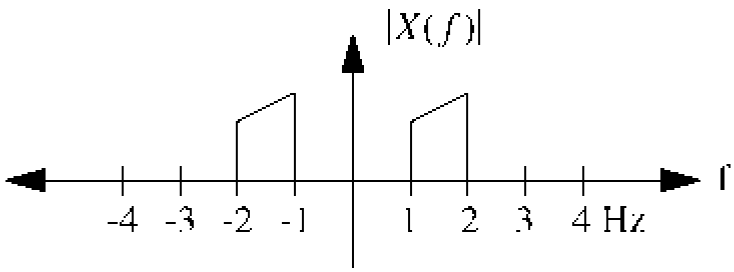
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

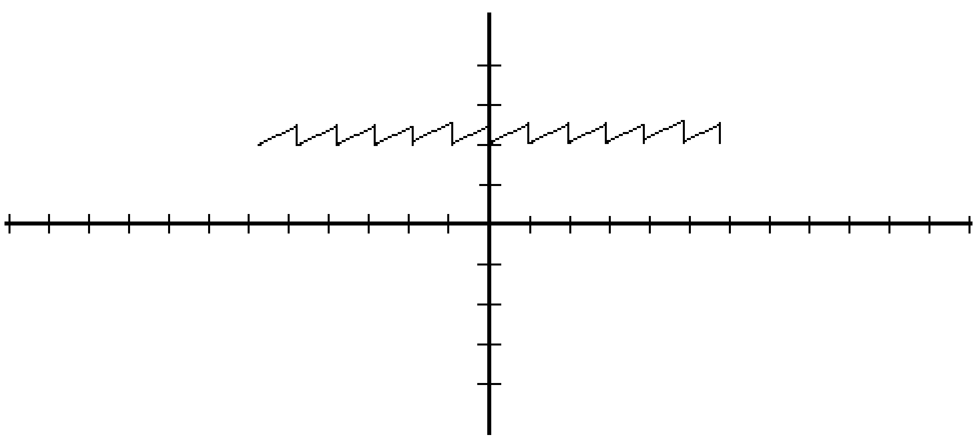
Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.

This signal is complex because it is not symmetric. Only real signals are symmetric.

(b) Draw the spectrum of the sampled signal if $f_s = 1$ Hz



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

There is no frequency response from 0 to 1 Hz in this signal, so according to the Band Pass Theorem, a sampling frequency of about 1 Hz is all that is needed. This is the minimal sampliung frequency from which the signal can be recovered.

Problem No. 2: Given the signal and impulse response shown below:

x(n) = δ(n) + δ(n+2) – δ (n+4) – δ (n+6)
h(n) = $3^{-1/2}$(n+1) + $3^{-1/2}$ (n) +$3^{-1/2}$ (n – 1)

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

$$y(n) = x(n) * h(n)$$
$$Z\{ x(n) * h(n) \} = X(z) \cdot H(z)$$

The signal is an energy signal because x(n) and h(n) are not periodic. Therefore, y(n) will only have a value for a finite number of N. It can be confirmed in part (b) that y(n) is not periodic.

(b) Compute described in (a) as the convolution of these two functions.

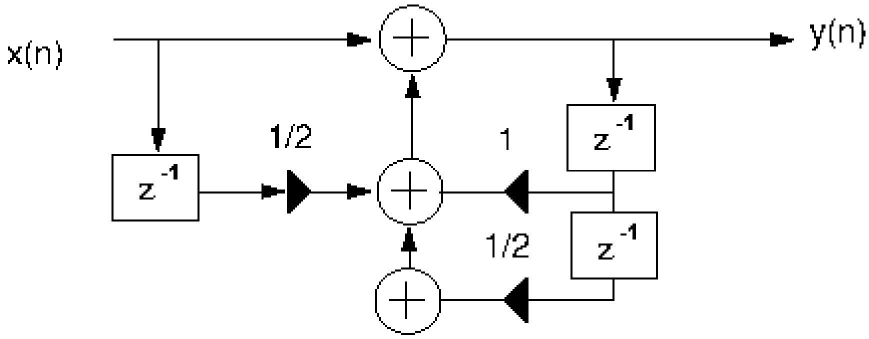
$$X(z) = 1 + z^{2} - z^{4} - z^{6}$$
$$H(z) = 3^{-1/2}[z^{1} + 1 + z^{-1}]$$
$$X(z) \cdot H(z) = 3^{-1/2}[z^{1} + 1 + z^{-1}] \cdot [1 + z^{2} - z^{4} - z^{6}]$$
$$= 3^{-1/2}[z^{1} + 1 + z^{-1} + z^{3} + z^{2} + z^{1} - z^{5} - z^{4} - z^{3} - z^{7} - z^{6} - z^{5}]$$
$$= 3^{-1/2}[z^{-1} + 1 + 2 \cdot z^{1} + z^{2} - z^{4} - 2 \cdot z^{5} - z^{6} - z^{7}]$$

y(n) = $3^{-1/2}$[δ(n –1) + δ(n) + 2δ(n+1) + δ(n+2) – δ(n+4) – 2δ(n+5) – δ(n+6) – δ(n+7)]

(c) Assume x(n) in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

If x(n) were periodic, y(n) would have about the same power of x(n) because y(n) is approximately just an average of x(n).

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

$$y(n) = x(n) + \frac{1}{2}x(n\text{-}1) + \frac{1}{2}y(n\text{-}2) + \frac{1}{2}y(n\text{-}1)$$

$$Y(z) = X(z) + \frac{1}{2}\cdot z^{-1}\cdot X(z) + \frac{1}{2}\cdot z^{-2}\cdot Y(Z) + z^{-1}\cdot Y(Z)$$

$$\frac{Y(z)}{X(z)} = \frac{X(z)}{X(z)} + \frac{1}{2}\cdot z^{-1}\cdot\frac{X(z)}{X(z)} + \frac{1}{2}\cdot z^{-2}\cdot\frac{Y(z)}{X(z)} + z^{-1}\cdot\frac{Y(Z)}{X(z)}$$

$$H(z) = 1 + \frac{1}{2}\cdot z^{-1} + \frac{1}{2}\cdot z^{-2}\cdot H(z) + z^{-1}\cdot H(z)$$

$$H(z) - \frac{1}{2}\cdot z^{-2}\cdot H(z) - z^{-1}\cdot H(z) = 1 + \frac{1}{2}\cdot z^{-1}$$

$$H(z)[1 - \frac{1}{2}\cdot z^{-2} - z^{-1}] = 1 + \frac{1}{2}\cdot z^{-1}$$

$$H(z) = \frac{1 + \frac{1}{2}\cdot z^{-1}}{1 - \frac{1}{2}\cdot z^{-2} - z^{-1}} = \frac{2 + z^{-1}}{2 - z^{-2} - 2\cdot z^{-1}}$$

(b) Find the impulse response.

Since the impulse response of a system outputs the transfer function, we must use part (a):

$$H(z) = \frac{2 + z^{-1}}{2 - z^{-2} - 2 \cdot z^{-1}} = \frac{2 + z^{-1}}{-(z^{-2} + 2 \cdot z^{-1} - 2)}$$

The roots of the polynomial in the denominator must first be found by using the quadratic equation.

$$z^{-2} + 2 \cdot z^{-1} - 2 \qquad \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$$a = 1$$

$$b = 2 \qquad \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot -1}}{2 \cdot 1} = -1 \pm \sqrt{3}$$

$$c = -1$$

Partial fractions must now be used.

$$\frac{-2 - z^{-1}}{(z^{-1} - (-1 + \sqrt{3}))(z^{-1} - (-1 - \sqrt{3}))} = \frac{A}{(z^{-1} - (-1 + \sqrt{3}))} + \frac{B}{(z^{-1} - (-1 - \sqrt{3}))}$$

$$-2 - z^{-1} = A \cdot (z^{-1} - (-1 - \sqrt{3})) + B \cdot (z^{-1} - (-1 + \sqrt{3}))$$

$$-2 - z^{-1} = A \cdot z^{-1} - A \cdot (-1 - \sqrt{3}) + B \cdot z^{-1} - B \cdot (-1 + \sqrt{3})$$

$$-1 = A + B$$

$$-2 = -A \cdot (-1 - \sqrt{3}) - B \cdot (-1 + \sqrt{3})$$

$$A = -0.788675$$

$$B = -0.211325$$

$$H(z) = \frac{-0.7887675}{(z^{-1} - (-1 + \sqrt{3}))} + \frac{-0.211325}{(z^{-1} - (-1 - \sqrt{3}))} = \frac{0.7887675}{((-1 + \sqrt{3}) - z^{-1})} + \frac{0.211325}{((-1 - \sqrt{3}) - z^{-1})}$$

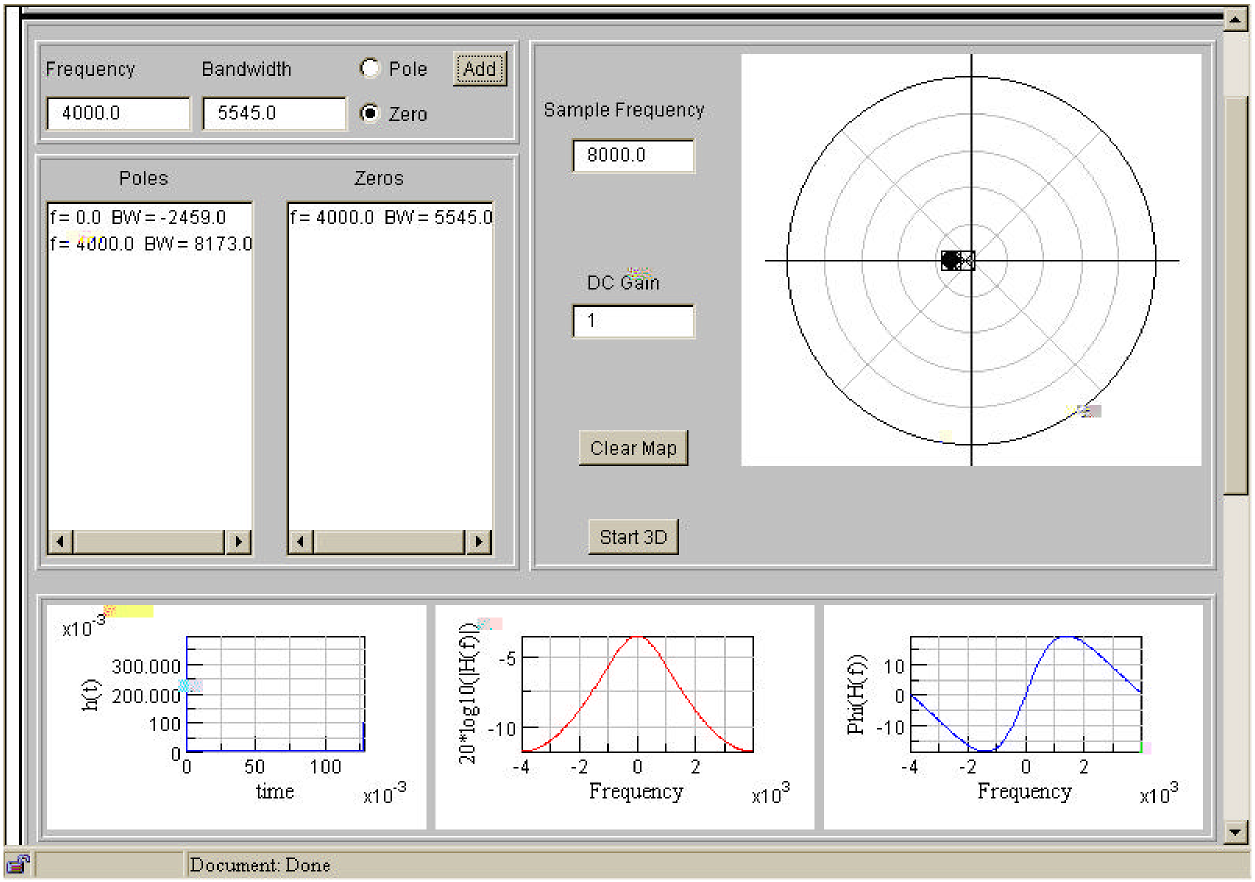
$$H(z) = \frac{0.7887675}{(-1 + \sqrt{3})(1 - \frac{1}{(-1 + \sqrt{3})} z^{-1})} + \frac{0.211325}{(-1 - \sqrt{3})(1 - \frac{1}{(-1 - \sqrt{3})} z^{-1})}$$

$$H(n) = \mathrm{Z}^{-1}\{H(z)\}$$

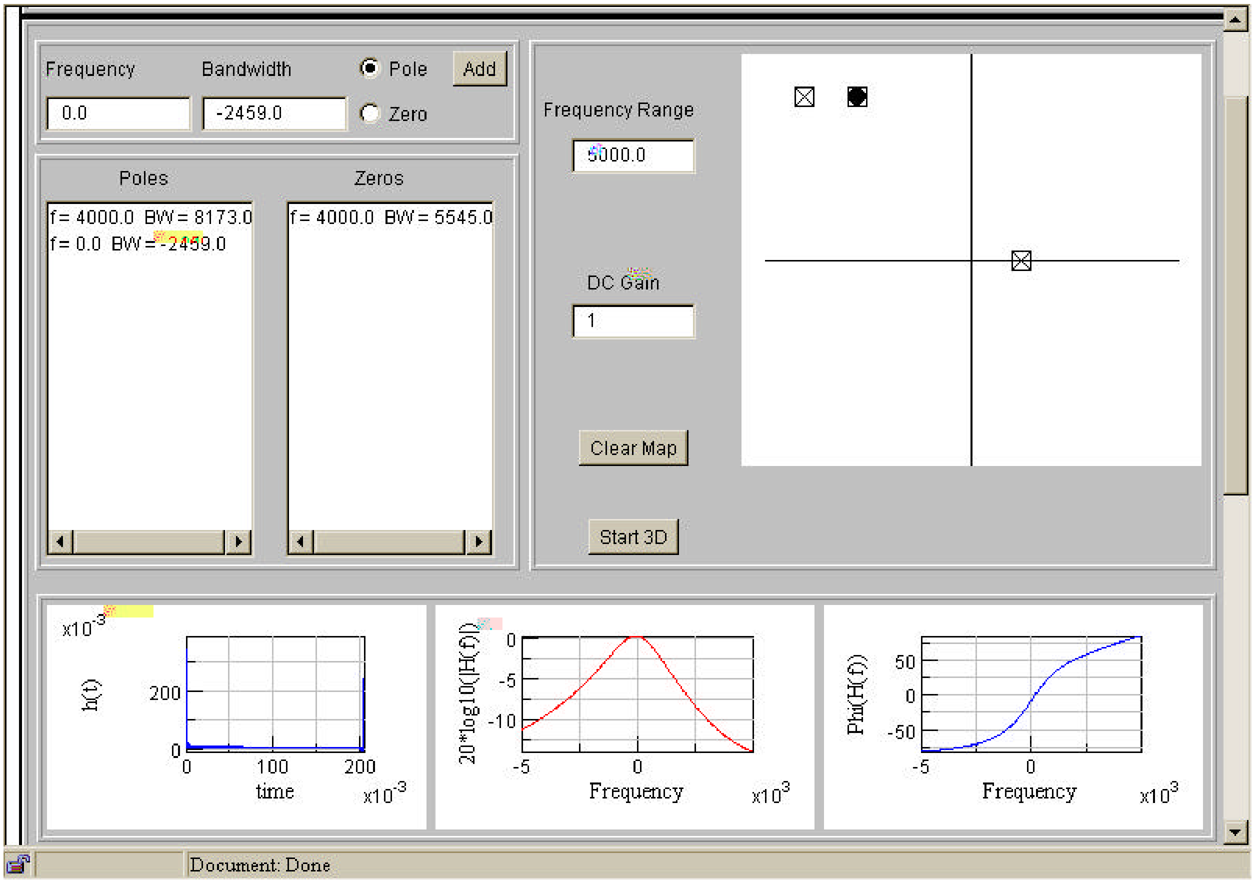
$$H(n) = \frac{0.788765}{-1 + \sqrt{3}} \cdot (\frac{1}{-1 + \sqrt{3}})^N + \frac{0.211325}{-1 - \sqrt{3}} \cdot (\frac{1}{-1 - \sqrt{3}})^N$$

$$H(n) = 1.077473 \cdot (1.366025)^N + -0.073503 \cdot (-0.366025)^N$$

(c) Sketch the magnitude of the frequency response.



(d) Convert to by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.



The pole has more of an effect on the frequency response in the s-plane, causing a sharper drop at $f_S/2$.