Name: Eddie McDaniel

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

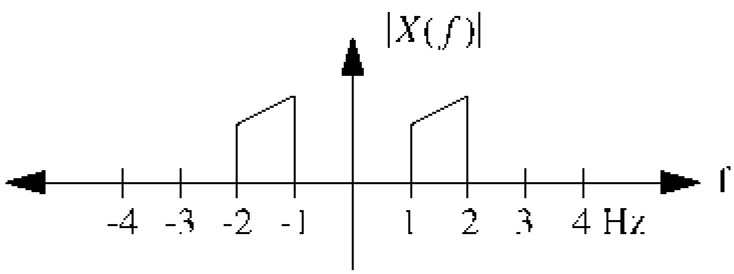
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can’t read it (and I am the judge of legibility), it is wrong. If I can’t follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

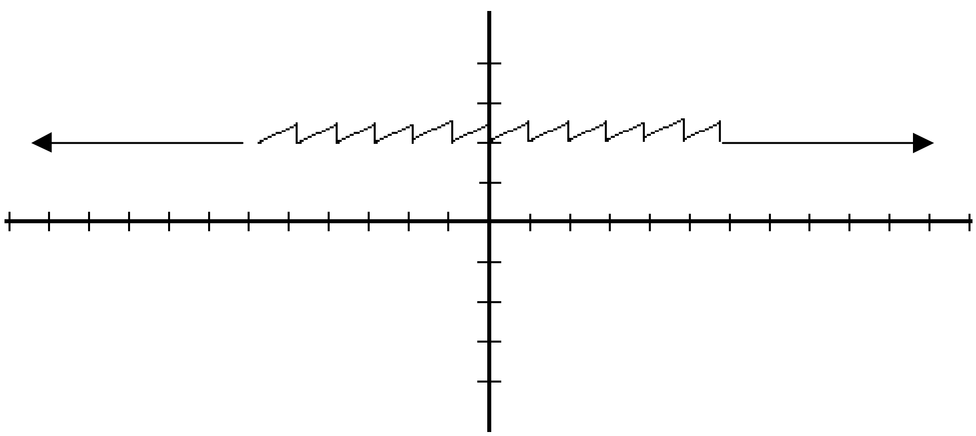
Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.

In order for this signal to be real it must be symmetric about zero frequency. Obvously this is not the case, therefore, the signal is complex.

(b) Draw the spectrum of the sampled signal if $f_s = 1$ Hz



The signal will appear as a sawtooth from over infinite frequencies. The sawtooth will be twice the height of the original signal.

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

The Sampling Theorem says that we should sample at twice the highest frequency which would mean we should sample at 2 Hz. This, however, only applies to real signals. For this signal, since each of the two pieces is the same only with a frequency shift, we are able to use the Band Pass Theorem and use the width of the pulse or 1 Hz.

Problem No. 2: Given the signal and impulse response shown below:

x(n) = δ(n) + δ(n+2) – δ (n+4) – δ (n+6)
h(n) = $3^{-1/2}$(n+1) + $3^{-1/2}$ (n) +$3^{-1/2}$ (n – 1)

(a) Define y(n) as the output of the convolution of these two functions. Is it an energy or power signal? Prove this.

As you can see below, y(n) is not periodic. This makes sense also because neither x(n) or h(n) are periodic. Since this is the case we can say definitely that y(n) is not a power signal and, thus, is an energy signal.

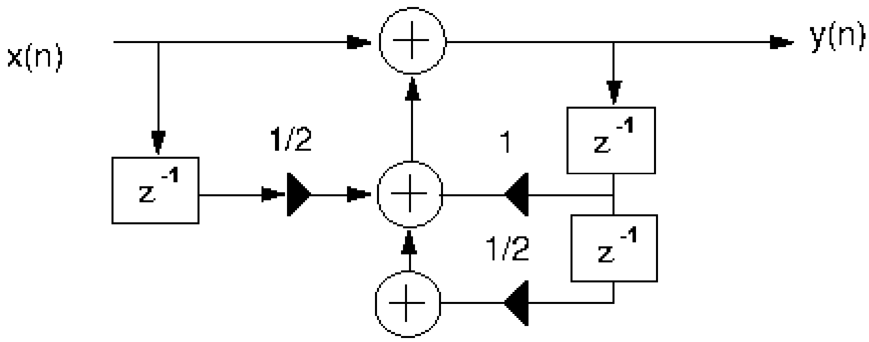
(b) Compute y(n) described in (a) as the convolution of these two functions.

$$y(n) = h(n) * x(n)$$
$$Y(z) = X(z) \times H(z)$$
$$X(z) = 1 + z^2 - z^4 - z^6$$
$$H(z) = 3^{-\left(\frac{1}{2}\right)}[z^1 + 1 + z^{-1}]$$
$$X(z) \times H(z) = 3^{-\left(\frac{1}{2}\right)}[z^1 + 1 + z^{-1}] \times [1 + z^2 - z^4 - z^6]$$
$$Y(z) = 3^{-\left(\frac{1}{2}\right)}[z^{-1} + 1 + 2z^1 + z^2 - z^4 - 2z^5 - z^6 - z^7]$$
$$y(n) = 3^{-\left(\frac{1}{2}\right)}\left[\delta(n-1) + \delta(n) + 2\delta(n+1) + \delta(n+2) - \delta(n+4) - 2\delta(n+5) - \delta(n+6) - \delta(n+7)\right]$$

(c) Assume x(n) in (a) was a periodic signal. Will the power in the output be different than the power in the input? Explain.

h(n) is an averager. Therefore, when we convolve it and x(n) we will get approximately the average of x(n). Obviously, the average of a signal will have a power very close the the power of the original signal.

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

$$y(n) = x(n) + \tfrac{1}{2}x(n\text{-}1) + \tfrac{1}{2}y(n\text{-}2) + \tfrac{1}{2}y(n\text{-}1)$$

$$Y(z) = X(z) + \frac{z^{-1}X(z)}{2} + \frac{z^{-2}Y(Z)}{2} + z^{-1}Y(Z)$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{1 + z^{-1}/2}{1 - z^{-2}/2 - z^{-1}} = \frac{2 + z^{-1}}{2 - 2\cdot z^{-1} - z^{-2}}$$

(b) Find the impulse response.

For the impulse response we need the inverse z-transform of the transfer function.

$$H(z) = \frac{2 + z^{-1}}{2 - 2\cdot z^{-1} - z^{-2}} = \frac{2 + z^{-1}}{-(z^{-2} + 2\cdot z^{-1} - 2)}$$

Using the quadratic equation we find the roots of the denominator to be

$$\frac{-2 \pm \sqrt{2^2 - 4\cdot 1\cdot -2}}{2\cdot 1} = -1 \pm 3^{\frac{1}{2}}$$

Now we use partial fractions to obtain the form of a transform in the table.

$$A(z^{-1} - (-1 - 3^{\frac{1}{2}})) + B(z^{-1} - (-1 + 3^{\frac{1}{2}})) = -2 - z^{-1}$$

$$Az^{-1} - A(-1 - 3^{\frac{1}{2}}) + Bz^{-1} - B(-1 + 3^{\frac{1}{2}}) = -2 - z^{-1}$$

$$A + B = -1$$

$$-A(-1 - 3^{\frac{1}{2}}) - B(-1 + 3^{\frac{1}{2}}) = -2$$

$$A = -0.788675$$

$$B = -0.211325$$

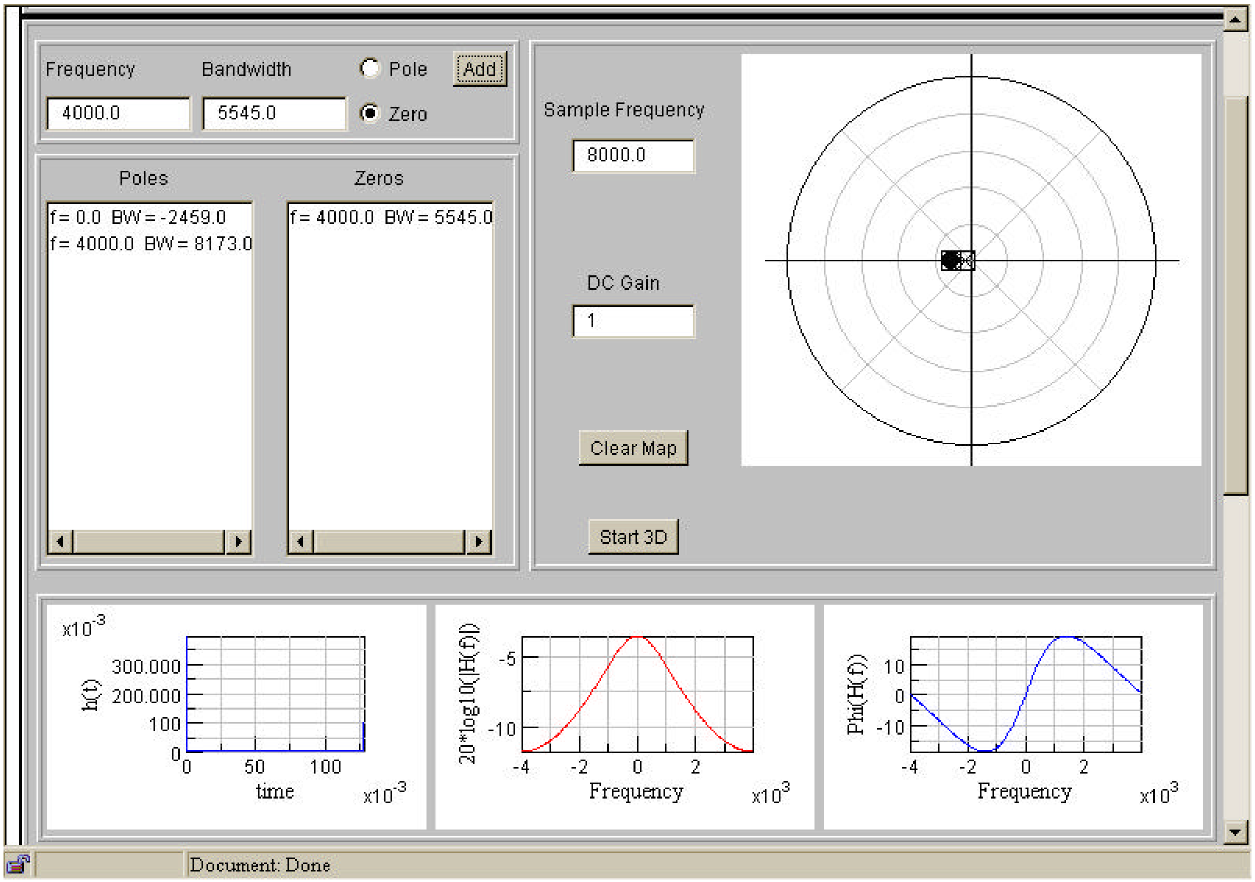
$$H(z) = \frac{-0.7887675}{(z^{-1} - (-1 + 3^{\frac{1}{2}}))} + \frac{-0.211325}{(z^{-1} - (-1 - 3^{\frac{1}{2}}))} = \frac{0.7887675}{(-1 + 3^{\frac{1}{2}} - z^{-1})} + \frac{0.211325}{(-1 - 3^{\frac{1}{2}} - z^{-1})}$$

$$H(z) = \frac{0.7887675}{(-1 + 3^{\frac{1}{2}})(1 - \frac{1}{(-1 + 3^{\frac{1}{2}})} z^{-1})} + \frac{0.211325}{(-1 - 3^{\frac{1}{2}})(1 - \frac{1}{(-1 - 3^{\frac{1}{2}})} z^{-1})}$$

$$h(n) = Z^{-1}\{H(z)\}$$

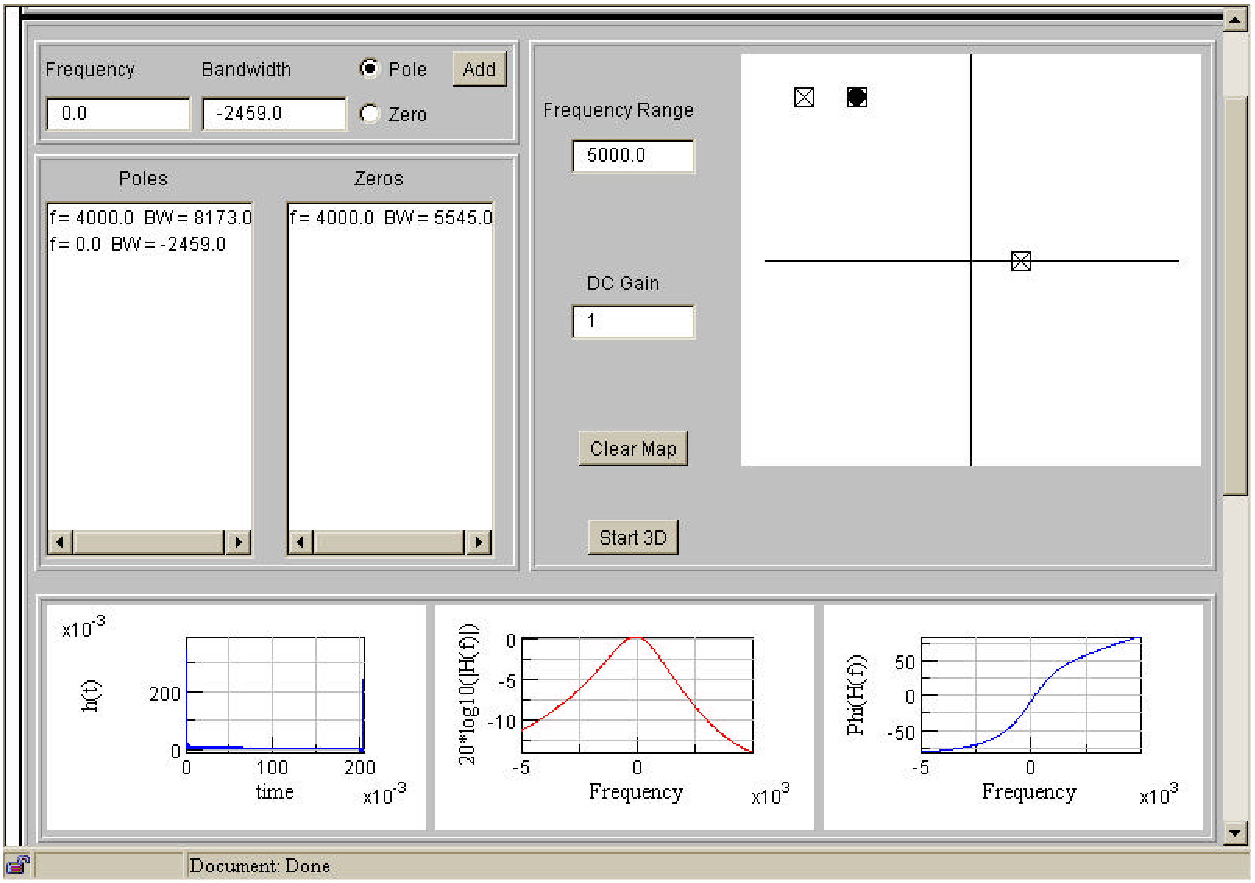
$$h(n) = 1.077473 \cdot (1.366025)^{n} + -0.073503 \cdot (-0.366025)^{n}$$

(c) Sketch the magnitude of the frequency response.



(d) Convert to by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

In order to convert poles and zeros from the z-plane to the s-plane, we use the relationship $z = e^{sT}$ where $T = 1/f_s$. Thus we get $s = 8000 \ln(z)$. We also know by the z-plane that we have a pole at 0 frequency and at $f_s/2$. The hole also exists at $f_s/2$. The plots are shown below using the pole-zero analysis tool.



The response is different between the two planes because of the nature of the way we map the response in each plane. In the z-plane, we walk around the unit circle to obtain the frequency response, but we walk along the imaginary axis in the s-plane for the response. Obviously, this difference will cause distortions between the two responses. Generally the bandwidth of response in the z-plane will be wider. This would seem to be a problem that would have to be addressed for things such as digital to analog converters.