Name: **Mark Mckinney**

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

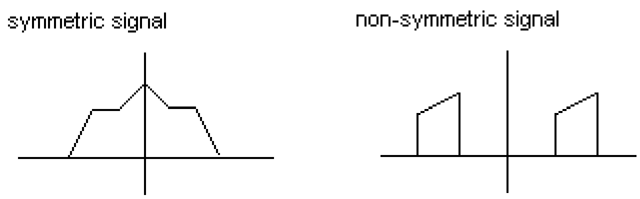
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

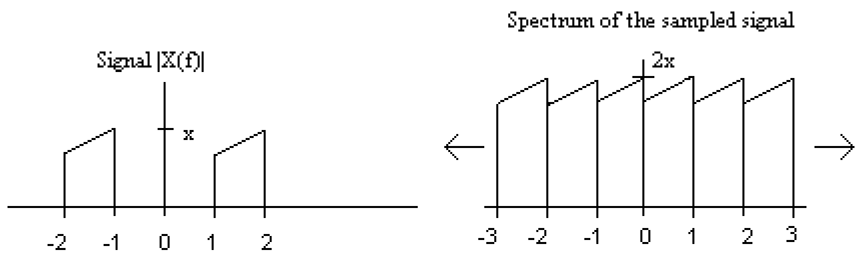
Problem No. 1: Sampling

(a) Is this signal real or complex? Justify your answer.

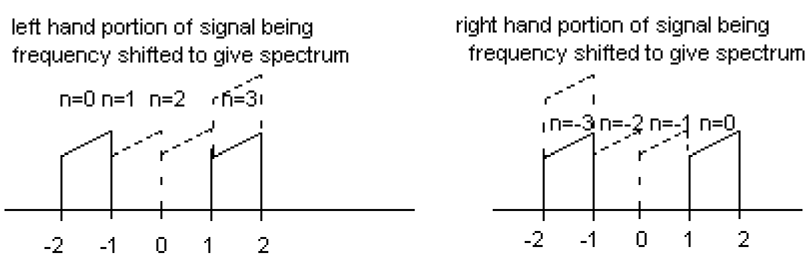
The signal is complex. A signal is considered real if the signal is symmetric. The signal in question is not symmetric so therefore the signal is complex.



(b) Draw the spectrum of the sampled signal if .



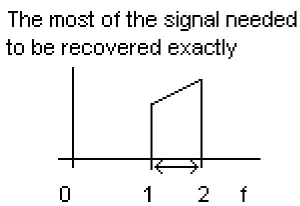
Since the expression to get the spectrum of the sampled signal is $X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$ the signal is shifted in frequency and produces a periodic spectrum. When the portion of the signal on the left hand side is shifted by n=1,2,3 the signal overlaps with the portion of the signal on the right hand side being shifted by n=-1,-2,-3 and causes the magnitude of this portion of the signal to be doubled. This effect can more readily be seen below.



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

To recover the signal you would use an ideal low pass filter whose cutoff frequency is $f_c = \frac{f_s}{2}$

Therefore $f_c = \frac{1}{2}\text{Hz}$. This cutoff is if the signal is center at the orign so the cutoff freq of our low pass filter is at one and two which the bandwidth of the low pass filter is one hertz so the sampling theorm was not violated. It can also be seen by the graph that the signal is modulated about the orgin to recover the signal the most of the signal you have to have is the portion on the right hand side from 1 to 2 hertz this is one hertz wide and thus the sampling frequency of one hertz does not violate the sampling theorem



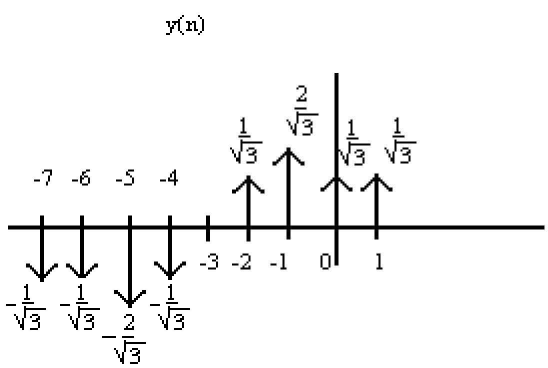
Problem No. 2: Given the signal and impulse response shown below:

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

The output of the convolution of these two functions is an energy signal.
The convolution of these two functions is not periodic therefore the signal has finite energy ,so that the power is zero. This proves the definition of an energy signal which is a signal is an energy signal if and only if 0<E<∞, so that P=0.

(b) Compute described in (a) as the convolution of these two functions.

By graphically convoluting x(n) and x(n) you get y(n) to be



You graphically convolute x(n)and h(n) by fliping h(n) along y axis and sliding the graph from positive infinity to negitive infinity.

You can also get y(n) analytically and it comes out to be the same and this can be seen below:

$$X(z) = 1 + z^2 - z^4 - z^6$$

$$H(z) = 3^{\frac{-1}{2}}\left[z^1 + 1 + z^1\right]$$

$$Y(z) = X(z) \bullet Y(z)$$

$$Y(z) = 3^{\frac{-1}{2}}\left[z^1 + 1 + z^1\right] \bullet \left[1 + z^2 - z^4 - z^6\right]$$

$$Y(z) = 3^{\frac{-1}{2}}\left[z^{-1} + 1 + 2z^1 + z^2 - z^4 - 2z^5 - z^6 - z^7\right]$$

$$Y(n) = 3^{\frac{-1}{2}}\left[\delta(n-1) + \delta(n) + 2\delta(n+1) + \delta(n+2) - \delta(n+4) - 2\delta(n+5) - \delta(n+6) - \delta(n+7)\right]$$

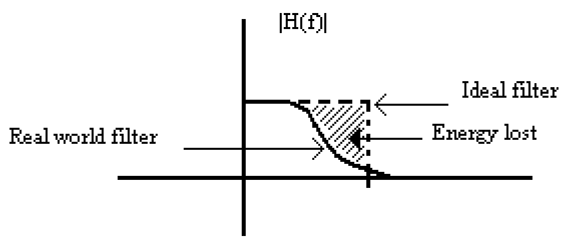
(c) Assume in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

When x(n) is periodic the frequency spectrum of the signal will be a line spectrum. When you multiply X(f) by H(f) you get Y(f). the function h(n) which can be seen below is just a filter. The filter works as an averager.

$$h(n) = \frac{1}{\sqrt{3}}x(n+1) + \frac{1}{\sqrt{3}}x(n) + \frac{1}{\sqrt{3}}x(n-1)$$

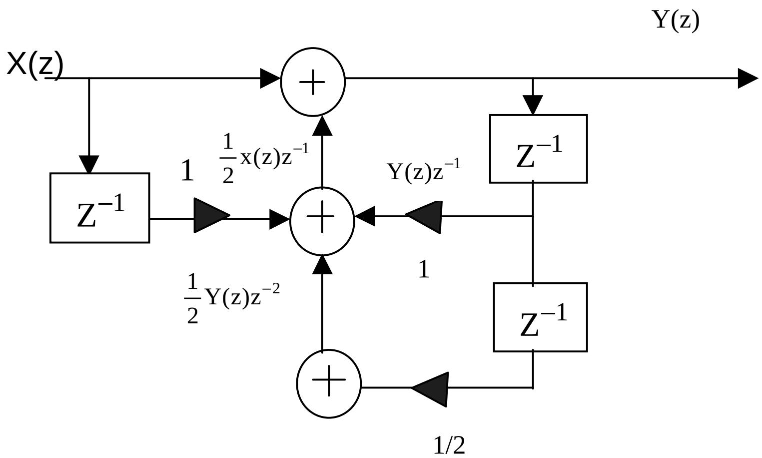
If an ideal filter was used the coeffiecients of $\frac{1}{\sqrt{3}}$ should have been $\frac{1}{3}$ but as seen below by running the signal through the filter you lose the shaded energy so the is replaced by the square root term which is a scale factor to make up for the lost energy. So the power of the output should be very close to the same as the input.

Figure showing the loss of energy with real world filters.



Problem No. 3: Z-Transforms

(a) Find the transfer function of the system shown above.



The transfer function of a discrete system is the z transform of the output over the z transform of the input.

$$Y(z) = X(z) + \frac{1}{2}X(z)z^{-1} + Y(z)z^{-1} + \frac{1}{2}Y(z)z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{X(z) + \frac{1}{2}X(z)z^{-1} + Y(z)z^{-1} + \frac{1}{2}Y(z)z^{-2}}{X(z)}$$

$$H(z) = 1 + \frac{1}{2}z^{-1} + H(z)z^{-1} + \frac{1}{2}H(z)z^{-2}$$

$$H(z) - H(z)z^{-1} - \frac{1}{2}H(z)z^{-2} = 1 + \frac{1}{2}z^{-1}$$

$$H(z)\left[1 - z^{-1} - \frac{1}{2}z^{-2}\right] = 1 + \frac{1}{2}z^{-1}$$

$$H(z) = \frac{1 + \frac{1}{2}z^{-1}}{\left[1 - z^{-1} - \frac{1}{2}z^{-2}\right]}$$

(b) Find the impulse response.

The impulse response is the inverse z transform of the transfer function therefore:

$$h(n) = Z^{-1}\{H(z)\}$$

$$H(z) \bullet \frac{2}{2} = \frac{2 + z^{-1}}{2 - 2z^{-1} - z^{-2}}$$

By using the quadratic equation to the denominator the roots can be obtained to be

$$\text{roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} = -1 \pm \sqrt{3}$$

$$H(z) = \frac{2 + z^{-1}}{(z^{-1} - (-1 + \sqrt{3}))(z^{-1} - (-1 - \sqrt{3}))}; \text{let\_}M = -1 + \sqrt{3}; N = -1 - \sqrt{3}$$

$$H(z) = \frac{2 + z^{-1}}{(z^{-1} - M)(z^{-1} - N)}$$

Now use partial fraction to get the transfer function into a useful condition to get the inverse z transform

$$\frac{-2 - z^{-1}}{(z^{-1} - M)(z^{-1} - N)} = \frac{A}{(z^{-1} - M)} + \frac{B}{(z^{-1} - N)}$$

$$-2 - z^{-1} = +A(z^{-1} - N) + B(z^{-1} - M)$$

$$-2 - z^{-1} = +Az^{-1} - AN + Bz^{-1} - BM$$

$$-2 = -AN - BM$$

$$-z^{-1} = Az^{-1} + Bz^{-1} \Rightarrow -1 = A + B$$

$$\therefore A = -1 - B$$

$$\therefore -2 = -(-B - 1)N - BM$$

$$-2 = N + BN - BM$$

$$\frac{-2 - N}{(N - M)} = B$$

$$B = -.21132486553$$

$$A = -1 - B = -.78867513447$$

Now by using some algebra it is possible to get H(z) into form which can be taken off the inverse z-transform table.

$$K^n \Leftrightarrow \frac{1}{1 - Kz^{-1}}$$

$$H(z) = \frac{-A}{M - z^{-1}} - \frac{B}{N - z^{-1}}$$

$$= \frac{-A}{M} \bullet \frac{1}{1 - \frac{1}{M} z^{-1}} - \frac{B}{N} \bullet \frac{1}{1 - \frac{1}{N} z^{-1}}$$

$$\therefore K_1 = \frac{1}{M}; K_2 = \frac{1}{N}$$

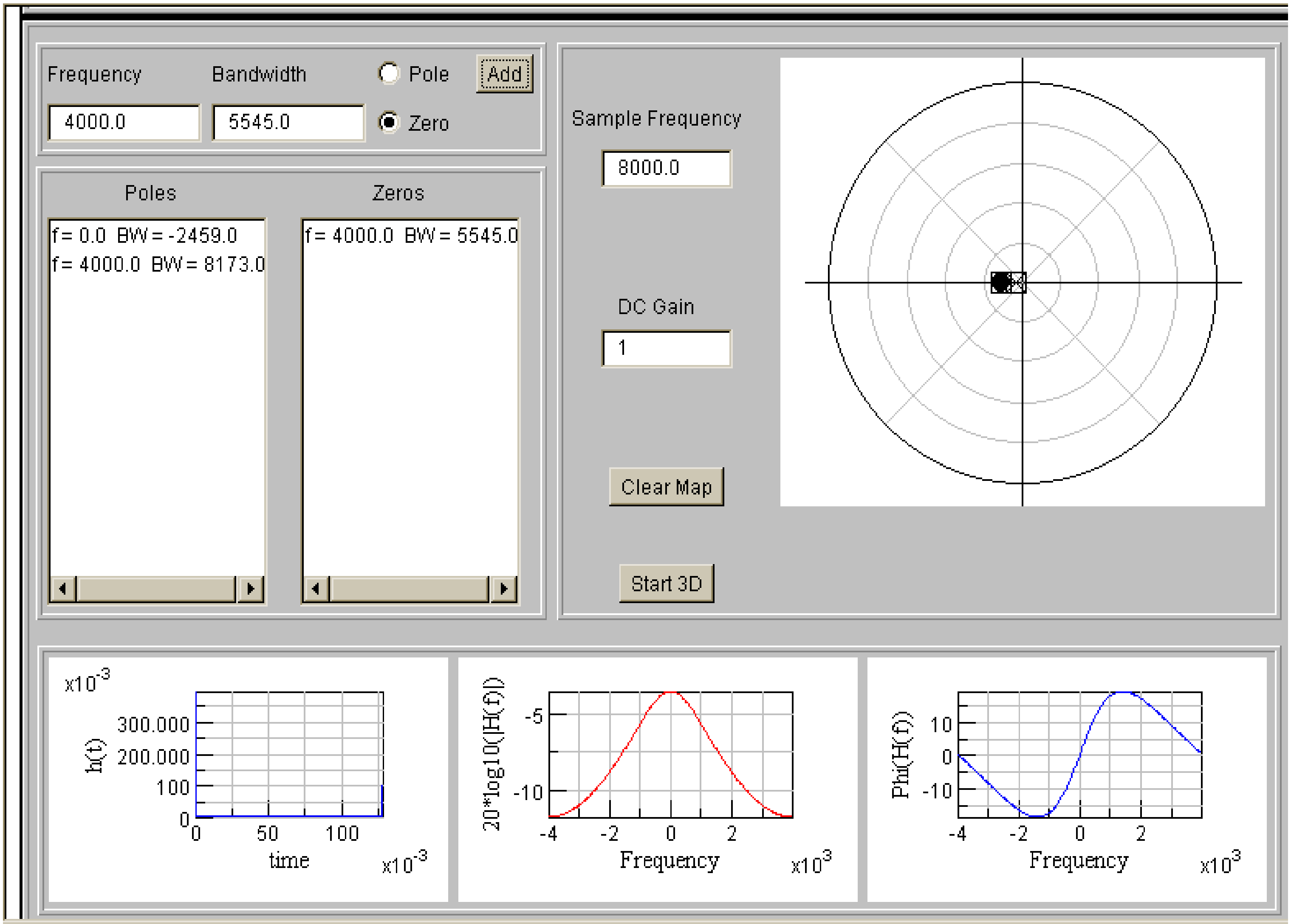
$$h(n) = -\frac{A}{M} \bullet \left(\frac{1}{M}\right)^n - \frac{B}{N} \bullet \left(\frac{1}{N}\right)^n$$

substituting for M, N, A, and B, the answer is:

$$h(n) = 1.07735(1.3660254)^n - 7.73503 \times 10^{-2} (-.366025403727)^n$$

(c) Sketch the magnitude of the frequency response.

The magnitude of the frequency response can be seen below. By implementation of the pole zero tool on the java applet you can position the poles and zero in the right location in the z-plane. This can also be seen below. The reason one of the poles can't be seen is the fact that the pole is in fact outside the unit circle which also means that the system is unstable. The frequency response is what you get by walking along the unit circle.



(d) Convert to by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

The frequency response in the s-plane was accomplished by transfering the poles and zeroes in the z-plane to there equivalents in the s-plane using the same bandwidths which were calculated with help by the pole zero applet. The plots of the s-plane and the frequency response was what was seen on the applet when poles and zeroes were transferred over. The reason for the drop on the right hand side of the frequency response was caused by the transfer from the z to the s plane. The info as the frequency response approached fs was aliased since the frequency response in the z plane goes from 0 to fs. You get the frequency response in the s plane by walking along the imaginary axis.