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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

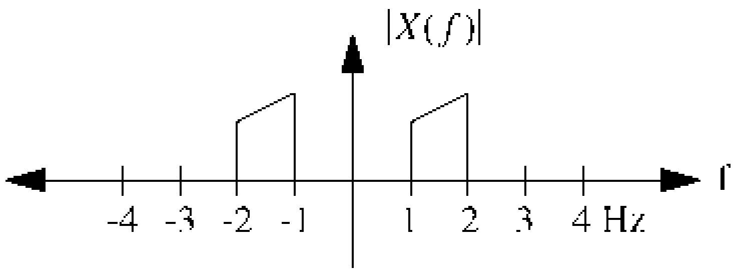
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

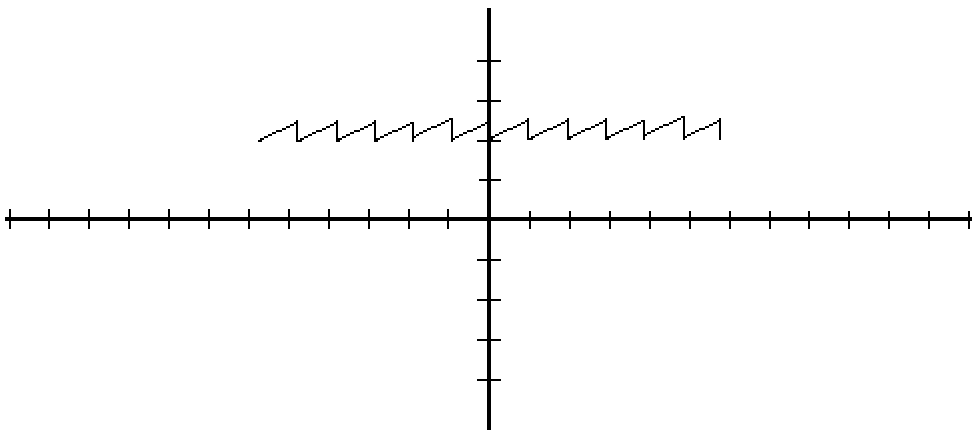
Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.

Complex. If this signal were a real signal, then it would be symmetric.

(b) Draw the spectrum of the sampled signal if $f_s = 1$ Hz



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

According to the Band Pass Theorem, about 1 Hz is all that is needed since there is no frequency response from 0 to 1 Hz in this signal. But 1 Hz provides a slight overlap in the spectrum. My suggestion would be to sample at something slightly larger than 1 Hz.

Problem No. 2: Given the signal and impulse response shown below:

$x(n) = \delta(n) + \delta(n+2) - \delta(n+4) - \delta(n+6)$
$h(n) = 3^{-1/2}(n+1) + 3^{-1/2}(n) + 3^{-1/2}(n-1)$

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

$$y(n) = x(n) * h(n)$$
$$Z\{ x(n) * h(n) \} = X(z) \cdot H(z)$$

It is an energy signal because neither x(n) nor h(n) are periodic. y(n) will only have values at a limited number of N. Look at part (b) to see wheree y(n) will have those values. You can also see in part (b) that y(n) isn't periodic.

(b) Compute described in (a) as the convolution of these two functions.

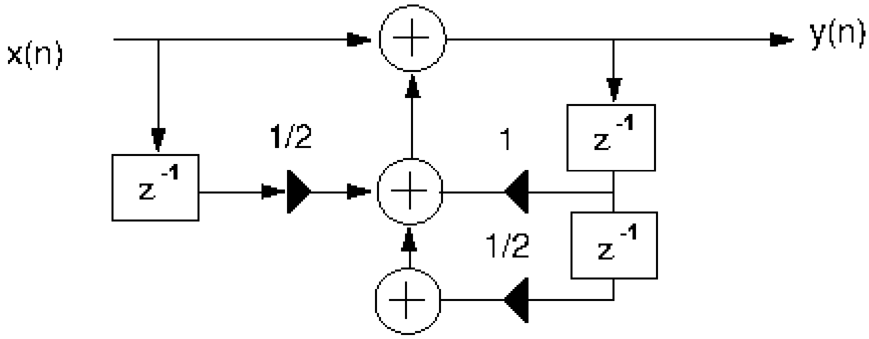
$$X(z) = 1 + z^2 - z^4 - z^6$$
$$H(z) = 3^{-1/2}[z^1 + 1 + z^{-1}]$$
$$X(z) \cdot H(z) = 3^{-1/2}[z^1 + 1 + z^{-1}] \cdot [1 + z^2 - z^4 - z^6]$$
$$= 3^{-1/2}[z^1 + 1 + z^{-1} + z^3 + z^2 + z^1 - z^5 - z^4 - z^3 - z^7 - z^6 - z^5]$$
$$= 3^{-1/2}[z^{-1} + 1 + 2 \cdot z^1 + z^2 - z^4 - 2 \cdot z^5 - z^6 - z^7]$$

$y(n) = 3^{-1/2}[\delta(n-1) + \delta(n) + 2\delta(n+1) + \delta(n+2) - \delta(n+4) - 2\delta(n+5) - \delta(n+6) - \delta(n+7)]$

(c) Assume x(n) in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

If x(n) were periodic, then x(n) and y(n) would have roughly roughly the same amount of power since y(n) is just a rough average of x(n).

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

$$y(n) = x(n) + \frac{1}{2}x(n\text{-}1) + \frac{1}{2}y(n\text{-}2) + \frac{1}{2}y(n\text{-}1)$$

$$Y(z) = X(z) + \frac{1}{2}\cdot z^{-1}\cdot X(z) + \frac{1}{2}\cdot z^{-2}\cdot Y(Z) + z^{-1}\cdot Y(Z)$$

$$\frac{Y(z)}{X(z)} = \frac{X(z)}{X(z)} + \frac{1}{2}\cdot z^{-1}\cdot\frac{X(z)}{X(z)} + \frac{1}{2}\cdot z^{-2}\cdot\frac{Y(z)}{X(z)} + z^{-1}\cdot\frac{Y(Z)}{X(z)}$$

$$H(z) = 1 + \frac{1}{2}\cdot z^{-1} + \frac{1}{2}\cdot z^{-2}\cdot H(z) + z^{-1}\cdot H(z)$$

$$H(z) - \frac{1}{2}\cdot z^{-2}\cdot H(z) - z^{-1}\cdot H(z) = 1 + \frac{1}{2}\cdot z^{-1}$$

$$H(z)[1 - \frac{1}{2}\cdot z^{-2} - z^{-1}] = 1 + \frac{1}{2}\cdot z^{-1}$$

$$H(z) = \frac{1 + \frac{1}{2}\cdot z^{-1}}{1 - \frac{1}{2}\cdot z^{-2} - z^{-1}} = \frac{2 + z^{-1}}{2 - z^{-2} - 2\cdot z^{-1}}$$

(b) Find the impulse response.

The impulse response of a system outputs the transfer function. So using part (a):

$$H(z) = \frac{2 + z^{-1}}{2 - z^{-2} - 2 \cdot z^{-1}} = \frac{2 + z^{-1}}{-(z^{-2} + 2 \cdot z^{-1} - 2)}$$

Here you must use the quadratic formula to determine the roots of the polynomial in the denominator.

$$z^{-2} + 2 \cdot z^{-1} - 2 \qquad \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$$a = 1$$

$$b = 2 \qquad \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot -1}}{2 \cdot 1} = -1 \pm \sqrt{3}$$

$$c = -1$$

$$\frac{-2 - z^{-1}}{(z^{-1} - (-1 + \sqrt{3}))(z^{-1} - (-1 - \sqrt{3}))} = \frac{A}{(z^{-1} - (-1 + \sqrt{3}))} + \frac{B}{(z^{-1} - (-1 - \sqrt{3}))}$$

$$-2 - z^{-1} = A \cdot (z^{-1} - (-1 - \sqrt{3})) + B \cdot (z^{-1} - (-1 + \sqrt{3}))$$

$$-2 - z^{-1} = A \cdot z^{-1} - A \cdot (-1 - \sqrt{3}) + B \cdot z^{-1} - B \cdot (-1 + \sqrt{3})$$

$$-1 = A + B$$

$$-2 = -A \cdot (-1 - \sqrt{3}) - B \cdot (-1 + \sqrt{3})$$

$$A = -0.788675$$

$$B = -0.211325$$

$$H(z) = \frac{-0.7887675}{(z^{-1} - (-1 + \sqrt{3}))} + \frac{-0.211325}{(z^{-1} - (-1 - \sqrt{3}))} = \frac{0.7887675}{((-1 + \sqrt{3}) - z^{-1})} + \frac{0.211325}{((-1 - \sqrt{3}) - z^{-1})}$$

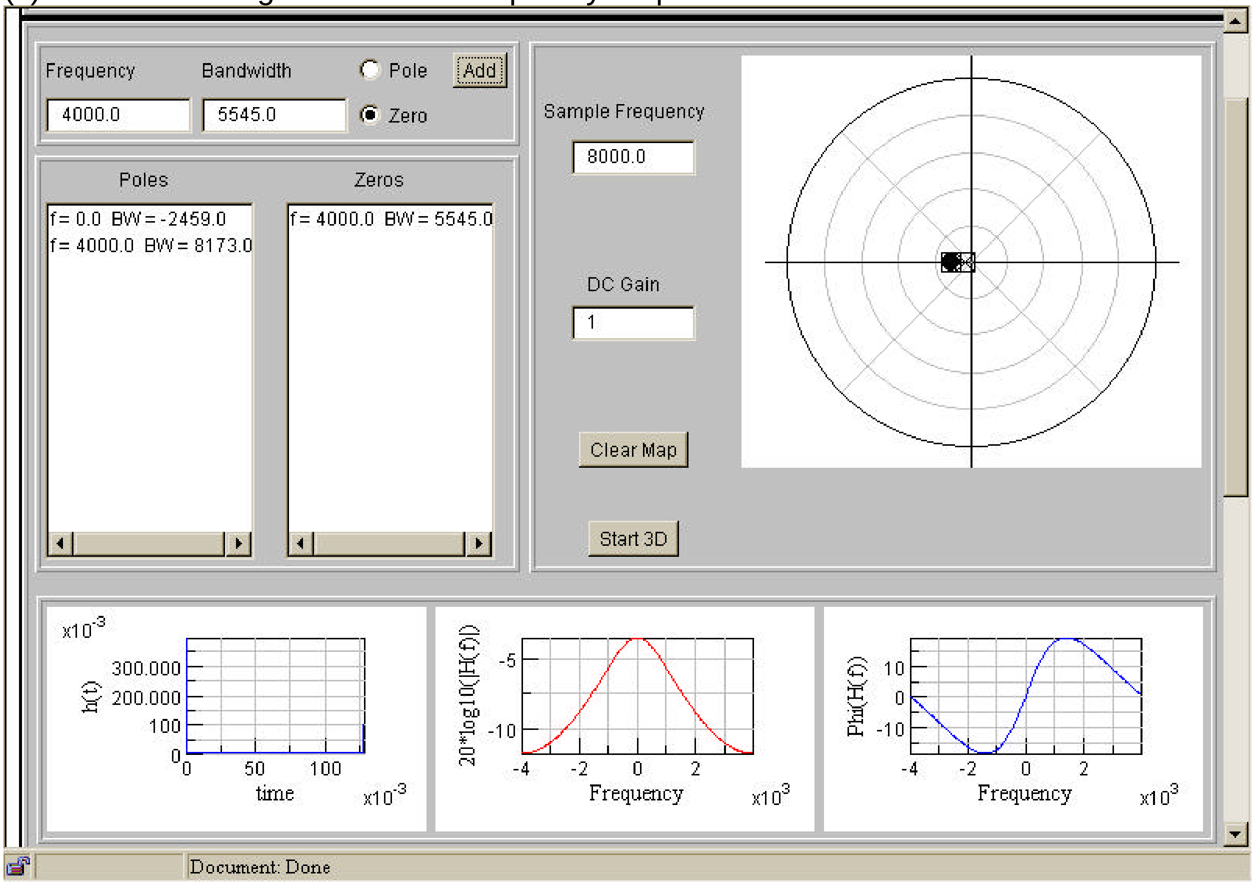
$$H(z) = \frac{0.7887675}{(-1 + \sqrt{3})(1 - \frac{1}{(-1 + \sqrt{3})} z^{-1})} + \frac{0.211325}{(-1 - \sqrt{3})(1 - \frac{1}{(-1 - \sqrt{3})} z^{-1})}$$

$$H(n) = \mathrm{Z}^{-1}\{H(z)\}$$

$$H(n) = \frac{0.788765}{-1 + \sqrt{3}} \cdot (\frac{1}{-1 + \sqrt{3}})^N + \frac{0.211325}{-1 - \sqrt{3}} \cdot (\frac{1}{-1 - \sqrt{3}})^N$$

$$H(n) = 1.077473 \cdot (1.366025)^N + \ - 0.073503 \cdot (-0.366025)^N$$

(c) Sketch the magnitude of the frequency response.



$$H(z) = \frac{-2 - z^{-1}}{(z^{-1} - (-1+\sqrt{3}))(z^{-1} - (-1-\sqrt{3}))}$$

This means that there is a zero at z=-2, a pole at

$z = (-1+\sqrt{3}) = 0.73$ and at $z = (-1-\sqrt{3}) = -2.73$

To find out the bandwidth(which is needed for the pole zero-tool). Use:

$$z = e^{sT}$$

$$\ln\ (-1+\sqrt{3}) = s \cdot T\ (T = \frac{1}{fs} = \frac{1}{8000})$$

$$s = 8000 \cdot \ln(-2.73) = 8040 + j25132$$

The real part of s is what determines bandwidth, so the imaginary part is ignored.

Poles: Bandwidth = -2459 and 8173
Zeros: Bandwidth = 5545

(d) Convert to by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

As you can see, the frequency response of this signal has a sharper cutoff close to $f_s/2$. This is because in the z-plane, as you travel around the unit circle, the frequency response gradually moves from "seeing" the effects of the pole to "seeing" the effects of the hole. In the s-plane, the pole is always "hidden" behind the pole. This means that the hole has more of an effect on the frequency response.