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Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

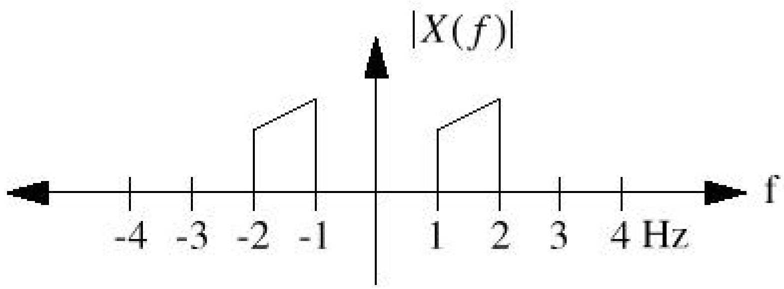
Notes:

1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

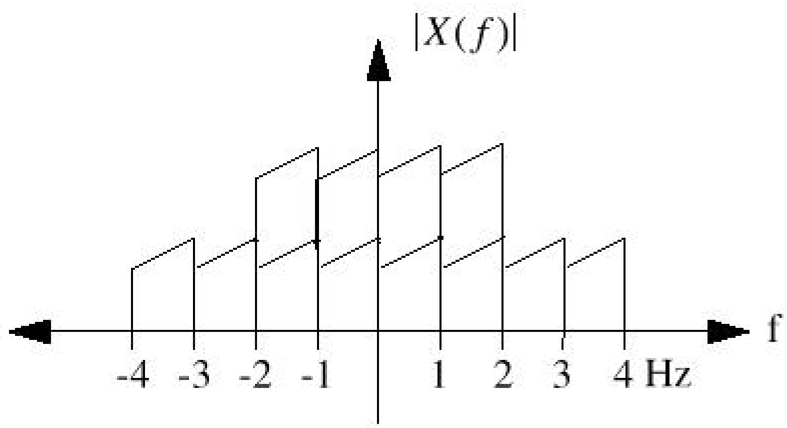
Problem No. 1: Sampling



(a) Is this signal real or complex? Justify your answer.

This is a complex signal, because it's not an even function.

(b) Draw the spectrum of the sampled signal if .



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

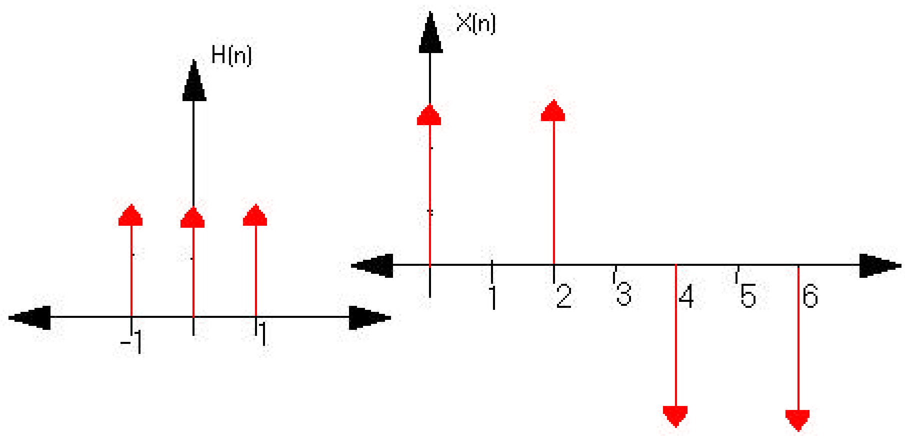
x(t) must be sampled at a frequency exceeding the Nyquist rate, then sampled signal $x_x(t)$ is passed through an ideal low-pass filter, with bandwidth W, where W is equal to on-half the sampling frequency. This method is equivaltent to weighting each sample by a sinc function and summing the contributions of the individual sinc functions. No the sampling theorem is not violated because the time between samples is no greater than ½f seconds.

Problem No. 2: Given the signal and impulse response shown below:

$$x(n) = d(n) + d(n+2) - d(n+4) - d(n+6$$

$$h(n) = 3^{\frac{-1}{2}} d(n+1) + 3^{\frac{-1}{2}} d(n) + 3^{\frac{-1}{2}} d(n-1)$$

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.



Y(n) = x(n) * h(n) is an energy signal because x(n) and h(n) are energy signals (they are non-periodic), and the convolution of two energy signal give energy signal.

(b) Compute described in (a) as the convolution of these two functions.

$$x(z) = 1 + z^2 - z^4 - z^6 \qquad h(z) = \frac{1}{\sqrt{3}} z^1 + \frac{1}{\sqrt{3}} z^0 + \frac{1}{\sqrt{3}} z^{-1} = \frac{1}{\sqrt{3}}(z^1 + 1 + z^{-1})$$

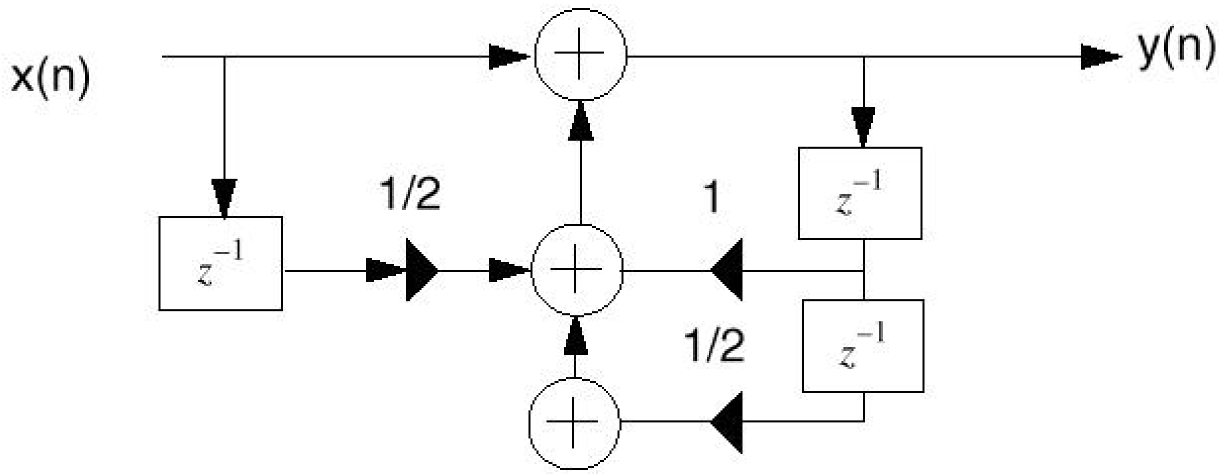
$$y(z) = x(z) \cdot h(z) = \frac{1}{\sqrt{3}}(z^{-1} + 1 + 2z + z^2 - z^4 - 2z^5 - z^6 - z^7)$$

$$y(n) = \frac{1}{\sqrt{3}}\left(d(n-1) + d(n) + 2d(n+1) + d(n+2) - d(n+4) - 2d(n+5) - d(n+6) - d(n+7)\right)$$

(c) Assume in (a) was a periodic signal. Will the power in the output, be different than the power in the input? Explain.

Yes, the power in the output will be different than the power in the input because the output signal is filtered by the h(n), therefore y(n) periodic signal is different than x(n) likewise the power.

Problem No. 3: Z-Transforms



(a) Find the transfer function of the system shown above.

$$y(n) = \frac{1}{2}x(n)z^{-1} + y(n)z^{-1} + \frac{1}{2}y(n)z^{-2} + x(n)$$

$$y(n) - y(n)z^{-1} - \frac{1}{2}y(n)z^{-2} = x(n) + \frac{1}{2}x(n)z^{-1}$$

$$Y(z)\left(1 - z^{-1} - \frac{1}{2}z^{-2}\right) = X(z)\left(1 + \frac{1}{2}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{\left(1 - z^{-1} - \frac{1}{2}z^{-2}\right)}$$

(b) Find the impulse response.

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+\frac{1}{2}z^{-1}}{\left(1-z^{-1}-\frac{1}{2}z^{-2}\right)}==\frac{2+z^{-1}}{2-2.z^{-1}-z^{-2}}\quad=\frac{-2-z^{-1}}{\left(z^{-2}+2.z^{-1}-2\right)}$$

$$roots=\frac{-b\pm\sqrt{b^2-4ac}}{2.a}\qquad=\frac{-2\pm\sqrt{4-4(-2)}}{2}\qquad=-1\pm\sqrt{3}$$

$$H(z)=\frac{-2-z^{-1}}{\left(z^{-1}-(-1+\sqrt{3})\right).\left(z^{-1}-(-1-\sqrt{3})\right)}$$

Let M $=\left(-1+\sqrt{3}\right)$ N $=\left(-1-\sqrt{3}\right)$

Partial fraction solve for A & B

$$H(z)=\frac{-2-z^{-1}}{\left(z^{-1}-M\right).\left(z^{-1}-N\right)}=\frac{A}{\left(z^{-1}-M\right)}+\frac{B}{\left(z^{-1}-N\right)}$$

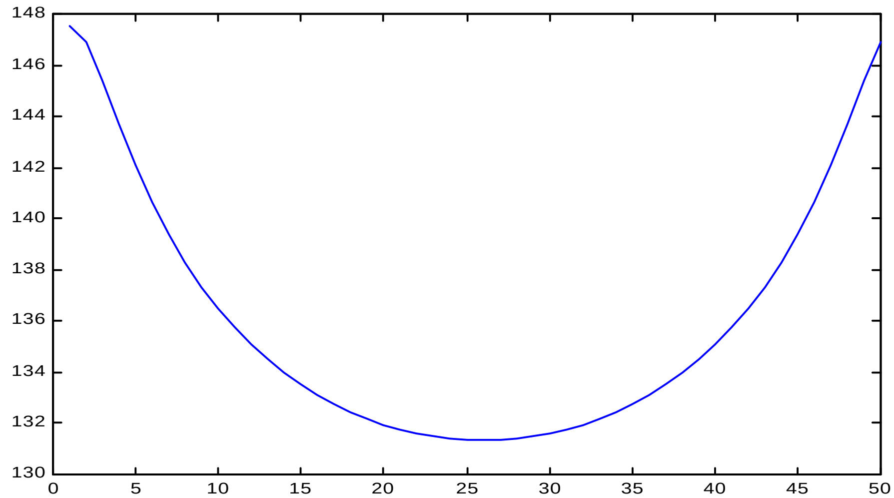
A $=\frac{-1-\sqrt{3}}{2.\sqrt{3}}$ = -0.7887 B $=\frac{-1+\sqrt{3}}{-2.\sqrt{3}}=-0.2113$

$$H(z)=\frac{-A}{\left(M-z^{-1}\right)}-\frac{B}{\left(N-z^{-1}\right)}\quad=\frac{\frac{-A}{M}}{1-\frac{1}{M}.z^{-1}}-\frac{\frac{B}{N}}{1-\frac{1}{N}.z^{-1}}$$

$$h(n)=\frac{-A}{M}.\left(\frac{1}{M}\right)^n-\frac{B}{N}.\left(\frac{1}{N}\right)^n=\frac{0.7887}{\left(-1+\sqrt{3}\right)}\left(\frac{1}{\left(-1+\sqrt{3}\right)}\right)^n+\frac{0.2113}{\left(-1-\sqrt{3}\right)}\left(\frac{1}{-1-\sqrt{3}}\right)$$

$$h(n)=\left(1.0774.(1.3660)^n\right)-\left(0.0773.(-0.3660)^n\right)$$

(c) Sketch the magnitude of the frequency response.



(d) Convert H(z) to H(s) by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

$$\mathrm{H}(z = e^{-sT}) = \frac{-2 - e^{-sT}}{(e^{-sT} - (-1+\sqrt{3}))(e^{-sT} - (-1-\sqrt{3}))}$$

zeros :

$$-2 - e^{-sT} = 0$$
$$e^{-sT} = -2$$
$$-sT = \ln(-2)$$
$$s = \frac{-\ln(-2)}{T}$$

poles :

$$e^{-sT} = -1+\sqrt{3}$$
$$-sT = \ln(-1+\sqrt{3})$$
$$s = \frac{-\ln(-1\pm\sqrt{3})}{T}$$

Magnitude and Phase Plot in S plane.