Name: Tan Ngiap Teen

| Problem | Points | Score |
|---|---|---|
| 1a | 10 | |
| 1b | 10 | |
| 1c | 10 | |
| 2a | 10 | |
| 2b | 10 | |
| 2c | 10 | |
| 3a | 10 | |
| 3b | 10 | |
| 3c | 10 | |
| 3d | 10 | |
| Total | 100 | |

Notes:

1. The exam is closed books/closed notes - except for one page of notes.

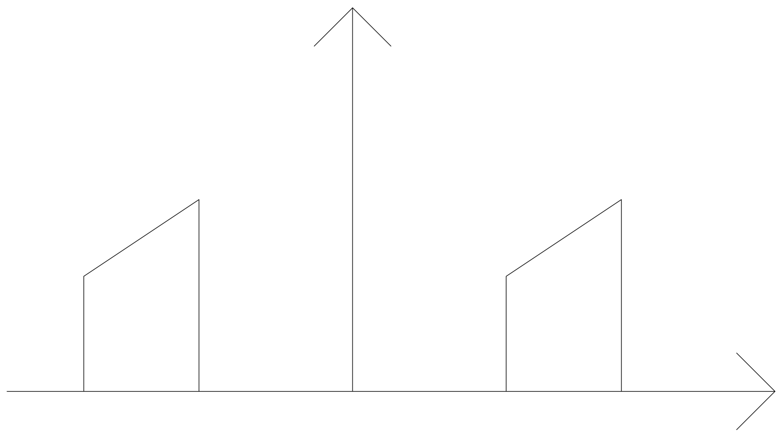
2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

**Problem No. 1**: Sampling

(a) Is this signal real or complex? Justify your answer.

**Answer:**

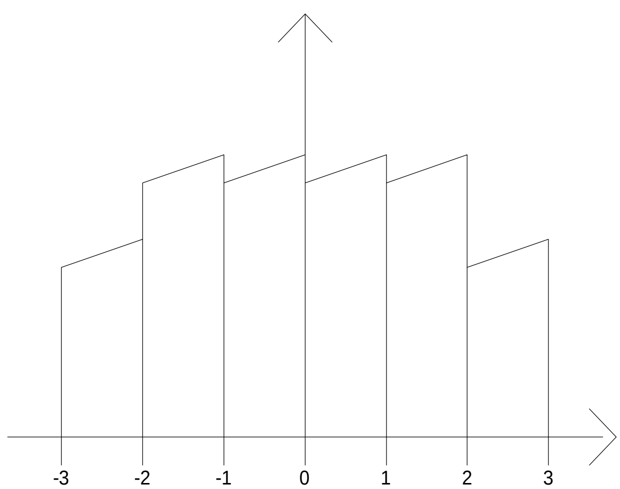


Graph: Non-Symmetric Signal

This is a complex signal since it is not a symmetric signal. A signal is considered real only if the signal is symmetric.

(b) Draw the spectrum of the sampled signal if .

**Answer:**

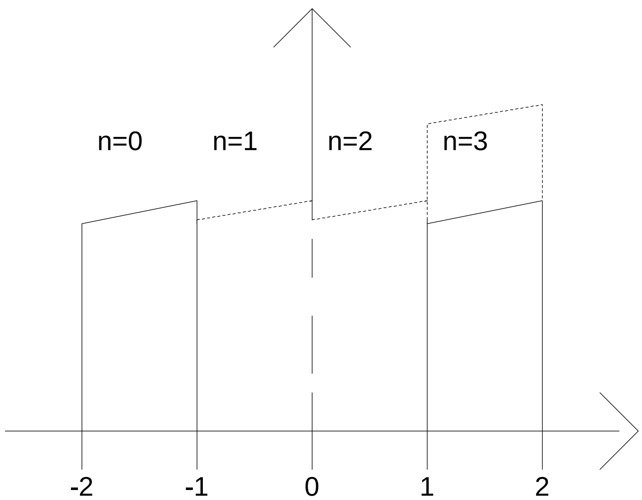



Graph: Spectrum of The Sampled Signal

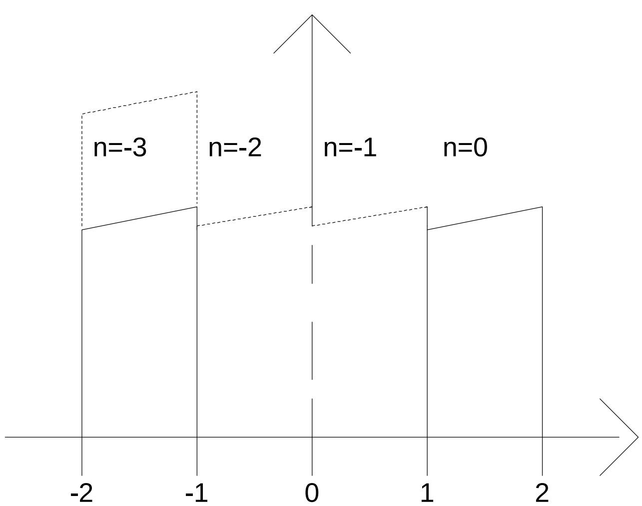
The expression for the spectrum of the sampled signal is:

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$$

When the portion of the signal on the right is shifted by n= -1, -2, and –3, it overlaps with the portion of the signal on the left which is shifted by n=1,2 and 3. The magnitude of the sampled signal is doubled. The spectra extends from $f = -\infty \text{ to} + \infty$ .




Graph: Shifting the left hand portion of the signal



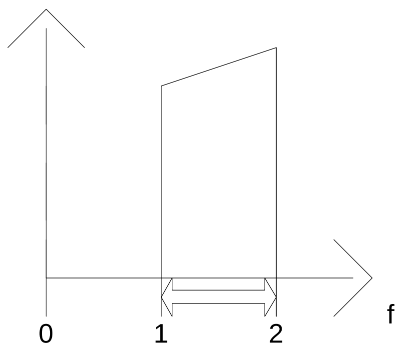

Graph: Shifting the right hand portion of the signal

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

**Answer:**

For a signal to be reconstructed from its sampled values, the sampled signal is passed through an ideal low pass filter. The bandwidth of the reconstruction filter is $f_c = \frac{f_s}{2}$.

In this case, $f_c = \frac{1}{2}$ Hz . From the graph it can be seen that the sampled signal is modulated about the origin. Looking at the right hand side portion of the graph, the signal from 1to 2 Hz is 1 Hz wide. The original signal is recovered. The sampling frequency of 1 Hz does not violate the sampling theorem.




Graph: The signal needed to be recovered exactly.

**Problem No. 2**: Given the signal and impulse response shown below:

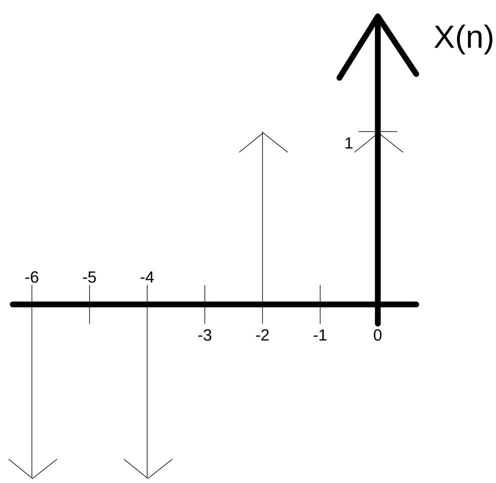
(a) Define  as the output of the convolution of these two functions. Is  an energy or power signal? Prove this.

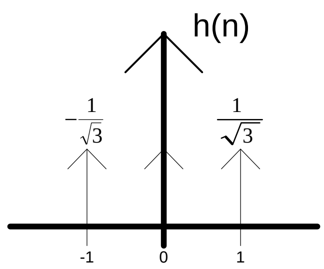
**Answer:**

The output of the convolution of these two functions is an energy signal. The convolution of these two functions does not result in a periodic signal. Therefore the signal has finite energy and the power is zero. This proves the definition of an energy signal: A signal is an energy signal if and only if $0 < E < \infty$, so that P=0.

(b) Compute  described in (a) as the convolution of these two functions.

**Answer:**



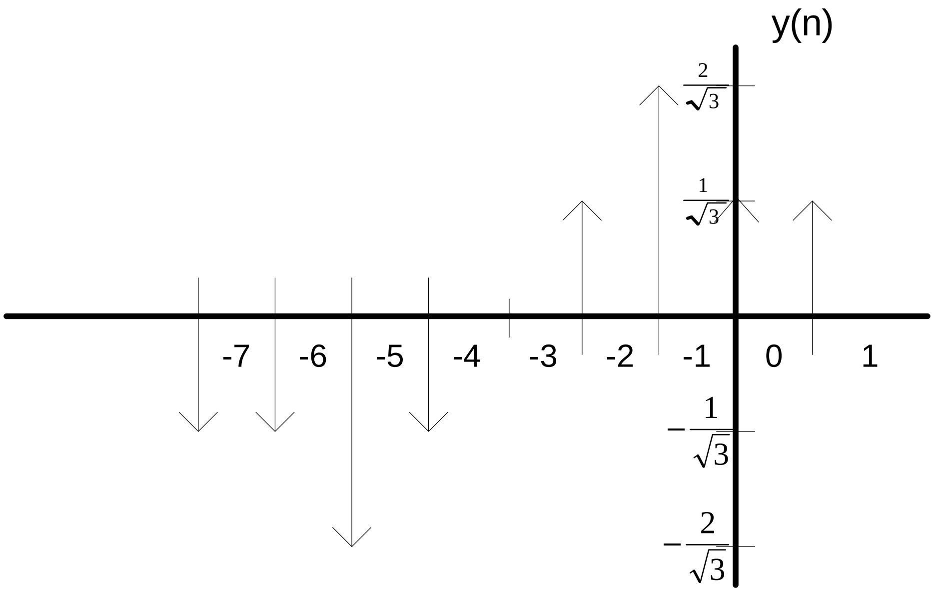



The value of the convolution sum is determined by multiplying and summing the two sets of sample values for each value of n.

| n | | | Samples x(mT) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | -1 | 0 | -1 | 0 | 1 | 0 | 1 |
| n= -7 | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | | | | | | |
| -6 | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | | | | | |
| -5 | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | | | | |
| -4 | | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | | | |
| -3 | | | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | | |
| -2 | | | | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | |
| -1 | | | | | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| 0 | | | | | | | | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ |
| 1 | | | | | | | | | $\frac{1}{\sqrt{3}}$ |

Table for evaluating summation.

Output:




(c) Assume  in (a) was a periodic signal. Will the power in the output, , be different than the power in the input? Explain.

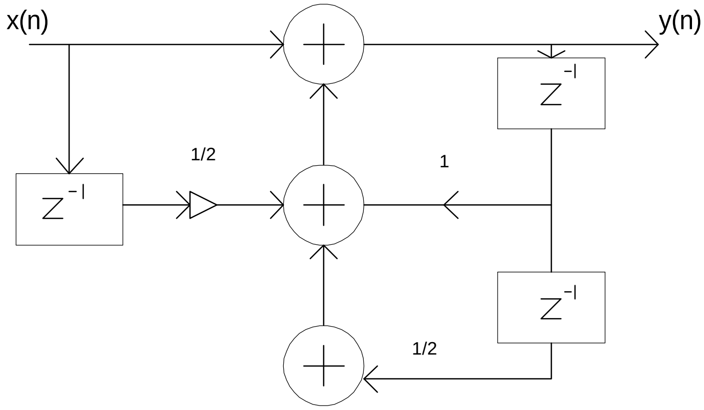
**Answer:**

Yes. The power of the output signal will be greater than the power in the input. Now both the signals are periodic and periodic signals are power signals.

**Problem No. 3**: Z-Transforms

(a) Find the transfer function of the system shown above.

**Answer:**




$$y(n) = x(n) + \frac{1}{2}\left(x(n).z^{-1}\right) + \left(y(n).z^{-1}\right) + \frac{1}{2}\left(y(n).z^{-2}\right)$$

$$y(n).\left[1 - z^{-1} - (\frac{1}{2}.z^{-2})\right] = x(n).\left[1 + (\frac{1}{2}.z^{-1})\right]$$

$$\frac{y(n)}{x(n)} = \frac{\left(1 + (\frac{1}{2}.z^{-1})\right)}{\left(1 - z^{-1} - (\frac{1}{2}.z^{-2})\right)}$$

$$H(z) = \frac{\left(1 + (\frac{1}{2}.z^{-1})\right)}{\left(1 - z^{-1} - (\frac{1}{2}.z^{-2})\right)}$$

(b) Find the impulse response.

**Answer:**

$$H(z)=\frac{\left(1+\left(\frac{1}{2}.z^{-1}\right)\right)}{\left(1-z^{-1}-\left(\frac{1}{2}z^{-2}\right)\right)}$$

$$=\frac{2+z^{-1}}{2-2.z^{-1}-z^{-2}}$$

$$=\frac{-2-z^{-1}}{\left(z^{-2}+2.z^{-1}-2\right)}$$

The roots of the denominator can be found by using the following equation:

$$roots=\frac{-b\pm\sqrt{b^2-4ac}}{2.a}$$

$$=\frac{-2\pm\sqrt{4-4(-2)}}{2}$$

$$=-1\pm\sqrt{3}$$

$$H(z)=\frac{-2-z^{-1}}{\left(z^{-1}-(-1+\sqrt{3})\right)\left(z^{-1}-(-1-\sqrt{3})\right)}$$

$$\text{Let P}=\left(-1+\sqrt{3}\right)$$
$$\text{Q}=\left(-1-\sqrt{3}\right)$$

$$H(z)=\frac{-2-z^{-1}}{\left(z^{-1}-P\right)\left(z^{-1}-Q\right)}$$

Partial fraction expansion is used for performing inverse z-transform operations.

$$\frac{-2-z^{-1}}{\left(z^{-1}-P\right).\left(z^{-1}-Q\right)}=\frac{A}{\left(z^{-1}-P\right)}+\frac{B}{\left(z^{-1}-Q\right)}$$

$$-2-z^{-1}=A(z^{-1}-Q)+B(z^{-1}-P)$$

Let $\mathrm{z}^{-1}=P$

$$\left(-2-\left(-1+\sqrt{3}\right)\right)=A.\left(-1+\sqrt{3}+1+\sqrt{3}\right)$$

$$\mathrm{A}=\frac{-1-\sqrt{3}}{2.\sqrt{3}}$$

$$\mathrm{A}=-0.7887$$

Let $\mathrm{z}^{-1}=Q$

$$\left(-2-\left(-1-\sqrt{3}\right)\right)=B\left(-1-\sqrt{3}+1-\sqrt{3}\right)$$

$$\mathrm{B}=\frac{-1+\sqrt{3}}{-2.\sqrt{3}}$$

$$\mathrm{B}=-0.2113$$

$$H(z)=\frac{-A}{\left(P-z^{-1}\right)}-\frac{B}{\left(Q-z^{-1}\right)}$$

$$=\frac{\frac{-A}{P}}{1-\frac{1}{P}.z^{-1}}-\frac{\frac{B}{Q}}{1-\frac{1}{Q}.z^{-1}}$$

The inverse transform of H(z) is found by using the transform pair 3 in Table 8-1 in the text book.

$$h(n)=\frac{-A}{P}.\left(\frac{1}{P}\right)^{n}-\frac{B}{Q}.\left(\frac{1}{Q}\right)^{n}$$

Thus, substituting the value for P,Q,A, and B

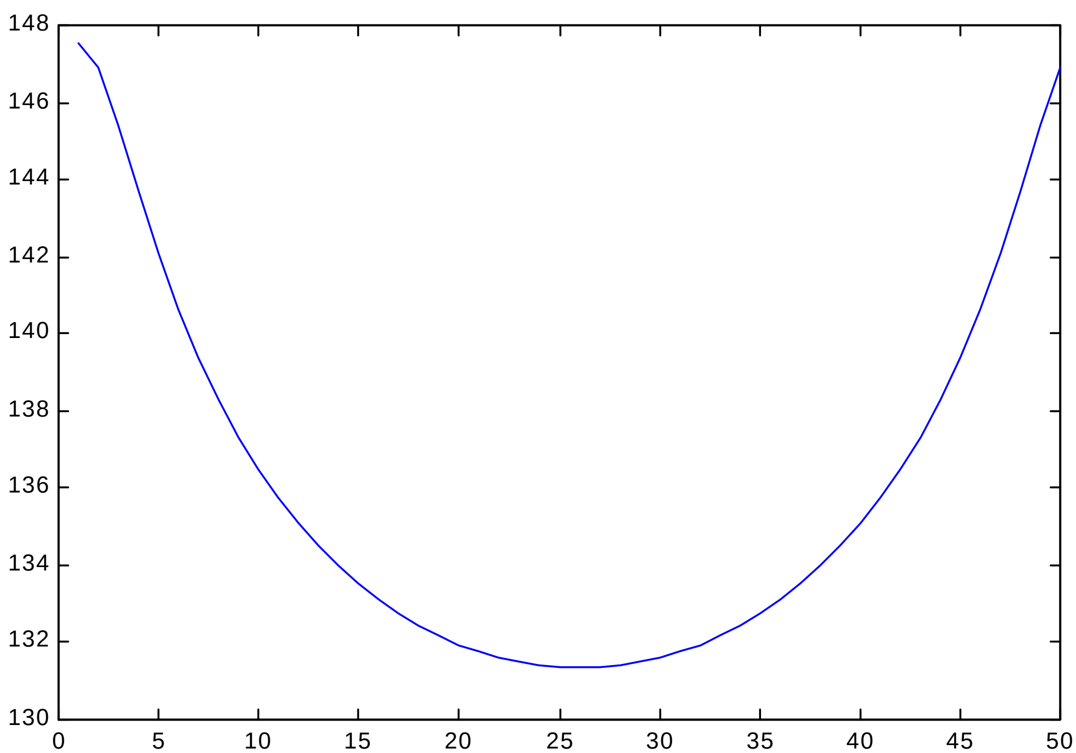
$$h(n) = \frac{0.7887}{\left(-1+\sqrt{3}\right)}\left(\frac{1}{\left(-1+\sqrt{3}\right)}\right)^n + \frac{0.2113}{\left(-1-\sqrt{3}\right)}\left(\frac{1}{-1-\sqrt{3}}\right)^n$$

Therefore, $h(n) = \left(1.0774.(1.3660)^n\right) - \left(0.0773.(-0.3660)^n\right)$

(c) Sketch the magnitude of the frequency response.

**Answer:**

This plot was obtained by plotting the h(n) equation in matlab. The |H(f)| is a low pass filter.



(d) Convert  to  by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

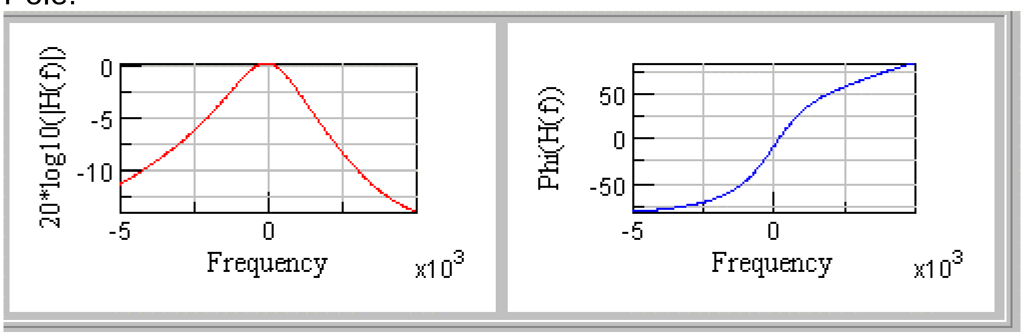
**Answer:**

$$\mathrm{H}(\mathrm{z}=e^{-sT})=\frac{-2-e^{-sT}}{(e^{-sT}-(-1+\sqrt{3}))(e^{-sT}-(-1-\sqrt{3}))}$$

$$zeros:$$
$$-2-e^{-sT}=0$$
$$e^{-sT}=-2$$
$$-sT=\ln(-2)$$
$$s=\frac{-\ln(-2)}{T}$$

$$poles:$$
$$e^{-sT}=-1+\sqrt{3}$$
$$-sT=\ln(-1+\sqrt{3})$$
$$s=\frac{-\ln(-1+\sqrt{3})}{T}\,or\,\frac{-\ln(-1-\sqrt{3})}{T}$$

Pole:

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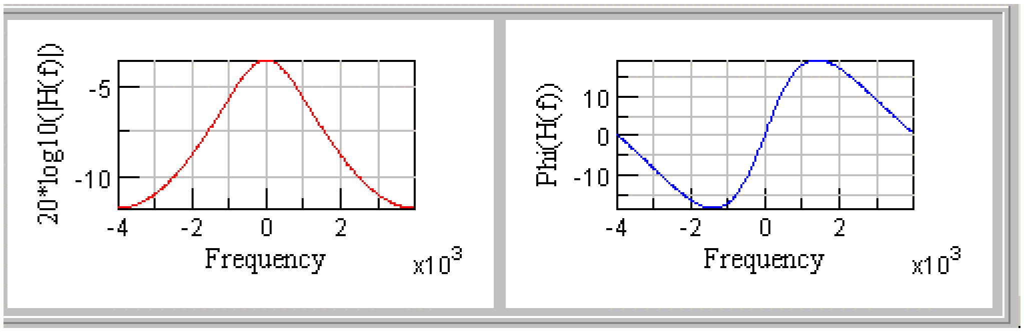
Zero:


20*log10(|H(f)|)
-5
-10
-4
-2
0
2
Frequency
x10^3
Phi(H(f))
10
0
-10
-4
-2
0
2
Frequency
x10^3