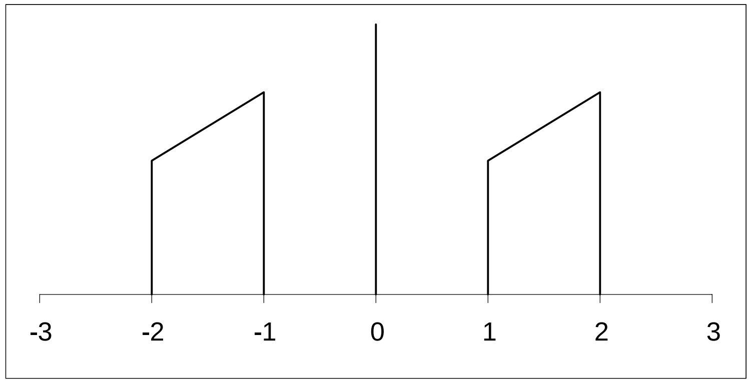
Test #3 Rework – Andrew Tomlinson

Problem #1. Sampling.



(a) Is this signal real or complex? Justify your answer.

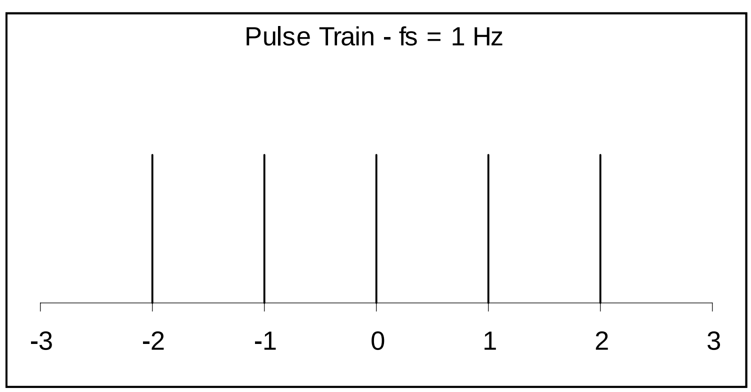
Complex. In order for a signal to be real :

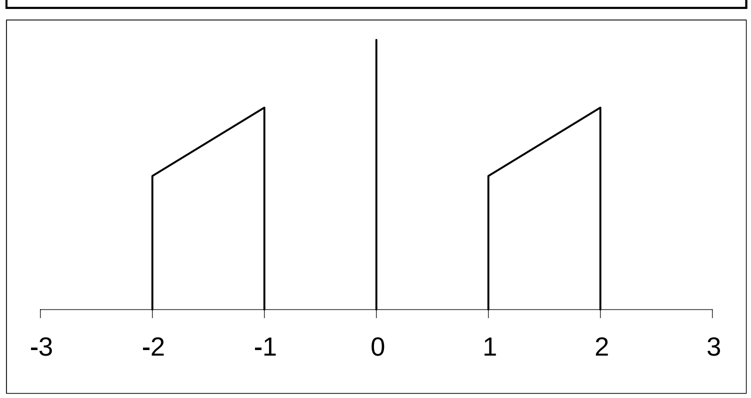
$$|x(k)| = |x(-k)|$$

The signal must also by symmetric about the center frequency

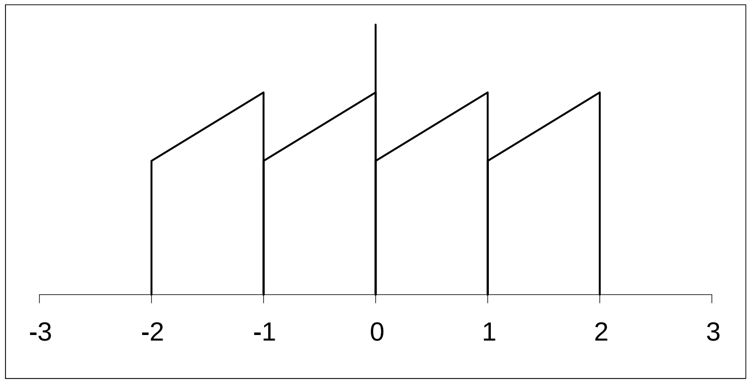
$$f_0/2$$

(b) Draw the spectrum of the sampled signal if fs = 1Hz.





The spectrum of the sampled signal would be copies of the one-sided spectrum of X(f) repeated every 1 hertz. This spectrum would be at a maximum of twice the magnitude of the original signal at the end of the sample period. It would appear as:



Obviously this signal runs from minus infinity to plus infinity (or as long as the pulse train is defined.) As noted before, this signal is twice as large (tall, if you will) as the original signal.

(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

The original signal can be recovered by passing the sampled signal, Xs(f) , through a bandpass filter to remove the facsimiles created by the sampling process. This is an appropriate place to utilize the bandpass sampling theorem which states that the minimum sampling frequency required to sample a signal with no aliasing is a function of the lowest and highest frequency for which the signal has spectral energy. The signal has bandwidth of 1 Hz and the upper and lower frequencies are multiples of a common factor.

Problem #2. Given the signal and impulse response shown below:

(a) Define y(n) as the output of the convolution of these two functions. Is y(n) an energy or power signal? Prove this.

y(n) = x(n) * h(n)

It is an energy signal because it is aperiodic (there is a finite area under the curve.)

(b) Compute y(n) described in (a) as the convolution of these two functions.

Note: The function h(n) is an averaging function that looks at the sample behind and ahead of the current (n)th sample. It weights each piece by a factor of:

$$1/\sqrt{3}$$

Using the Discrete Convolution Theorem:

$$y(n) = \sum_{k=-\infty}^{\infty} x(n)\, h(n-k)$$

gives an answer of:

$$y(n)=\left\{-1/\sqrt{3}\,,-2/\sqrt{3},-1/\sqrt{3}\,,0,1/\sqrt{3}\,,2/\sqrt{3}\,,1/\sqrt{3}\right\}$$

This function acts as a sliding window as it tracks along with the input signal, x(t). Since there were seven samples in the input signal, x(t), there will be seven parts in the output signal, y(t).

(c) Assume x(n) in (a) was a periodic signal. Will the power in the output, y(n), be different than the power in the input? Explain.

The spectrum of a periodic signal is a line spectrum, which has half of its power (A^2/2) located at plus the fundamental frequency, fo, and the other half of its power (also A^2/2)located at minus the fundamental frequency, -fo.

A power signal, one that is periodic, by definition has finite power and infinite energy. An energy signal, one that is aperiodic, by definition has finite energy and zero power.

Because the "averager" weights each sample by one over the square root of three, the power in the output will most certainly be different than that in the input. The energy in h(n) is however, unity(=1.)

Problem #3: Z-Transforms

(a) Find the transfer function of the system shown above.

Upon initial inspection of the system, the following equation was formed.

$$x(z)+\tfrac{1}{2}x(z)z^{-1}+(1)y(z)z^{-1}+\tfrac{1}{2}y(z)z^{-2}=y(z)$$

After grouping like terms on each side of the equal sign, the equation is ready to be solved for in terms of the output, y(z), over the input, x(z).

$$h(z)=\frac{1+\tfrac{1}{2}z^{-1}}{1-z^{-1}-\tfrac{1}{2}z^{-2}}$$

(b) Find the impulse response.

The impulse response is the inverse z transform of the transfer function. Solving for h(z)/z and P.F.E. to get A and B yields:

A=1.077
B=0.077

Such that,

$$\frac{H(z)}{z}=\frac{1.077}{z-1.366}+\frac{0.077}{z+0.366}$$

$$H(z)=1.077\left(\frac{1}{1-1.366z^{-1}}\right)+0.077\left(\frac{1}{1+0.366z^{-1}}\right)$$

$$h(n)=z^{-1}\{H(z)\}$$

Finally we have,

$$h(n) = \left[1.077\left(1.366\right)^{n} + 0.077\ (-0.366)^{n}\right]u(n)$$

Note: The system is unstable because there is a pole that exists outside of the unit circle in the z-plane.

(c) Sketch the magnitude response.

(d) Convert H(z) to H(s) by converting poles and zeros in the z-plane to their equivalents in the s-plane. Plot the frequency response in the s-plane. Explain any differences.

Using the fact that z = exp^(st), substituting into the H(z) equation and combining like terms gives:

$$H(?) = 1.15 - 1.44 e^{-st}$$

In z-plane analysis, the frequency portion of the plane is limited to n*pi multiples of the frequency. This is to say that the transformation from the s-plane to the z-plane is nonlinear. Mapping from s to z warps the frequency content of the signal.