Name: Reggie Warren

Problem	Points	Score
1a	10	
1b	10	
1c	10	
2a	10	
2b	10	
2c	10	
3a	10	
3b	10	
3c	10	
3d	10	
Total	100	

Notes:

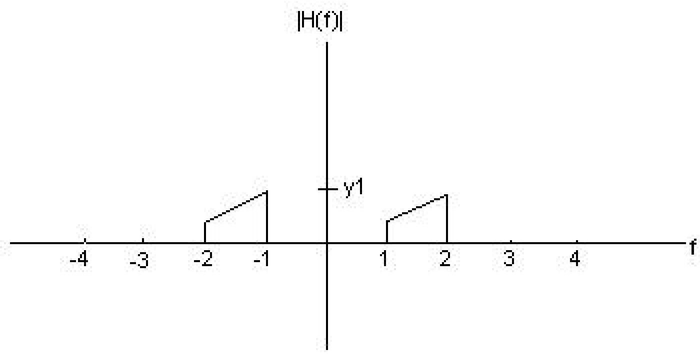
1. The exam is closed books/closed notes - except for one page of notes.

2. Please show ALL work. Incorrect answers with no supporting explanations or work will be given no partial credit.

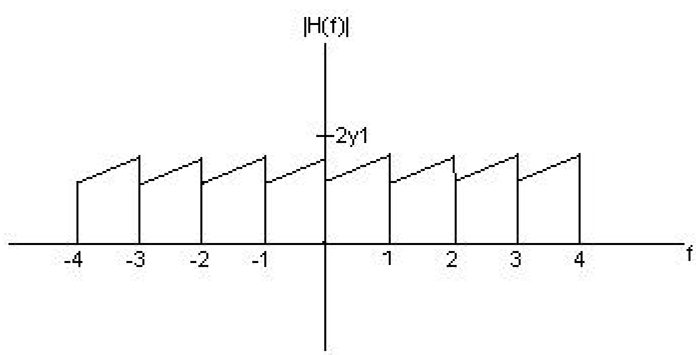
3. Please indicate clearly your answer to the problem. If I can't read it (and I am the judge of legibility), it is wrong. If I can't follow your solution (and I get lost easily), it is wrong. All things being equal, neat and legible work will get the higher grade:)

Problem No. 1: Sampling

(a) Is this signal real or complex? Justify your answer.

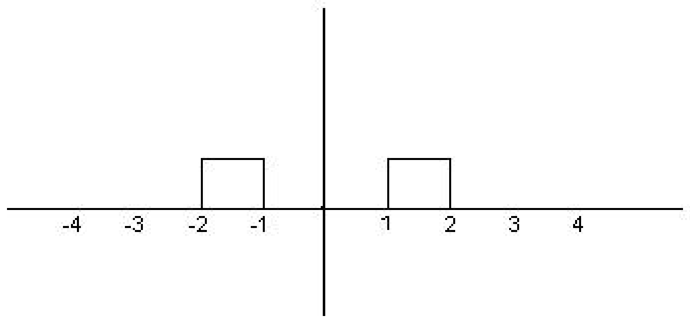


(b) Draw the spectrum of the sampled signal if $f_s = 1$ Hz.



(c) Explain in great detail how you would recover the signal. Was the Sampling Theorem violated?

To recover the signal, would use a filter similar to the following:



Given the sampling theory statement: $f_s \geq 2f_h$, where f_s is the sampling frequency and f_h is the signal's highest frequency, the sampling theory is violated in this case (1 Hz < 4Hz); however, because of the way the signal is constructed, this does not necessarily prevent us from "accurately" reconstructing the signal. Instead of using the filter above, one could also use an "ideal" filter twice, recovering a single "sawtooth" each time.

Problem No. 2: Given the signal and impulse response shown below:

(a) Define as the output of the convolution of these two functions. Is an energy or power signal? Prove this.

The input signal, x(n), is time-limited, which implies that it is not a power signal. To prove that it is an energy signal:

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 \Rightarrow$$
$$E = (-6)^2 + (-4)^2 + 2^2 + 2^2 \Rightarrow$$
$$E = 60\ J$$

and

$$P = \sum_{n=-\infty}^{T_o - 1} |x(n)|^2 = 0$$

Assuming a linear, time-invariant system, the output of this system is also an energy signal.

(b) Compute described in (a) as the convolution of these two functions.

Taking the z-transform of input and impulse response signals:

$$X(z) = 1 + z^2 - z^4 - z^6$$
$$H(z) = \frac{1}{\sqrt{3}} z + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} z^{-1}$$

y(n) will then be found in the following manner:

$$y(n) = x(n) * h(n) = X(z)H(z)$$

$$X(z)H(z) = (1 + z^2 - z^4 - z^6)(\frac{1}{\sqrt{3}} z + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} z^{-1}) \Rightarrow$$

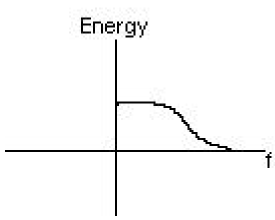
$$X(z)H(z) = \frac{1}{\sqrt{3}} z + \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} z^{-2} - \frac{1}{\sqrt{3}} z^4 - \frac{2}{\sqrt{3}} z^5 - \frac{1}{\sqrt{3}} z^6 - \frac{1}{\sqrt{3}} z^7 + \frac{1}{\sqrt{3}} z^{-1}$$

Now taking the inverse z-transform to obtain y(n):

$$y(n) = \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}(n+1) + \frac{1}{\sqrt{3}}(n+2) - \frac{1}{\sqrt{3}}(n+4) - \frac{2}{\sqrt{3}}(n+5) + \frac{1}{\sqrt{3}}(n+6) - \frac{1}{\sqrt{3}}(n+7) + \frac{1}{\sqrt{3}}(n-1)$$

(c) Assume in (a) was a periodic signal. Will the power in the output be different than the power in the input? Explain.

Real-world filters exhibit this type of behavior:



from which it can be inferred that a small portion of the signal's energy is lost each time the signal is filtered. This particular filter, however, is designed in such a way as to minimize this effect (this is accomplished via the $3^{-1/2}$ scale factor). As a result, the output signal will have almost the same power as the input signal.

Problem No. 3: Z-Transforms

(a) Find the transfer function of the system shown above.

$$y(n) = x(n) + y(n-1) + \frac{1}{2}y(n-2) + \frac{1}{2}x(n-1)$$

Taking the z-transform of y(n) to obtain the transfer function, H(z):

$$Y(z) = X(z) + z^{-1}Y(z) + \frac{1}{2}z^{-2}Y(z) + \frac{1}{2}z^{-1}X(z)$$

$$\frac{Y(z)}{X(z)} = 1 + \frac{Y(z)}{X(z)}z^{-1} + \frac{1}{2}\frac{Y(z)}{X(z)}z^{-2} + \frac{1}{2}z^{-1}$$

$$\frac{Y(z)}{X(z)} - \frac{Y(z)}{X(z)}z^{-1} - \frac{1}{2}\frac{Y(z)}{X(z)}z^{-2} = 1 + \frac{1}{2}z^{-1}$$

$$\frac{Y(z)}{X(z)}\left(1 - z^{-1} - \frac{1}{2}z^{-2}\right) = 1 + z^{-1}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{2}z^{-1}}{1 - z^{-1} - \frac{1}{2}z^{-2}} = \frac{2 + z^{-1}}{2 - 2z^{-1} - z^{-2}}$$

(b) Find the impulse response.

The impulse response can be found by taking the inverse transform of the transfer function, H(z). H(z) can be written as:

$$H(z) = \frac{2 + z^{-1}}{\left[1 - \left(\frac{1+\sqrt{3}}{2}\right)z^{-1}\right]\left[1 - \left(\frac{1-\sqrt{3}}{2}\right)z^{-1}\right]}$$

for which the partial fraction expansion gives

$$H(z) = \frac{2 + z^{-1}}{\left[1 - \left(\frac{1+\sqrt{3}}{2}\right)z^{-1}\right]\left[1 - \left(\frac{1-\sqrt{3}}{2}\right)z^{-1}\right]} = \frac{A}{\left[1 - \left(\frac{1+\sqrt{3}}{2}\right)z^{-1}\right]} + \frac{B}{\left[1 - \left(\frac{1-\sqrt{3}}{2}\right)z^{-1}\right]} \Rightarrow$$

$$2 + z^{-1} = A\left[1 - \left(\frac{1-\sqrt{3}}{2}\right)z^{-1}\right] + B\left[1 - \left(\frac{1+\sqrt{3}}{2}\right)z^{-1}\right] \Rightarrow$$

$$2 + z^{-1} = A - A\left(\frac{1-\sqrt{3}}{2}\right)z^{-1} + B - B\left(\frac{1+\sqrt{3}}{2}\right)z^{-1} \Rightarrow$$

$$2 = A + B \textit{ and}$$

$$z^{-1} = -A\left(\frac{1-\sqrt{3}}{2}\right)z^{-1} - B\left(\frac{1+\sqrt{3}}{2}\right)z^{-1} \Rightarrow$$

$$1 = -A\left(\frac{1-\sqrt{3}}{2}\right) - B\left(\frac{1+\sqrt{3}}{2}\right) \Rightarrow$$

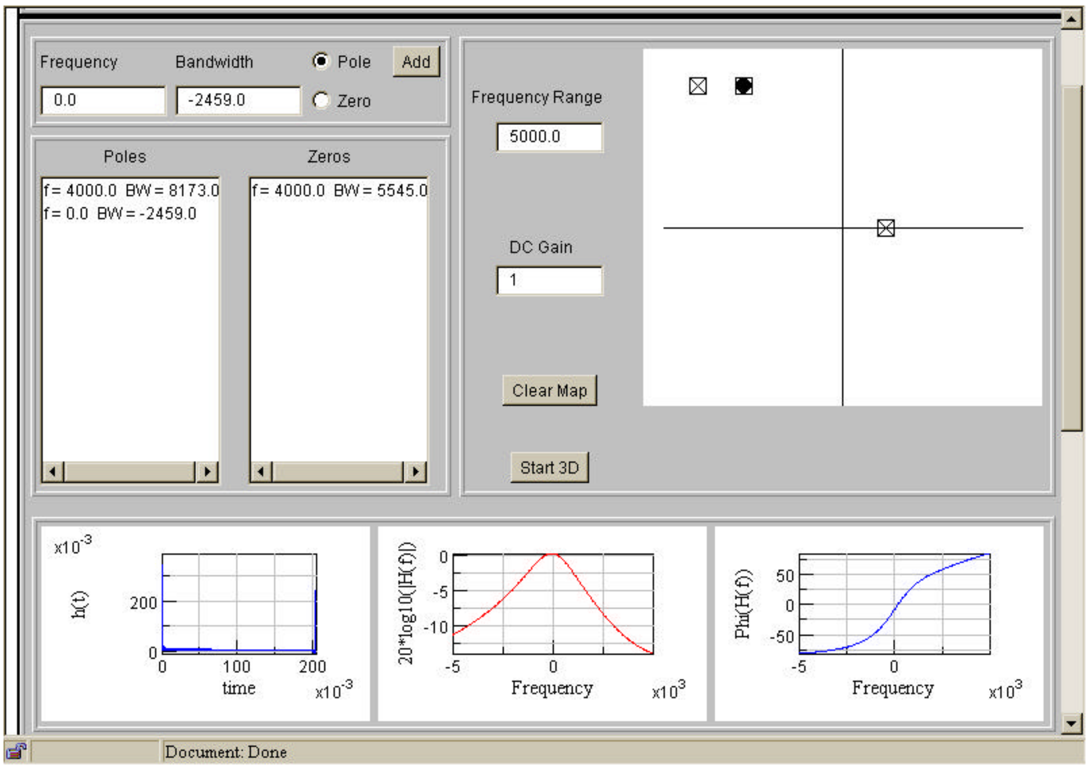
$$A = 2.1547;\ B = -0.1547 \Rightarrow$$

$$H(z) = \frac{2.1547}{\left[1 - \left(\frac{1+\sqrt{3}}{2}\right)z^{-1}\right]} - \frac{0.1547}{\left[1 - \left(\frac{1-\sqrt{3}}{2}\right)z^{-1}\right]}$$

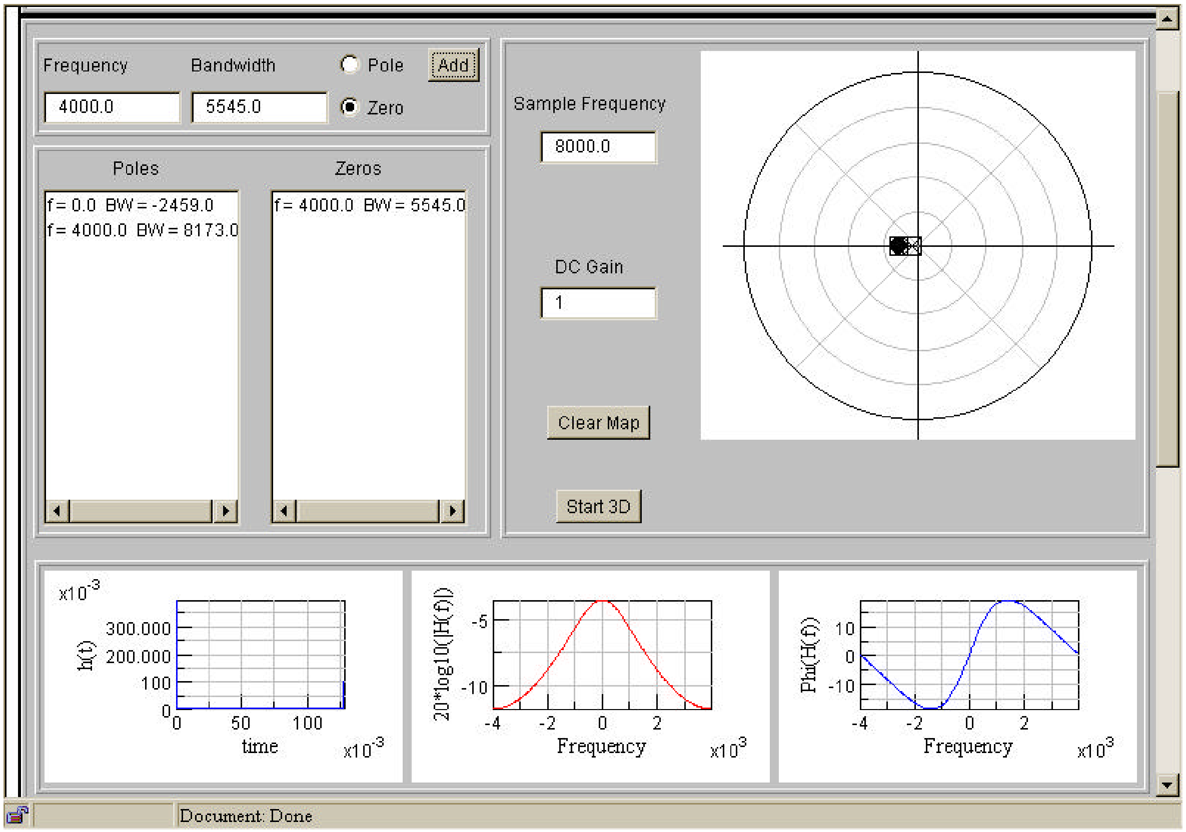
Now taking the inverse z-transform to obtain the impulse response, h(n), we obtain

$$h(n) = 2.1547\left(\frac{1+\sqrt{3}}{2}\right)^n - 0.1547\left(\frac{1-\sqrt{3}}{2}\right)^n$$

(c) Sketch the magnitude of the frequency response.



(d) Convert to the s-plane by converting poles and zeros in the z-plane to their equivalents (same frequency and bandwidth) in the s-plane. Plot the frequency response in the s-plane. Explain any differences.



In the z-plane, the signal closely resembles a sine wave. The magnitude is also reduced by a factor greater than 4. In the z-plane it appears that poles and zeros are much closer together than they were in the s-plane. Poles on the jw axis in the s-plane correspond to poles on the unit circle in the z-plane, and imply a time-domain function that oscillates at a frequency determined by the angle of the pole.