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Test 2 Extra Credit Problem

Part 1: Analytical Solution

$$h[n] = 1.9(-0.9)^n u[n]$$

$$\text{Input: } x[n] = 1 + \sin(\pi n / 4) + \sin(\pi n / 2)$$

Since the input is a sinusoid the output response,  $y[n]$ , will be a sinusoid in the form:

$$y[n] = 1 |H(0)| + |H(\pi/4)| \sin\left(\frac{\pi}{4}n + \angle H(\pi/4)\right) + |H(\pi/2)| \sin\left(\frac{\pi}{2}n + \angle H(\pi/2)\right)$$

$$h[n] \longleftrightarrow H(\omega) = \frac{1.9}{1 + 0.9e^{-j\omega}} = \frac{1.9e^{j\omega}}{e^{j\omega} + 0.9}$$

$$H(0) = \frac{1.9}{1 + 0.9e^0} = \frac{1.9}{1 + 0.9} = 1 \angle 0^\circ$$

$$\begin{aligned} H(\pi/4) &= \frac{1.9e^{j\pi/4}}{e^{j\pi/4} + 0.9} = \frac{1.9(\cos(\pi/4) + j\sin(\pi/4))}{\cos(\pi/4) + j\sin(\pi/4) + 0.9} \\ &= \frac{1.9(0.707 + j0.707)}{0.707 + j0.707 + 0.9} = \frac{1.34 + j1.34}{1.607 + j0.707} = \frac{1.90 \angle 45^\circ}{1.76 \angle 23.75^\circ} \\ &= 1.08 \angle 21.27^\circ = 1.08 \angle 0.371 \text{ rad} \end{aligned}$$

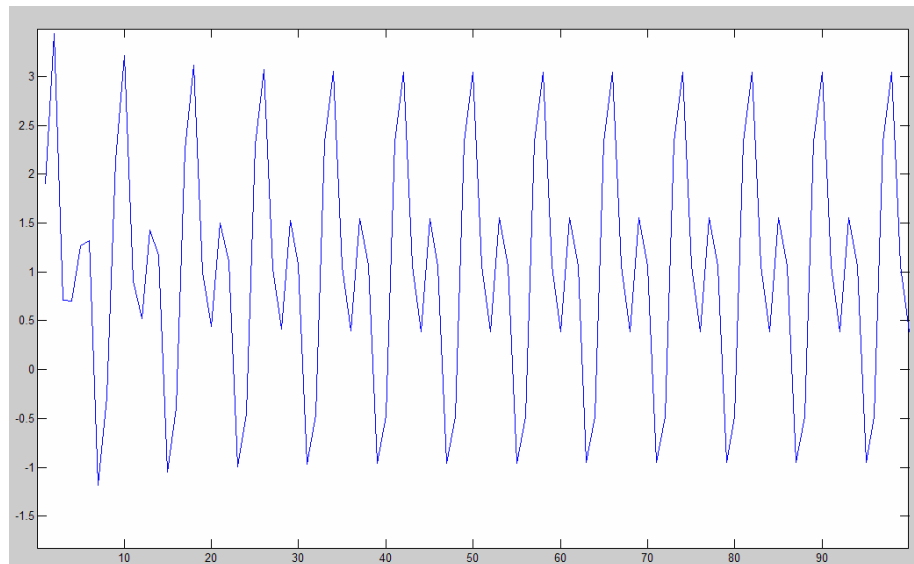
$$\begin{aligned} H(\pi/2) &= \frac{1.9e^{j\pi/2}}{e^{j\pi/2} + 0.9} = \frac{1.9(\cos(\pi/2) + j\sin(\pi/2))}{\cos(\pi/2) + j\sin(\pi/2) + 0.9} \\ &= \frac{1.9(0 + j)}{0 + j + 0.9} = \frac{1.9j}{0.9 + j} = \frac{1.90 \angle 90^\circ}{1.35 \angle 48^\circ} \\ &= 1.41 \angle 42^\circ = 1.08 \angle 0.733 \text{ rad} \end{aligned}$$

Therefore the final output response can be written as:

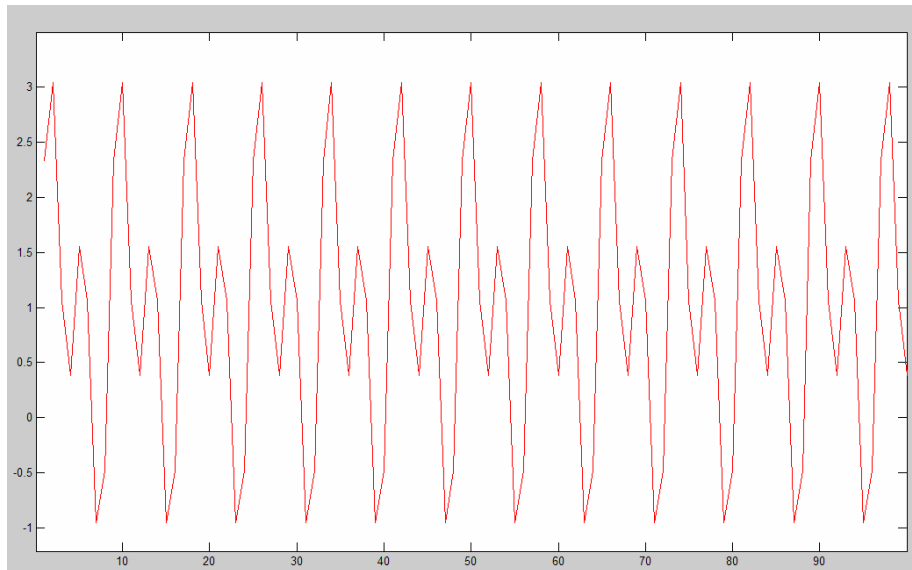
$$y[n] = 1 + 1.08 \sin\left(\frac{\pi}{4}n + 0.371\right) + 1.41 \sin\left(\frac{\pi}{2}n + 0.733\right)$$

## Part 2: Verification of solution with MATLAB

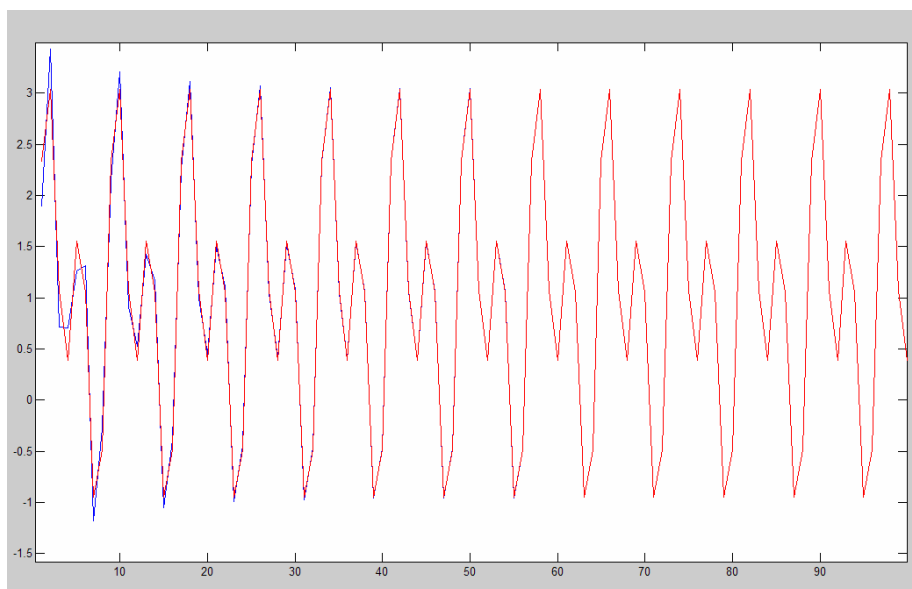
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syms x y h n a b
n = (0:1:100);
a = pi/4;
b = pi/2;
x = 1 + sin(a*n) + sin(b*n); %input function
h = (1.9*(-0.9).^n); %impulse function
y = conv(x,h) %output response by convolution
a = 1+1.08*sin(a*n+.371)+1.41*sin(b*n+.733); %analytical answer
plot(y, 'b') %plot of MATLAB result
hold
plot(a, 'r') %plot of analytical result
```



**Figure 1: Plot of MATLAB solution**



**Figure 2: Plot of analytical solution**



**Figure 3: Plot of MATLAB solution (blue) and analytical solution (red) together**

### Part 3: Solution details

By definition the output response of a sinusoidal input is also a sinusoid. This fact is used to determine the output response for this problem. The input,  $x[n]$ , and impulse,  $h[n]$ , functions are given. For these two functions the output response can be written as:

$$y[n] = 1 |H(0)| + |H(\pi/4)| \sin\left(\frac{\pi}{4}n + \angle H(\pi/4)\right) + |H(\pi/2)| \sin\left(\frac{\pi}{2}n + \angle H(\pi/2)\right)$$

Therefore the first step in solving the problem is to find the Fourier transform of the impulse function. This function is  $H(\omega)$ . Now the values  $0$ ,  $\pi/2$ , and  $\pi/4$  are substituted into the frequency domain function and the magnitudes and angles are found. These values are then placed into the equation above to give a final answer of:

$$y[n] = 1 + 1.08 \sin\left(\frac{\pi}{4}n + 0.371\right) + 1.41 \sin\left(\frac{\pi}{2}n + 0.733\right)$$

The result could also be found by multiplying the Fourier transform of the input function,  $X(\omega)$  by  $H(\omega)$ . This gives the output response in the frequency domain,  $Y(\omega)$ . The inverse Fourier transform will yield the output response in the time domain,  $y[n]$ . Also  $y[n]$  could be found by convoluting  $x[n]$  and  $h[n]$ . In MATLAB the input and impulse functions were defined. Then the output response was defined to be the convolution of  $x[n]$  and  $h[n]$ . This function was then plotted and can be seen in Figure 1. Then the answer found in Part 1 was defined and plotted. This plot can be seen in Figure 2. Then the two graphs were placed on the same plot. It is easy to see that these two plots lay on each other. I therefore feel rather confident that this solution is correct.