

Given,

$$x[n] = 1 + \sin\left(\frac{\pi}{4} \cdot n\right) + \sin\left(\frac{\pi}{2} \cdot n\right) \quad \text{and} \quad h[n] = 1.9(-0.9)^n \cdot u[n]$$

taking the DTFT of $h[n]$ and using Euler's identity yields,

$$H(\Omega) = \frac{1.9}{1 - (-0.9)e^{-j\Omega}} = \frac{1.9 \cdot e^{j\Omega}}{e^{j\Omega} - (-0.9)} = \frac{1.9(\cos(\Omega) + j\sin(\Omega))}{0.9 + \cos(\Omega) + j\sin(\Omega)}$$

Next compute the magnitude and phase of $H(\Omega)$ for $\Omega_0 = 0, \frac{\pi}{4}, \frac{\pi}{2}$

$$|H(0)| = \frac{1.9}{1.9} = 1 \quad \text{and} \quad \angle H(0) = 0$$

$$H\left(\frac{\pi}{4}\right) = \frac{1.9 \frac{\sqrt{2}}{2} + j1.9 \frac{\sqrt{2}}{2}}{0.9 + \frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2}} = 1.08 \angle 0.371 \quad \text{or}$$

$$\left|H\left(\frac{\pi}{4}\right)\right| = 1.08 \quad \text{and} \quad \angle H\left(\frac{\pi}{4}\right) = 0.371$$

$$H\left(\frac{\pi}{2}\right) = \frac{j1.9}{0.9 + j} = 1.41 \angle 0.733 \quad \text{or}$$

$$\left|H\left(\frac{\pi}{2}\right)\right| = 1.41 \quad \text{and} \quad \angle H\left(\frac{\pi}{2}\right) = 0.733$$

Using equation 5.65 from section 5.5.1 on page 250 in the text book,

$A = 1$

$$y[n] = |H(0)| + \left|H\left(\frac{\pi}{4}\right)\right| \sin\left(\frac{\pi}{4}n + \angle H\left(\frac{\pi}{4}\right)\right) + \left|H\left(\frac{\pi}{2}\right)\right| \sin\left(\frac{\pi}{2}n + \angle H\left(\frac{\pi}{2}\right)\right)$$

$$y[n] = 1 + 1.08 \sin\left(\frac{\pi}{4}n + 0.371\right) + 1.41 \sin\left(\frac{\pi}{2}n + 0.733\right), \quad n = 0, \pm 1, \pm 2, \dots$$